Refutation of the axiom of dependent choices (DC) on supercompactness of ω_1

Abstract: The axiom of dependent choices (DC) is evaluated in two equations on supercompactness of ω_1, with *neither* tautologous and hence refuting DC. Therefore DC equations are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ikegami, D.; Trang, N. (2019). On supercompactness of ω_1.
arxiv.org/pdf/1904.01815.pdf ikegami@shibaura-it.ac.jp nam.trang@unt.edu

This paper studies structural consequences of (full) supercompactness of ω1 under ZF. We first show the following basic structural consequences.

Theorem 1. Assume that ω1 is supercompact. Then
1. the Axiom of Dependent Choices (DC) holds, while ... (1.0)

3 Choice principles and supercompactness of ω1

In this section, we prove Theorem 1.
Since ω_1 is supercompact, there is a fine normal measure on $P\omega_1 A$. We fix such a measure μ.
Claim. For μ-measure one many elements σ of $P_{\omega 1}A$, the following holds:

$(\forall x \in \sigma)\ (\exists y \in \sigma)\ (x, y) \in R$ (3.1.1)

LET p, q, r, s: x, y, R, σ .

(((%p<s)&(#q<s))&(p&q))<r ; **FFF**N **FFFF FFFF FFFF** (3.1.2)

Suppose not. $(\exists x \in \sigma)\ (\forall y \in \sigma)\ (x, y) \notin R$

(((%p<s)&(#q<s))&(p&q))>r ; TTTC TTTT TTTT TTTT (3.1.3)

Eqs. 3.1.2 and 3.1.3 are *not* tautologous, hence refuting the axiom of dependent choices (DC) for (full) supercompactness of ω_1 under ZF.