Confirmation of the collapse of the Buss hierarchy of bounded arithmetics

Abstract: Two seminal rules of inference evaluated as *not* tautologous. This means the following are also refuted: Buss's hierarchy of bounded arithmetics does not entirely collapse; Takeuti's argument implies P ≠ NP; and systems PV and PV⁻. What follows is that separation of bounded arithmetic using a consistency statement is not viable. Therefore the above are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
 > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
 < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
 = Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
 % possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
 (z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
 (%z>#z) `N` as non-contingency, Δ, ordinal 1;
 (%z<#z) `C` as contingency, ∇, ordinal 2;
 ~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yamagata, Y. (2019). Separation of bounded arithmetic using a consistency statement. arxiv.org/pdf/1904.06782.pdf yoriyuki.y@gmail.com, yoriyuki.yamagata@aist.go.jp

Abstract. This paper proves Buss's hierarchy of bounded arithmetics ... does not entirely collapse... . Further, we can allow any finite set of true quantifier free formulas for the BASIC axioms … . By Takeuti's argument, this implies P ≠ NP.

3. PV and related systems
3.2. Equality axioms. The identity axiom is formulated as (15) t = t
The remaining equality axioms are formulated as inference rules rather than axioms.

(19) $$\frac{t(x) = u(x)}{t(r) = u(r)}$$ for any term r. (3.2.19.1)

LET p, q, s, t , u, v, x, w :
 ε, i, s, t_1, u, v, x, t_2 .
 ((t&x)=(u&x))>((t&r)=(u&r)) ;

```
TTTT TTTT TTTT TTTT( 1), TTTT FFFF TTTT FFFF( 2),
TTTT TTTT TTTT TTTT( 2), TTTT FFFF TTTT FFFF( 2),
TTTT TTTT TTTT TTTT( 2), TTTT FFFF TTTT FFFF( 2),
TTTT TTTT TTTT TTTT( 2), TTTT FFFF TTTT FFFF( 2),
TTTT TTTT TTTT TTTT(17)
```
(3.2.19.2)

3.3. Induction.

(20) $$\frac{t1(\varepsilon) = t2(\varepsilon) \; t1(six) = vi(t1(x)) \; t2(six) = vi(t2(x)) \; (i = 0, 1)}{t1(x) = t2(x)} \qquad (3.3.20.1)$$

((((t&p)=(w&p))&((t&((s&q)&x))=((v&q)&(t&x)))) &
(((w&((s&q)&x))=((v&q)&(w&x)))&(q=((q@q)+(q=q))))) >
((t&x)=(w&x)) ;

```
                              TTTT TTTT TTTT TTTT(16),
TTTT TTTT TTTT TTTT( 1), TTFT TTFT TTTT TTTT( 1),
TTTT TTTT TTTT TTTT( 1), TTFT TTFT TTTT TTTT( 1),
TTTT TTTT TTTT TTTT( 1), TTTT TTTT TTFT TTFT( 1),
TTTT TTTT TTTT TTTT( 1), TTTT TTTT TTFT TTFT( 1),
TTFT TTFT TTTT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
TTFT TTFT TTTT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTFT TTFT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTFT TTFT( 1), TTTT TTTT TTTT TTTT( 1)
```
(3.3.20.2)

Remark 3.3.20.2: The term (i=0,1) is written as i=**F** or T, not as ordinals.

The system PV contains defining axioms, equality axioms, and induction as axioms and inference rules. By contrast, the system PV^{-} contains only defining axioms and equality axioms as axioms and inference rules.

Eqs. 3.2.19.2 and 3.3.20.2 as rendered are *not* tautologous. This refutes two inference rules, whereby the following are also refuted: Buss's hierarchy of bounded arithmetics does not entirely collapse; Takeuti's argument implies P ≠ NP; and systems PV and PV^{-}. What follows is that separation of bounded arithmetic using a consistency statement is not viable.