

Refutation of Skolem axiom form

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Abstract: "A *Skolem axiom* has the form $\forall x,y(\varphi(x,y) \rightarrow \varphi(x,f(x)))$, where f is a new function symbol introduced to denote a "Skolem function" for φ ." The Skolem axiom form is *not* tautologous, hence refuting it. Therefore the Skolem axiom form is a *non* tautologous fragment of the universal logic $\forall\exists$.

We assume the method and apparatus of Meth8/ $\forall\exists$ with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \sqcap , \cdot ; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \rightsquigarrow ;
 $<$ Not Imply, less than, \in , $<$, \subset , \prec , $\#$, \ll , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , $\hat{=}$, \approx , \cong ; @ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists , \diamond , M ; # necessity, for every or all, \forall , \square , L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, \emptyset , Null, \perp , zero;
 $(\%z>\#z)$ **N** as non-contingency, Δ , ordinal 1;
 $(\%z<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); $(A=B)$ ($A \sim B$); $(B>A)$ ($A \vdash B$); $(B>A)$ ($A \neq B$).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Avigad, J. (2004). Forcing in proof theory. andrew.cmu.edu/user/avigad/Papers/definitions.pdf

5 Point-free model theory

5.3 Eliminating Skolem functions

A *Skolem axiom* has the form $\forall x,y(\varphi(x,y) \rightarrow \varphi(x,f(x)))$, where f is a new function symbol introduced to denote a "Skolem function" for φ . (5.3.1.1)

LET $p, q, r, s: \varphi, x, y, f$

$(p \& (\#q \& \#r)) > (p \& (\#q \& (s \& \#q)))$; TTTT TTCT TTTT TTTT (5.3.1.2)

Remark 5.3.1.2: Eq. 5.3.1.2 is not tautologous, though nearly so. This means the Skolem axiom form is denied.