Refutation of Ramsey's theorem via Pythagorean triple of integers

Abstract: We evaluate the lemma proffered to prove Ramsey's theorem, and the strengthened lemma in a note. Neither are tautologous. We evaluate the proof of Pythagorean triple of integers as colored. It also is not tautologous. In fact, the coloring or non-coloring produces a logically equivalent result, meaning the Ramsey theorem is neither a tautology nor a contradiction. This implies "the inductive hypothesis" is suspicious. What follows is that the HOL proof assistant for the Ramsey theorem is an historical *enormity* in its 200 TB computer program with a certified prize result. Therefore the Ramsey theorem and HOL proof assistants are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Ramsey's_theorem

By the inductive hypothesis $R(r-1, s)$ and $R(r, s-1)$ exist. (0.1.1), (0.2.1)

LET p, q, r, s: p, R, r, s

%(q&((r-(%p>#p))&s)) = (p=p) ; CCCC CCCC CCCC CCCC (0.1.2)

%(q&(r&(s-(%p>#p)))) = (p=p) ; CCCC CCCC CCCC CCCC (0.2.2)

Remark 0.: Eqs. 0.1.2 and 0.2.2 are logically *equivalent*. This questions the efficacy of reliance on the inductive hypothesis.

Lemma 1. $R(r, s) \le R(r-1, s) + R(r, s-1)$: (L.1.1)

~((%(q&((r-(%p>#p))&s))+%(q&(r&(s-(%p>#p)))))<(q&(r&s))) = (p=p) ;
NNNN NNNN NNNN NNTT (L.1.2)

Note. In the 2-colour case, if $R(r-1, s)$ and $R(r, s-1)$ are both even, the induction inequality can be strengthened to: $R(r, s) \leq R(r-1, s) + R(r, s-1) - 1$. (N.1.1)

~((%(q&((r-(%p>#p))&s))+(%(q&(r&(s-(%p>#p))))-(%p>#p)))<(q&(r&s)))
= (p=p) ; `NNNN NNNN NNNN NNTT` (N.1.2)

Remark N.1.: Eqs. L.1.2 and N.1.2 are logically equivalent. This means "the induction inequality" is *not* strengthened as claimed.

Lemma 1 is *not* tautologous. The textual proof of Lemma 1 uses the case of two colors. Instead of stepping through the proof in that text, we evaluate Ramsey's two color theorem in equation 2 below.

We evaluate Ramsey's two color theorem framed as the Boolean Pythagorean triples problem from en.wikipedia.org/wiki/Boolean_Pythagorean_triples_problem .

The Boolean Pythagorean triples problem is a problem relating to Pythagorean triples which was solved using a computer-assisted proof in May 2016.

This problem is from Ramsey theory and asks if it is possible to color each of the positive integers either red or blue, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ [(2.1)] are all the same color. For example, in the Pythagorean triple 3, 4 and 5 ($3^2 + 4^2 = 5^2$), if 3 and 4 are colored red, then 5 must be colored blue.

Remark 2.1: We simplify the two colored theorem into four-variables as follows, assigning Blue as the negation of Red.

For a, b, c as 1, 2, 3:

LET p, q, r, s: $a^2=1^2=1$, $b^2=2^2=4$, $c^2=3^2=9$, Red.

If (1 + 4) ≠ 9, then if (1 and 4 are Red), then 9 is Not Red. (2.1.1)

((p+q)@r)>(((p&q)=s)>(r=~s)) ; T**FF**T TTTT TTTT TTTT (2.1.2)

If (1 + 9) ≠ 4, then if (1 and 9 are Red), then 4 is Not Red. (2.2.1)

((p+r)@q)>(((p&r)=s)>(q=~s)) ; T**F**TT **F**TTT TTTT TTTT (2.2.2)

If (4 + 9) ≠ 1, then if (4 and 9 are Red), then 1 is Not Red. (2.3.1)

((q+r)@p)>(((q&r)=s)>(p=~s)) ; TT**F**T **F**TTT TTTT TTTT (2.3.2)

Eqs. 2.1.2-..3.2 are *no*t tautologous. This means the answer to "if it is possible to color each of the positive integers either red or not red, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ are all the same color" is no.

Remark 2.4: The answer to the contra-question of "if it is possible to color each of the positive integers in the same color, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ are all not the same color" is also no. For example from Eq. 2.3.2:

((q+r)@p)>(((q&r)=s)>(p= s)) ;   `TT`**`F`**`T` **`F`**`TTT TTTT TTTT`   (2.4.2)

In fact, Eqs. 2.3.2 and 2.4.2 are logically equivalent, meaning the Ramsey theorem is neither a tautology nor a contradiction.

This speaks to the fact that injection of exponentiation results in a probabilistic vector space which abandons bivalency. What follows is that HOL proof assistants are not bivalent and hence produce unpredictable results, such as the alleged proof of the Ramsey theorem via Pythagorean triple of integers in the historical *enormity* of a 200 TB propositional logic computer program with result of a certified and paid prize.