Solution of Horty's puzzles in stit logic

Abstract: In see-to-it-that logic (stit logic), three deontic examples are presented of Horty's coin betting puzzle with two agents. The form of the examples is tautologous. However, a profitability analysis by contrasting outcome for the agents shows none is tautologous. The example for the agent initiating the state of the coin as more profitable than the other agent is more closely aligned to tautology and hence the more profitable strategic outcome. What follows is that stit logic is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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2.1 Horty's Puzzles

The 3 puzzles ... that pose a problem for formalizing epistemic oughts just with the epistemic extension of act utilitarian logic, can be summarized as follows.

Example 1. Agent β places a coin on top of a table –either heads up or tails up– but hides it from agent α. Agent α can bet that the coin is heads up, that it is tails up, or it can refrain from betting. If α bets and chooses correctly, it wins €10. If it chooses incorrectly, it does not win anything, and if it refrains from betting, it wins €5. (2.1.1.1)

LET p, q, r, s: α , β, heads-up, prize.

(q>(r+~r)) >
(((((~r&(p>~r))+(r&(p> r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p> r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) ;
`TTTT TTTT TTTT TTTT` (2.1.1.2)

Example 2. With the same scheme as in the previous example, if α bets and chooses correctly, it wins €10. If it refrains from betting, it *also* wins €10. If it bets incorrectly, it does not win anything. (2.1.2.1)

(q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)> (s=(s=s))))) ;
TTTT TTTT TTTT TTTT (2.1.2.2)

Example 3. With the same scheme as in the previous examples, if α bets and chooses correctly, it wins €10. If it bets incorrectly or refrains from betting, it does not win anything. (2.1.3.1)

(q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))) ;
TTTT TTTT TTTT TTTT (2.1.3.2)

Remark 2.1: Eqs. 2.1.1.2-2.1.3.2 as rendered are tautologous. This is because the respective main antecedent and consequent are tautologous.

Profitability is evaluated where each example implies the three cases for agent α has more, less, or the equivalent of agent β. Because the examples are theorems, the respective results are identical. We present the truth tables for Example 3.

Agent α has the equivalent of agent β: (3.1.1)

((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)=(q&s)) ;
TTTT TTTT T**FF**T T**FF**T (3.1.2)

Agent α has less than agent β: (3.2.1)

((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)<(q&s)) ;
FFFF **FFFF** **F**T**FF** **F**T**FF** (3.2.2)

Agent α has more than agent β: (3.3.1)

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((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)>(q&s)) ;
```

TTTT TTTT T**F**TT T**F**TT (3.3.2)

Remark 3: Eqs. 3.1.2-3.3.2 are *not* tautologous. However, Eq. 3.3.2 is *closest* to a tautologous state with the fewest **F** values present in the resulting truth table. Hence, example 3 is the superior choice for a winning strategy.