Algorithm Capable of Proving Goldbach's Conjecture- An Unconventional Approach

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1. Abstract

I created an algorithm capable of proving Goldbach's Conjecture. This is not a claim to have proven the conjecture. The algorithm and all work contained in this document is original, so no outside sources have been used. This paper explains the algorithm then applies the algorithm with examples. The final section of the paper contains a series of proof-like reasoning to accompany my thoughts on why I believe Goldbach's Conjecture can be proven with the use of my algorithm.

Keywords— Goldbach; Algorithm; Number Theory; Mathematics

2. Introduction

Goldbach's Conjecture states that all even integers greater than 2 are the sum of two prime integers (hereafter, referred to as "prime" or "primes"). This is one of the oldest unproven conjectures because, although even integers have been tested to large values, it takes one example of an even integer that is not the sum of two primes to prove the conjecture false.

3. Algorithm Description

The purpose of the algorithm is to eliminate all even integers that are not the sum of one particular prime plus any other prime.

4. Definitions

Algorithm prime: The particular prime to which the algorithm is being applied.

Eliminated: Not the sum of the algorithm prime plus any other prime.

Prime + composite = eliminated

Prime + prime = not eliminated

Strand: Having a starting point from which numbers are systematically eliminated. Strands are exclusive to the algorithm prime.

Starts with: The starting point of each strand. Do not eliminate this point.

Skips by: The increment by which even integers are eliminated. Strand 1 always skips by 6.

Significance: The first even integer that is not eliminated after applying previous strands. These points can be found by using:

$$8 * [NumberofCurrentStrand] + [PreviousSignificance] = [CurrentSignificance] \tag{1}$$

The "previous significance" for the first significance point is the (algorithm prime +1); Call this the "base significance".

The number of significance points equals the number of strands needed to eliminate all sums that are not the sum of the algorithm prime plus any other prime. When applying the algorithm one strand at a time, some even integers are eliminated more than once. Significance points can act as a shortcut for the algorithm (see "Note 2").

Note 1: 2 cannot be an algorithm prime; besides 2+2, 2+ any other prime has an odd sum and the focus of this work is on even sums.

Note 2: To use significance points as a shortcut, apply strand 1 and then simply eliminate even integers from each strand's significance point onward.

5. Algorithm

Strand 1 starts with an integer (determined by adding 3 to the chosen algorithm prime) and eliminates the even integers that skip by 6 from the starting point. The strands thereafter are the previous "starts with" +2 and the previous "skips by" +4.

6. Algorithm Example

Our example desired maximum integer is 53 and the example algorithm prime is 3. First determine how many strands are needed:
Significance point 1: 8*1 (because this is the first significance point) + (algorithm prime +1)
In the example I chose 3 as the algorithm prime
The second significance point onward is 8* number of significance + previous significance point= new significance.

Significance point 1: (8*1) + (3+1) = 12
Now to find the second significance point, 12 is the "previous significant point".

Significance point 2: (8*2) + 12 = 28
Now to find the third significance point, 28 is the "previous significant point".

Significance point 3: (8*3) + 28 = 52
To ensure we do not need one additional strand to reach our desired max integer (53), one can see where the 4th strand would become significant:
To find the fourth significance point, 52 is the "previous significant point".

Significance point 4: (8*4) + 52 = 84
This means we only need three strands of the algorithm to find all the even integers that are not the sum of 3 + another prime between 0 and 53.

The first strand starts with 3+ the chosen algorithm prime (3) and skips by 6. So starting with 6, but not including the starting point, we eliminate 12, 18, 24, 30, 36, 42, 48. Strand 2 starts with +2 and skips by +4 more than the last: starting on 8, skip by every 10th number, eliminating 18, 28, 38, 48. The third strand starts on 10 and skips by 14, eliminating 24, 38, 52. Some even integers are eliminated more than once, so look to "Note 2". The remaining even integers are: 6,8,10,14,16,20,22,26,32,34,40,44,46,50. Each of these remaining even integers minus 3 (the chosen algorithm prime) will equal consecutive primes; aka prime + prime.
Notice: the number 4 is not visually included on either list. Technically 4 is belongs on the eliminated list because it is the "previous significance" for the first significance point. Think of the "previous significance" for the first significance point as "point 0" or the base point. This would make the base significance point: (8*0) + (3+1), or 4.
Recall: significance is the first point that has not been eliminated by other strands and is not the sum of prime + prime.

7. Discussion

As previously mentioned: for this example we chose the number 3 to be the algorithm prime. This means that after eliminating all even sums corresponding to the strands of the algorithm, the remaining even integers are the sums of 3+ one other prime. This does not account for the sums of 5+ any other prime or 7+ any other prime, etc.
To find the sums of alternate prime + prime combinations, choose a different algorithm prime. The remaining even integers will always be the sum of the algorithm prime plus other primes.

8. Additional Notes

Points to keep in mind as we work through the body of work.

1: Evens ≥ 2 and < 12 are the sum of two primes because each prime doubled and $3 + 5 =$ 4,6,10 and 8. In order: 4,6,8,10.

2: Prime > 2 + odd = even.

- All primes > 2 are odd
  - o Odd + odd = even

3: Algorithm eliminates prime + composite sums by algorithm design.

4: Algorithm can be applied to any prime by algorithm design, with the exception of Note 1.

5: Algorithm can be applied to multiple primes simultaneously by algorithm design.

6: Algorithm applied to one prime will eliminate some evens.

- Some evens will be the sum of prime + prime; these will not be eliminated.
  - o Not all odds are composite (7)
- Some evens will be the sum of prime + composite; these will be eliminated.
  - o Not all odds are prime (9)

7: Prime + Prime and Prime + Composite sums follow the same pattern.

- Both types of sum are found using the same steps: Apply the algorithm. After subtracting the algorithm prime:
  - o Some evens = Algorithm prime + prime
  - o Some evens = Algorithm prime + composite (eliminated by algorithm design)
- The sums of consecutive primes + arbitrary z will take on prime characteristics (numerical distance between primes), therefore follow the same "pattern".

Example: If I add red to every fifth blue, every fifth blue will turn purple.

P = prime

z = arbitrary number

y = odd composite

List of odds with prime spacing:

P P P y P P y P P y P y P P...

P+z:

P+z, P+z, P+z, y, P+z, P+z, y, P+z, P+z, y, P+z, y, P+z, P+z...

P+z carries the same spatial distance/pattern as primes.

Direct number example (x= break in consecutive even integers):

List of primes:

3,5,7,11,13,17,19,23,29,31...

Prime + prime:

3+3, 5+3, 7+3, 11+3, 13+3, 17+3, 19+3, 23+3, 29+3, 31+3...

Sums: 6,8,10,x,14,16,x,20,22,x,26,x,x,32,34...

Prime + Composite:

3+9, 5+9, 7+9, 11+9, 13+9, 17+9, 19+9, 23+9, 29+9, 31+9...

Sums: 12,14,16,x,20,22,x,26,28,x,32,x,x,38,40...

Notice that in both types of sums the x is located in exactly the same spot. It is important to realize not only that a pattern exists, but that both types of addend follow the pattern. Along with this information, also keep in mind that according to the algorithm prime + prime and prime + composite have opposite definitions (prime + composite= eliminated, prime + prime= not eliminated).

o Whether its prime + prime or prime + composite, adding any number to consecutive primes will yield sums that take on the same numerical distance as primes. The spacing of primes doesn't change, so the sums preserve the prime spacing as well.

o The corresponding sums will always take on prime spacing because to see all possible addend combinations we always start with the smallest odd prime and work our way up, adding each consecutive prime to each consecutive odd number.

9. Body of Work: Why the Algorithm Can Prove Goldbach's Conjecture

Lemma 9.1. If prime + prime and prime + composite follow the same pattern but have opposite definitions by algorithm design, then what is true for one type of sum will have the equal but opposite truth for the other type of sum.

Proof. When the algorithm is applied to 3, the number 12 is eliminated because 12-3=9, which makes 3+9 a prime + composite. When the same algorithm is applied to 5, the number 12 is not eliminated because 12-5= 7, which makes 5+7 a prime + prime. Equal sum= 12, opposite truth: eliminated and not eliminated. □

Lemma 9.2. All evens ≥ 12 are the sum of prime + composite.

Proof. The largest number of consecutive odd primes is 3: 3,5,7. The largest gap in odd composites is 2 (where the twin primes occur). Using 3,5,7 alone, each composite can make three even sums per 1 prime + composite combination. When primes are added over the gap, there will be no gap in consecutive even sums.
There is one odd composite before the gap and one odd composite after. Over the course of 4 odd numbers with the gap on numbers 2 and 3, there are 6 consecutive sums.
Example: list of odd numbers with a twin prime gap: 15,17,19,21.
3+15=18, 5+15= 20, 7+15= 22
3+21= 24, 5+21= 26, 7+21= 28.
There is no break in the even sum between 7+15 and 3+21 because the number of consecutive primes is larger than the number in gap. This will always be true because every third odd number greater than 3 is divisible by 3. The gap will never exceed 2 and every composite starts with being added to 3,5, and 7 to account for all possible prime+ composite combinations. With no gap in even sums of prime + composite addends ≥ 12 , all evens ≥ 12 are the sum of prime + composite. □

Theorem 9.1. If evens ≥ 12 are the sum of prime + composite, then all evens ≥ 12 are eliminated.

Proof. All evens ≥ 12 can be expressed as a sum of prime + odd composite. Since 0 and 1 are not prime or composite, the first odd composite is 9. The first possible odd prime is 3. 9+3= 12.

Note that since the focus is on eliminating prime + composite combinations and 1 is neither prime nor composite, only odds > 1 will be considered.

Since the goal is to eliminate evens, the prime 2 + an odd will not be even, we only consider primes >2.

The number of even integers that are eliminated increases directly with the number of primes the algorithm is applied to. Prime y + set of odds = x number of combinations to eliminate.

Prime 1 + arbitrary number of consecutive odds: 3 + [3,5,7,9,11,13,15,17,19,21,23,25,27,29,31]. Eliminate 3 + composite: Eliminate 3 + [9,15,21,25,27]. § There are five, 3 + composite combinations to eliminate.

Prime 2 + same odds in congruence with prime 1 + same odds = 2*x number of combinations to eliminate. Prime 2 + same odds as prime 1: 5 + [3,5,7,9,11,13,15,17,19,21,23,25,27,29,31]. Eliminate 5 + composite: Eliminate 5 + [9,15,21,25,27]. § There are five, 5 + composite combinations to eliminate + the previous five from Prime1 = ten combinations to eliminate.

This work shows x number of combinations to eliminate increases directly with number of primes being added to composites in congruence with previous combinations. Since primes are infinite, there will be infinite prime + composite combinations to eliminate. Prime + composite = even integer. There will be infinite even integers to eliminate.

Applying the algorithm to one prime eliminates even integers that are not the sum of that one prime plus any other prime. Applying the algorithm to two primes simultaneously eliminates even integers that are not the sum of both primes plus any other prime.

Example: Apply the algorithm to 3 and 13 simultaneously, the numbers 16 and 26 will not be eliminated because each of them are sums of the algorithm primes + one other prime. 16 is 3+13 and 13+3; or prime + prime for both algorithm primes. 26 is 3+23 and 13+13; or prime + prime for both algorithm primes. Although 18 is an algorithm prime + prime sum (13+5), it is not a prime + prime element of both algorithm primes. When applied to 3, 18 is a prime + composite (3+15). Hence, 18 is eliminated when applying the algorithm to the primes 3 and 13 simultaneously.

Applying the algorithm to all primes simultaneously eliminates even integers that are not the sum of all primes plus any other prime.

No even is the sum of every prime plus any other prime. If such an even exists, then primes would be finite. Therefore, all even integers are eliminated.

Since all evens ≥ 12 can be written as the sum of a prime + composite, the algorithm can be applied to multiple primes simultaneously, the algorithm eliminates prime + composite addends by design, and no even is the sum of every prime plus any other prime: applying the algorithm to infinite primes will eliminate all evens ≥ 12. □

Lemma 9.3. If prime + prime and prime + composite follow the same pattern; yielding equal but opposite truths, all evens ≥ 12 are eliminated, and the equal but opposite truth is that all evens ≥ 12 are not eliminated, then all evens ≥ 12 can be expressed as the sum of prime + prime.

Proof. Algorithm definition of not eliminated: prime + prime. □

Theorem 9.2. If all evens $\geq$ to 12 are the sum of prime >2 + odd composite >1, then all evens > 2 are the sum of two primes.

Proof. Assuming all previous logic is true, then evens 4 through 10 are the sum of prime + prime and all evens $\geq$ to 12 are the sum of prime >2 + odd composite >1. The algorithm eliminates prime + composite combinations by design. When the algorithm is applied to infinite primes, infinite prime + composite combinations are eliminated. All evens ≥ 12 are the sum of prime + composite. No even is the sum of every prime plus any other prime because primes are infinite, so all evens ≥ 12 are eliminated. Prime + prime and Prime + composite follow the same pattern, yielding equal but opposite results (equal: sum, opposite results: eliminated and not eliminated). The equal but opposite result of eliminating all evens ≥ 12 is that all evens ≥ 12 are not eliminated. By algorithm definition, not eliminated = prime + prime. Therefore, all evens > 2 are the sum of two primes.

□

References/Bibliography

N/A- this is all original work.

Data Availability

N/A- Everything is provided in this paper. To figure out the algorithm, I did trial and error scratch work with pen and paper. From there I noticed patterns and made notes of them, which I put into this document.

Supplimentary Materials

N/A

Contact

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Conflict of Interest

N/A

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Biography

B.A. in Math. Artistic minded and loves finding patterns. Interested in primes and Goldbach’s Conjecture. Left handed. Loves spending time with her husband, two children and two furbabies. Does not like the word “impossible”.