

Refutation of projective determinacy via the ultrapower

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Abstract: We evaluate the definition of ultrapower as a convention. The two states equated to 1 as designated *proof* value and as ordinal value are *not* tautologous. This refutes the ultrapower and hence colors the subsequent exposition to deny projective determinacy. Therefore the ultrapower and projective determinacy are *non* tautologous fragments of the universal logic $V\mathbb{L}4$.

We assume the method and apparatus of Meth8/ $V\mathbb{L}4$ with \top tautology as the designated proof value, \mathbf{F} as contradiction, \mathbf{N} as truthity (non-contingency), and \mathbf{C} as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; $+$ Or, \vee , \cup , \sqcup ; $-$ Not Or; $\&$ And, \wedge , \cap , \sqcap , $;$; \setminus Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \rightsquigarrow ;
 $<$ Not Imply, less than, \in , $<$, \subset , \ll , \lesssim , \uparrow ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , \triangleq , \approx , \simeq ; $@$ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists , \diamond , \mathbf{M} ; $\#$ necessity, for every or all, \forall , \square , \mathbf{L} ;
 $(z=z)$ \top as tautology, \top , ordinal 3; $(z@z)$ \mathbf{F} as contradiction, \emptyset , Null, \perp , zero;
 $(\%z>\#z)$ \mathbf{N} as non-contingency, Δ , ordinal 1;
 $(\%z<\#z)$ \mathbf{C} as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); $(A=B)$ ($A \sim B$); $(B>A)$ ($A \vdash B$); $(B>A)$ ($A \neq B$).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Martin, D.A.; Steele, J.R. (1989). A proof of projective indeterminacy. Journal of the American Mathematical Society. 2:1. 01.1989. 71/125.
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 dam@math.ucla.edu steel@math.berkeley.edu

I. Extenders (pg 75), **Convention.**

Let D be a directed nonempty set of sets: if $a, b \in D$ then there is a $c \in D$ such that $a \cup b \subseteq c$.

Suppose that Z is a set and $\langle \mu_a \mid a \in D \rangle$ is such that (75.1.1)

- (1) each μ_a is a countably additive measure on ${}^a Z = \{f \mid f: a \rightarrow Z\}$;
- (2) the μ_a are *compatible*: if $a \subseteq b$ and $\mu_a(X) = 1$, then $\mu_b(\{f \mid f \upharpoonright a \in X\}) = 1$. (75.2.1)

We wish to define the *ultrapower* (of the universe V) by $\langle \mu_a \mid a \in D \rangle$. (This will really be a direct limit of ultrapowers rather than an ultrapower proper, but calling it an "ultrapower" is by now standard.)

Remark 75: We build the conjecture for ultrapower as Eqs. 75.1 implies 75.2. (75.3.0)

There are two states with 1 as \top as tautology (the designated *proof* value) (75.3.1.1)

LET $p, q, r, s, u, X:$
 $a, b, D, f, \mu, X;$

$$(((p < r) > (u \& p)) > (\sim (q < p) \& (((u \& p) \& x) = (p = p)))) > (((u \& q) \& ((s < (p < x)) > s)) = (p = p)) ;$$

\mathbf{TFTF} TTTT \mathbf{TFTF} TTTT (2), TTTT TTTT TTTT TTTT (2),
 \mathbf{TFTT} TTTT \mathbf{TFTT} TTTT (2), \mathbf{TFTF} \mathbf{TFTF} \mathbf{TFTT} \mathbf{TFTT} (2) $\times 4$ (75.3.1.2)

or 1 as ordinal one. (75.3.2.1)

$$(((p < r) > (u \& p)) > (\sim (q < p) \& (((u \& p) \& x) = (\%p \> \#p)))) > (((u \& q) \& ((s < (p < x)) > s)) = (\%p \> \#p)) ;$$

TCTC TTTT TCTC TTTT (2), TTTT TTTT TTTN TTTN (2),
 TCTC TTTT TCTC TTTT (2), TCTC TCTC TCTT TCTT (2) $\times 4$ (75.3.2.2)

Eqs. 75.3.1.2 and 3.2.2 as rendered are *not* tautologous. This refutes Eq. 75.1.1 $\langle \mu_a \mid a \in D \rangle$ as defining the ultrapower. What follows is the coloring and denial of the subsequent exposition.