REMOVING THE TENSION BETWEEN PLANCK H_0=66.93[1] AND RIESS ET AL H_0=73.24[2]

Bruce Wallman, PhD, Princeton, New Jersey, brucewallman@gmail.com

ABSTRACT

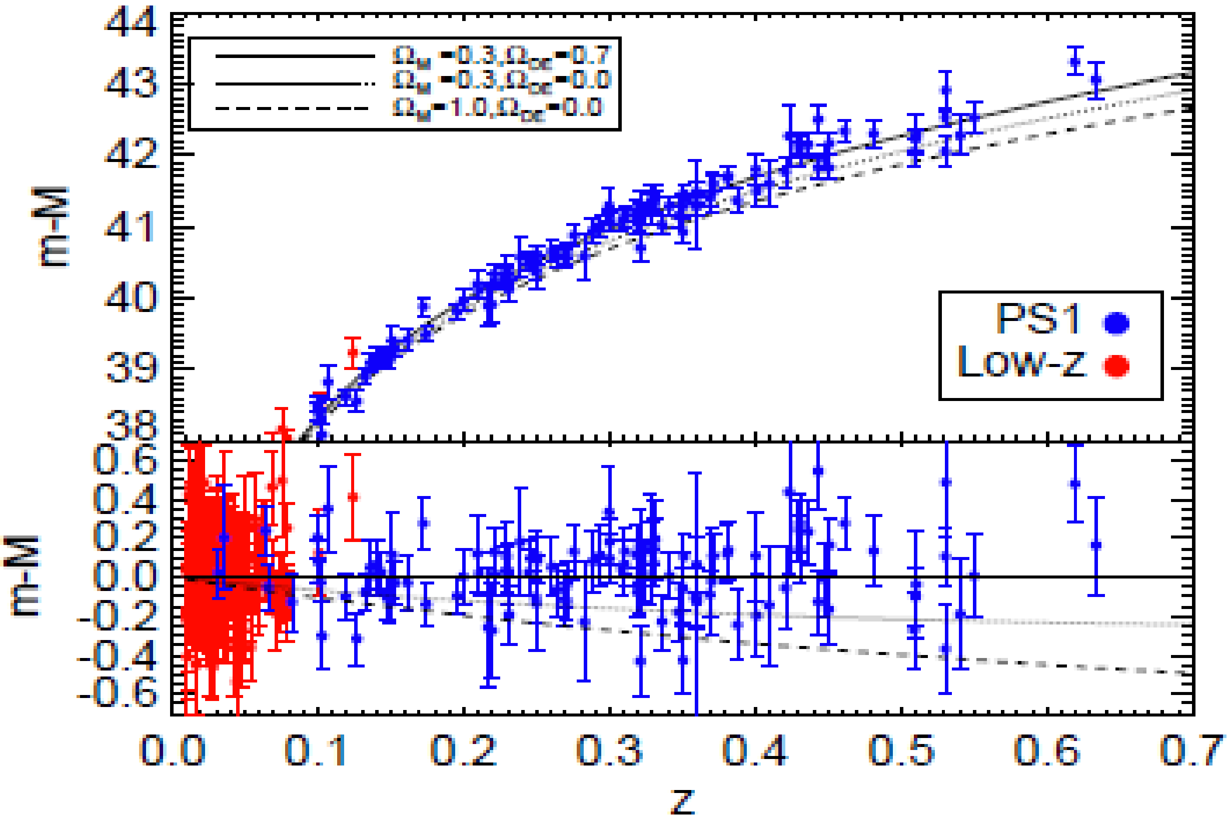
The value for the "Hubble constant" is referred to as the Hubble value in this paper. H_v is dealt with herein just as Edwin Hubble developed it in the 1920's. However, Hubble drew a straight line through his points assuming that there might be a straight line relationship between expansion or recessional velocity and proper distance. This assumption is no longer correct since Riess and Perlmutter showed with better data that there is a slight acceleration or increase in the velocity of expansion especially in the past several billion light-years. Given the data available today this paper does not assume a straight line relationship between recessional velocity and distance. It proves that this relationship is a power series with a slight downward bend. There is an open question in cosmology as of 2018 about the significance of the difference between the 2016 Planck collaboration value H_0=66.93±0.62 km/s/Mpc and the 2016 Riess et al value H_0=73.24±1.74 km/s/Mpc. This paper shows that both the Planck collaboration and Riess et al values are likely correct. Tracing the cosmological redshift z from SN1a several thousand megaparsecs in the past to those that exploded closer to today describes an increase in our perception of H_v as distance to us lessens. A best fit power function of $v_r=73.586*D_p^{-0.9909}$ (where v_r is recessional velocity and D_p is proper distance) for recessional velocity versus distance is derived connecting these two values of H_0 using 1836 SN1a data points in the NED database[3]. Taking a Hubble derivative of this formula gives the power function $H_v=dv_r/dD_p= 72.85014*D_p^{-0.01}$. This formula describes a change of over 5 km/s/Mpc in the Hubble value for the observable universe increasing smoothly from 66.8 to over 71.6 km/s/Mpc as the universe expands with most of the increase occurring in the last 1000 Mpc. This is consistent with the ΛCDM cosmology which gives an expansion to the observable universe that accelerates in the most recent several billion light-years[4]. The values 66.8 and 71.6 km/s/Mpc derived in this paper are affected by assumptions for the values of omega matter and omega lambda and are not meant to be taken as final numbers. They are calculated to show that there is a legitimate connection between the Planck collaboration and Riess et al values for H_0. Furthermore, by converting distance in megaparsecs to light-seconds, it is possible to show H_v is an acceleration that varies from 6.47 angstrom/s/s at the distance of the CMB to 6.93 angstrom/s/s in the local vicinity of the visible universe (approximately 8 Mpc) where the Hubble flow breaks down due to local gravity. This depiction of H_v as an acceleration is helpful in envisioning what we have learned from the Riess and Perlmutter SN1a studies versus the WMAP and Planck probes dating from 1998 to today.

1. INTRODUCTION

The starting data points for deriving a Hubble value are redshift z and distance modulus m-M. The recessional velocity V_r is derived from redshift, omega matter, omega Lambda, omega radiation, and the tightness of increments used to examine the expansion factor. The proper distance is calculated from the formula from the m-M distance modulus and redshift z as $D_p=10^{-6}x10^{((m-M)/5)+1)}/(1+z)$. The Hubble value H_v that is derived is the currently measured Hubble value from our location in space and time, not the value at the time the observed signal was emitted. The cosmological parameters used herein are commonly used in SN1a and Planck CMB papers.

A first chart is provided to give context for the format of other charts that follow. Chart 1, showing distance modulus (m-M) versus redshift, is copied from the Rest et al[5] (includes Riess) 2014 paper cited for the 2014ApJ...795...44R SN1a data. Chart 1 provides one validation point for m-M error bars and omega matter=0.295 and omega lambda=0.704913 values used in calculations herein later on.

Chart 1 - distance modulus (m-M) versus redshift z according to Rest et al in 2014.

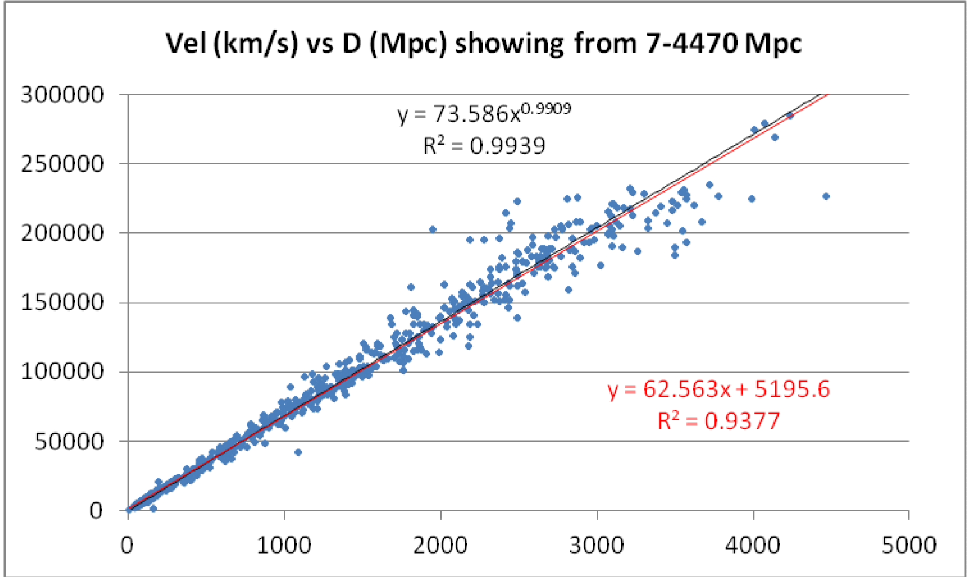


2. ANALYSIS

While relying on the data and analysis in the Rest et al paper for its backdrop, the remainder of this paper changes chart formats from the Rest et al axes in order to help visualize and compute the Hubble value. First, proper distance calculated from the m-M distance modulus redshift z for SN1a in the NED data base as $D_p=10^{-6}x10^{((m-M)/5)+1)}/(1+z)$ is used rather than the distance modulus. Second, redshift z is converted to recessional velocity (using a simulation derived from a cosmological calculator[6]) because recessional velocity in km/s, not z, is the numerator when calculating a Hubble value. Third, proper distance in Mpc is plotted along the x axis and recessional velocity in km/s is plotted on the y axis rather than vice versa. These changes allow the recessional velocity charts to visualize Hubble values similarly to Hubble's intuitive work in the 1920's. The derivative of recessional velocity over distance, originally seen as a straight line over a relatively short distance by Hubble, is shown to change over large distances according to a power function derived from the data.

A set of 1836 NED SN1a (reference 2014ApJ...795...44R and 2013MNRAS.433.2240G) data points was chosen because it comprises a low variance spectrographic data set that is consistent with the 2016 Riess et al paper. Two data points get added. One is for the Cepheid NGC 5128 at 7.00 Mpc and another is for the CMB at 14,400 Mpc. These are added to extrapolate for H_v comparisons to Riess in recent times and Planck at roughly 13.8 billion years ago. The recessional velocity computations for NGC 5128 at 7 Mpc and the CMB at 14,400 Mpc using the parameters selected end up at 458 km/sec and 959,380 km/sec respectively. This current recessional value for the CMB is calculated to be greater than the speed of light.

Chart 2 - recessional velocity versus proper distance.



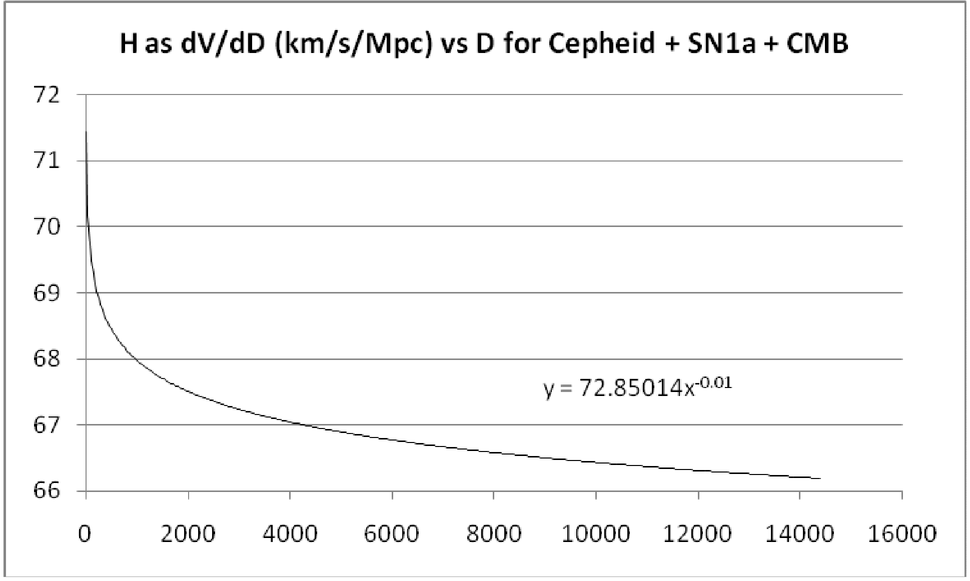
The regression squared for the recessional velocity power function is 0.9939. This value is better than alternatives tested such as linear regression (shown in red). Using the linear fit sets the Hubble value to a constant slope of 62.563 km/s/Mpc. Hubble originally made the assumption of a linear slope for the narrow set of data available in the 1920's. This is not a valid assumption given today's range of NED SN1a data. Accordingly, the analysis that follows uses the better fit power function $v_r=73.586*D_p^{-0.9909}$. The data points on the chart could be shown as short horizontal bars to emphasize that there is an average error measurement of 0.18 m-M in the distance modulus used to calculate proper distance. This is consistent with error bars shown vertically in chart 1. In the 310 SN1a from the 2014 Riess study there are 102 SN1a with err<=0.15 m-M, 232 with err<=0.20, and 283 with err<=0.25 m-M. Overall, there are 306 SN1a from that study with err<0.32 m-M and 4 SN1a with 0.32<=err<=0.64. There is no error for the y axis because the recessional velocity is calculated from observed redshift z and the observed redshift has no error listed in the NED database. This is consistent with the observed redshift being identical for every measurement of the same SN1a listed in the NED database.

The computation of recessional velocity (The rate at which proper distance to source increases per unit time expressed in km/s) used in chart 2 is done via a loop taken from the cosmological calculator and coded in Visual Basic for Applications to run in Excel. The loop continues for a large number of cycles at which point comoving redshift and the comoving redshift when the incremented expansion factor becomes greater than 1 / (1 + z) are both set. Recessional velocity is calculated via the difference between these two comoving redshifts multiplied by the speed of light. The Visual Basic for Applications code is provided in the Appendix.

The cosmological values in this recessional velocity simulation are omega matter=0.295, omega radiation=0.000087, and omega lambda=0.704913. These values were selected because they provide a flat universe, are close to the values given by the Planck collaboration and Rest et al, and provide a satisfactory solution to the problem of connecting the H_0=66.93±0.62 km/s/Mpc and H_0=73.24±1.74 values.

Given the power function best fit for recessional velocity $v_r=73.586*D_p^{-0.9909}$, the derivative of this power function at each proper distance is by definition the Hubble value for that proper distance. That derivative is $H_v=dv_r/dD_p= 72.85014*D_p^{-0.01}$. Plotting this derivative formula…

Chart 3 - proper Hubble derivative versus proper distance.



The line that is shown is the derivative at all points of the recessional velocity per unit distance using 7.00 Mpc as the nearest distance and 14,400 Mpc as the farthest to be consistent with both z=0.0015 used by Riess et al and z=1088 used by the Planck collaboration in 2016.

Using the two NED data sets the extended proper Hubble derivative reaches down to 66.8 km/sec/Mpc at 14,400 Mpc and up to 71.6 km/sec/Mpc at 7.00 Mpc. 66.8 is within the range given by the Planck collaboration and 71.6 within the range given by Riess et al.

These results are consistent with the 1998 Riess and Perlmutter conclusions when the relationship between the Hubble value and the acceleration of the universe over time is accepted. The Hubble value is typically expressed as km/s/Mpc. Using a constant speed of light the Hubble value can be expressed in km/s/light-sec or angstrom/s/light-sec. This is helpful in understanding why the derivative of the recessional velocity with respect to time gives the Hubble value. The following charts express the recessional velocity in angstrom/second versus distance in light-seconds and then the acceleration for the visible universe in angstrom/s/light-sec versus the proper distance using Mpc as was done for the Hubble value.

Chart 4 - recessional velocity versus proper distance converted to light-seconds.

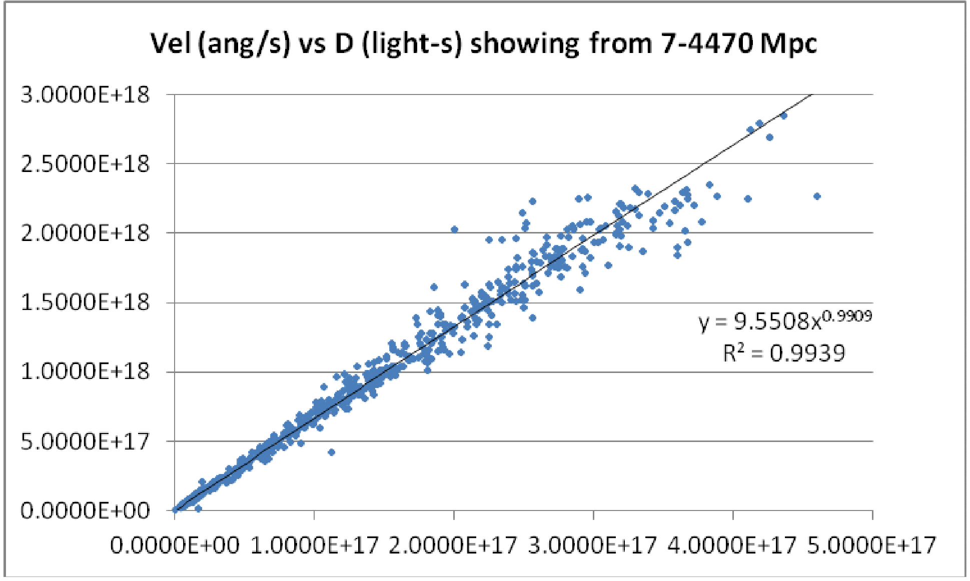
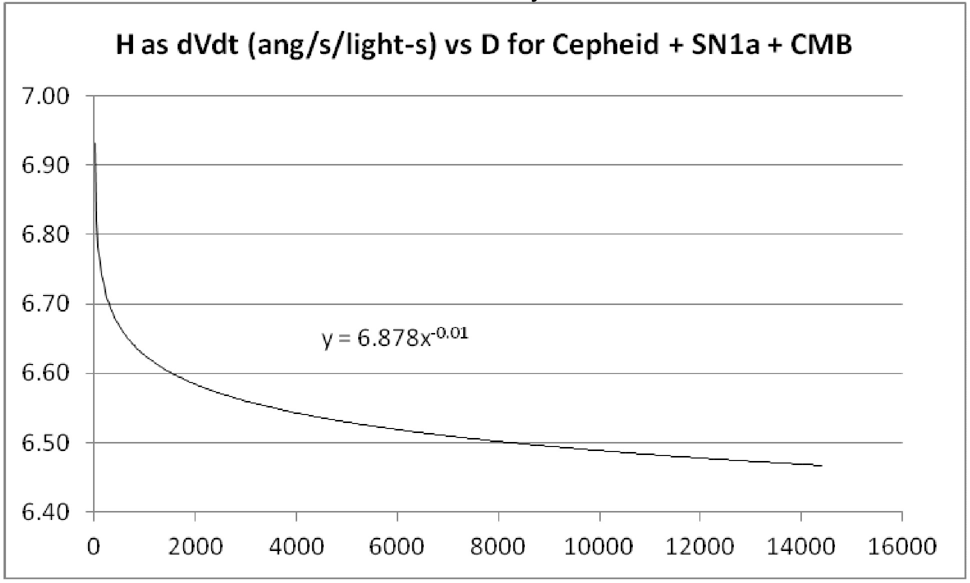


Chart 5 - derivative of recessional velocity versus time.



3. CONCLUSIONS

In order to reach the goal of connecting the Planck value of H_0=66.93±0.62 km/s/Mpc and the Riess et al value of H_0=73.24±1.74 km/s/Mpc, the analysis has used the following overarching concepts:

a. A consistent, low variance set of NED data and three omega cosmological parameters consistent with a flat universe are used.
b. Hubble acceleration $H_v=dv_r/dD_p= 72.85014*D_p^{-0.01}$ is calculated as a derivative of recessional velocity $v_r=73.586*D_p^{-0.9909}$ versus distance. This approach (versus plotting independent H_v values) smooths the variance seen in the imperfectly measured distance modulus. Either approach calculates the same Hubble values.

c. The use of a best fit power function rather than a lesser fit linear regression allows the slope (derivative of recessional velocity) to connect the range of H_0 values from Riess et al to the Planck collaboration by not forcing the Hubble value to be a constant.
d. Working with a sizeable number of unique SN1a provides low variance, statistically justifiable results.

4. FUTURE ANALYSIS

By connecting the dots for several hundred SN1a, this paper shows statistically that the Hubble values derived by the Planck collaboration and Riess et al are each valid for their respective distances. It does not address the physics of why this is so. The SN1a chart suggests a systemic change having to do with the nature of SN1a luminosity derived distances; however, this chart extrapolates smoothly to the Planck CMB value suggesting that the change may reflect a reality that the Hubble value as we see it today increases modestly in the late time visible universe. This interpretation of the Hubble value increasing as we see it for recent times is consistent with what Riess, Perlmutter, et al gave us as dark energy.

5. ACKNOWLEDGEMENT

I have had two departments (Physics and Astrophysics) mentoring me at Princeton University. These physicists have taught me much since I started retirement in 2012. This paper, right or wrong, acknowedges their help and open attitudes towards helping me learn.

REFERENCES

[1] Planck collaboration arXiv: 1502.01589v3, Planck 2015 results. XIII. Cosmological parameters selected by Riess et al as LambdaCDM with 3 neutrino flavors having a mass of 0.06 eV and the Planck CMB data (TT,TE,EE+SIMlow; 3.2 sigma for TT+SIMlow), other early universe (CMB) models range from 66.88 to 67.31.
[2] Riess et al arXiv:1604.01424v3, A 2.4% Determination of the Local Value of the Hubble Constant.
[3] NED Extragalactic database NED27.02.1-D-14.1.0-20170227 at https://ned.ipac.caltech.edu/.
[4] Noted in papers such as Paturel et al, Hubble Law: Measure and Interpretation, Foundation of Physics (2007):47: p1214.
[5] Rest et al arXiv: 1310.3828v2 Cosmological Constraints from Measurements of Type Ia Supernovae discovered during the first 1.5 years of the Pan-STARRS1 Survey.
[6] Relativity 4 Engineers cosmological calculator initially used available at http://www.einsteins-theory-of-relativity-4engineers.com/cosmocalc.htm served as basis for the VBA code shown.

APPENDIX - The Visual Basic for Applications code

```
Option Explicit
'Data must be sorted by z to allow incremental progression to work properly
Public OmegaMatter, OmegaRadiation, OmegaLambda, OmegaCurvature, ExpFactorA,
 zMeasured, ProperRecessionalVelocity
Public RowNumber, zEq, ComovingDist, ExpFactor, ComovingLoopControl,
 TimeVariableDensityFunction, NumberofIterations
Public Increment, SwOnceExpFactorGTExpFactorA,
 ComovingDistOnceExpFactorGTExpFactorA, FinalRow
Public ProperDistance, ComovingLoopControlOnceExpFactorGTExpFactorA, zRel,
 ProperRecessionalVelocityRel
Public Sub Main()
    OmegaMatter = Worksheets("Const").Cells(5, 7).Value
'Observed normal + dark matter fraction of the critical energy density.
    OmegaLambda = Worksheets("Const").Cells(6, 7).Value
'Observed cosmological constant as fraction of the critical energy density.
    zEq = Worksheets("Const").Cells(7, 7).Value
'Deduced redshift where radiation energy density and matter density are
'equal.
    OmegaRadiation = OmegaMatter / zEq
    Worksheets("Const").Cells(11, 7).Value = OmegaRadiation
    OmegaCurvature = 1 - OmegaMatter - OmegaLambda - OmegaRadiation
    Worksheets("Const").Cells(12, 7).Value = OmegaCurvature
    Worksheets("Const").Cells(13, 7).Value = OmegaMatter + OmegaLambda +
 OmegaRadiation + OmegaCurvature
    NumberofIterations = Worksheets("Const").Cells(3, 7).Value
'Ran with number of iterations set to 500,000 yielding an increment of 10^(-6)
'in the expansion factor used to detect the value of z. This value balances
'high accuracy in the charts versus a satisfactory computation speed.
    Increment = 1 / NumberofIterations
    ComovingLoopControlOnceExpFactorGTExpFactorA = 0
    For RowNumber = 1 To 2000
        If Worksheets("Data").Cells(RowNumber, 3).Value <> "" And
 IsNumeric(Worksheets("Data").Cells(RowNumber, 2).Value) Then
            zMeasured = Worksheets("Data").Cells(RowNumber, 3).Value
'Factor of increase in wavelength of light emitted from source, due to the
'cosmic expansion; z=1 means that wavelength and all cosmic scale proper
'distances have doubled since that light was emitted.
            ExpFactorA = 1 / (1 + zMeasured)
'ExpFactorA is expansion factor at zMeasured
            ComovingDist = 0
            ComovingDistOnceExpFactorGTExpFactorA = 0
            ExpFactor = 0
            TimeVariableDensityFunction = 0
            SwOnceExpFactorGTExpFactorA = 0
            ComovingLoopControl = 0
            Do While ComovingLoopControl < 1
                ExpFactor = ComovingLoopControl + (Increment / 2)
                TimeVariableDensityFunction = ((OmegaMatter / ExpFactor) +
 OmegaCurvature + (OmegaRadiation / (ExpFactor ^ 2)) + (OmegaLambda * ExpFactor ^
 2)) ^ (1 / 2)
                ComovingDist = ComovingDist + ((1 / (ExpFactor *
 TimeVariableDensityFunction)) * Increment)
'Comoving proper distance of the source like measuring cosmic distances
'using measuring rods.
                ComovingLoopControl = ComovingLoopControl + Increment
```

```
                If ExpFactor > ExpFactorA And SwOnceExpFactorGTExpFactorA = 0 Then
                    ComovingDistOnceExpFactorGTExpFactorA = ComovingDist
                    SwOnceExpFactorGTExpFactorA = 1
                    ComovingLoopControlOnceExpFactorGTExpFactorA =
 ComovingLoopControl
                End If
            Loop
            ProperRecessionalVelocity = (ComovingDist -
 ComovingDistOnceExpFactorGTExpFactorA) * Worksheets("Const").Cells(2, 1).Value
            Worksheets("Data").Cells(RowNumber, 4).Value =
 ProperRecessionalVelocity
            Worksheets("Data").Cells(RowNumber, 8).Value = ComovingDist -
 ComovingDistOnceExpFactorGTExpFactorA
            FinalRow = RowNumber - 1
            Worksheets("Data").Cells(2, 5).Value = FinalRow
        End If
    Next RowNumber
    Worksheets("Data").Cells(2, 5).Value = FinalRow & " fin"
End Sub
```