Refutation of coalgebraic geometric modal logic

Abstract: Two definitions are *not* tautologous, hence denying the monotone functor on KHaus. What follows is that the use of coalgebra to manufacture a geometric modal logic is refuted. Therefore the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⤞ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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Abstract: Using the theory of coalgebra, we introduce a uniform framework for adding modalities to the language of propositional geometric logic.

4 The monotone functor on KHaus: Definition 4.4. Let F be a frame. Let MF be the frame generated by □a,◇a, where a ranges over F, subject to the relations [... where a, b ∈F and A is a directed subset of F.] (4.4.0)

Remark 4.4.0: The clauses invoking F above are ignored because the equations below as consequents do not contain F.

□(a ∧ b) ≤ □a (M1) (4.4.1.1)

LET p, q: a, b

~(#p<#(p&q))=(p=p) ; TCTT TCTT TCTT TCTT (4.4.1.2)

◇a ≤ ◇(a ∨b) (M4) (4.4.4.1)

~(%(p+q)<%p)=(p=p) ;   TTCT TTCT TTCT TTCT   (4.4.4.2)

Eqs. 4.4.1.2 and 4.4.4.2 as rendered are *not* tautologous, hence denying the monotone functor on KHaus. What follows is that the use of coalgebra to manufacture a geometric modal logic is refuted.