Refutation of constructive proof of Craig's interpolation theorem using Maehara's technique

Abstract: We evaluate a constructive proof of Craig's interpolation theorem by way of Maehara's technique. Six equations are *not* tautologous, and serve as antecedents for respective conclusions of two- or four-sequents. Hence, the concluding consequents in any state of proof value will always return a tautology. This means the technique of Maehara does not produce a constructive proof of Craig's interpolation theory as applied to sequent logic for interpolation of non-normal logics. Therefore the approach forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Orlandelli, E. (2019). Sequent calculi and interpolation for non-normal logics. arxiv.org/pdf/1903.11342.pdf eugenio.orlandelli@unibo.it

4 Craig's interpolation theorem

[F]or each modal or deontic logic X which has neither C nor $D^{\Diamond}$ as axiom, a constructive proof of Craig's interpolation theorem by means of the well-known Maehara's technique [for respective conclusions of two- or four-sequents]

Remark 4: Rendering of the original text equations in pdf required too much format manipulation, so only the mappings into Meth8 script are below, and in reverse order from the text.

LET p, q, r, s, t: A, B, Γ, Δ, Π.

((p=p)>p)>(r>(s&#p)) ; TTTT T**F**T**F** TTTT NNNN (R-N.2)

t>((#t&r)>s) ; TTTT TTTT TTTT TTTT(1),
TTTT CCCC TTTT TTTT(1) (L-D*.2)

p>((#p&r)>s) ; TTTT TCTC TTTT TTTT (L-D$^{\perp}$.2)

(t>q)>((#t&r)>(s&#q)) ; TTTT TTTT TTTT TTTT(1),
TTTT TTCC TTTT TTTT(1) (LR-K.2)

((p&t)>q)>((#p&(#t&r))>(s&#q)) ;
TTTT TTTT TTTT TTTT(1),
TTTT TTTC TTTT TTTT(1) (LR-R.2)

(p>q)>((#p&r)>s) ; TTTT TTTC TTTT TTTT (LR-M.2)

((p>q)&(q>p))>((#p&r)>(s&#q)) ;
TTTT TTTC TTTT TTTT (LR-E.2)

Remark 4: The six equations above are *not* tautologous. Because these serve as antecedents for respective conclusions of two- or four-sequents, the concluding consequents in any state of proof value will always return a tautology. This means the technique of Maehara does not produce a constructive proof of Craig's interpolation theory.