Refutation of temporal type theory (TTT), temporal landscapes, and Scott's topology

Abstract: From a footnote, and then a three part definition, temporal type theory (TTT), and then temporal landscapes for open sets are *not* tautologous and hence refuted. That further refutes Scott's topology. These results therefore form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fong, B.; Speranzon, A.; Spivak, D.I. (2019). arxiv.org/pdf/1904.01081.pdf
Temporal landscapes: a graphical temporal logic for reasoning. (2019). bfo@mit.edu

[1]Note that the constant reals can be considered as a subtype R ⊆ R-hat˜ of the varying real numbers. (1.1.1)

LET p, q, r , s , t , u:
t_1, t_2, R, t_2', t_1', L

~(~r<r) = (p=p) ; **FFFF** TTTT **FFFF** TTTT (1.1.2)

Remark 1.2.2: The constant reals considered as a subtype of varying real numbers is not tautologous. This refutes temporal type theory (TTT) at its outset. However, we press on assuming that difficulty may be overcome by simply avoiding it.

Definition 2.1. A temporal landscape on R *is a set L of time intervals* [t1,t2] ⊆ R, where t1 ≤ t2, such that

(a) if [t_1,t_2] ∈ L, and t_1'≤ t_1 ≤ t_2 ≤ t_2', then [t_1',t_2'] ∈ L. (2.1.a.1)

(((p&q)<u)&~(~(s<q)<~(p<t)))>((t&s)<u) ;
TTTT TTTT TTTT TTTT(1),
TTT**F** TTT**F** TTTT TTTT(1),
TTTT TTTT TTTT TTTT(2) (2.1.a.2)

(b) if $[t_1,t_2] \in L$ *then there exists* $t_1' < t_1 \leq t_2 < t_2'$ *such that* $[t_1',t_2'] \in L$. (2.1.b.1)

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(((p&q)<u)>%(~((q<s)<(t<p))=(p=p)))>((t&s)<u) ;
                    FFFN FFFN FFFF FFFF(1),
                    FFFN FFFN TTTT TTTT(1),
                    FFFF FFFF FFFF FFFF(2)
```
(2.1.b.2)

We write Prop for the set of temporal landscapes. Together, requirements (a) and (b) state that temporal landscapes form the open sets of the Scott topology on the *interval domain* IR, a well-studied topological space in domain theory. (2.1.1.1)

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((u<r)>(~(r<(p&q))&~(s<t)))>(((((p&q)<u)&~(~(s<q)<~(p<t)))>((t&s)<u))&
((((p&q)<u)>%(~((q<s)<(t<p))=(p=p)))>((t&s)<u))) ;
                    FFFN FFFN FFFF FFFF(1),
                    FFFF FFFF TTTT TTTT(1),
                    FFFF FFFF TTTT FFFF(1),
                    FFFF FFFF FFFF FFFF(1)
```
(2.1.1.2)

Eqs. 2.1.1.2, ..b.2, and ..1.2 are not tautologous. This refutes Def. 2.1 and temporal landscapes for open sets, and hence denies Scott's topology.