Refutation of Parikh's axiomatization of game logic G and completeness of logic system Par

Abstract: Three equations of Parikh's axiomatization of game logic G are *not* tautologous. Hence, the extended logic system Par is refuted, and Parikh's completeness conjecture is also refuted. Therefore these artifacts are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Enqvist, S.; Hansen, H.H.; Kupke, C.; Marti, J.; Venema, Y. (2019). Completeness for Game Logic. arxiv.org/pdf/1904.07691.pdf
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Abstract- ... In this paper, we introduce a cut-free sequent calculus for game logic, and two cut-free sequent calculi that manipulate annotated formulas, one for game logic and one for the monotone μ-calculus, the variant of the polymodal μ-calculus where the semantics is given by monotone neighbourhood models instead of Kripke structures. We show these systems are sound and complete, and that completeness of Parikh's axiomatization follows.

Fig. 1. Par Axioms:

4) $\langle\gamma^*\rangle\varphi \leftrightarrow \varphi \vee \langle\gamma\rangle\langle\gamma^*\rangle\varphi$ (4.1)

LET p, q, r: φ, γ^*, γ
(q&p)=(p+((r&q)&p)) ; T**F**TT T**F**TT T**F**TT T**F**TT (4.2)

5) $\langle\psi?\rangle\varphi \leftrightarrow \psi \wedge \varphi$ (5.1)

LET p, q, r: φ, ψ, ψ?
(r&p)=(q&p) ; TT T**F** T**F**TT TT T**F** T**F**TT (5.2)

6) $\langle\gamma^{d}\rangle\varphi \leftrightarrow \neg\langle\gamma\rangle\neg\varphi$ (6.1)

LET p, q, r: φ, γ, γ^{d}
(r&p)=(~q&~p) ; **F**TTT **F**T**FF** **F**TTT **F**T**FF** (6.2)

The system Par is easily seen to be sound. A main contribution of our paper is that we confirm Parikh's completeness conjecture.

Axiom Eqs. 4.2, 5.2, and 6.2 as rendered are *not* tautologous. Hence, the logic system Par is easily seen not to be sound, and Parikh's completeness conjecture is denied.