EXPLAINING MAGNETISM BY RELATIVITY

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ABSTRACT : In this paper , equations of Magnetism are rediscovered by using Special Relativity and Coulomb's law with different method and various cases are studied . Magnetic field near a current carrying conductor is redefined as ratio of relativistic electric field experienced by reference frame of a moving charge near the conductor to velocity of that reference frame with respect to the conductor. Mathematically, it is found that the existing formulas of magnetism given by Biot- Savart Law are valid only for normal velocities and completely fail for higher or microscopic velocities of moving charge. 'Fleming's Right/Left Hand Rules' can easily be explained by Relativity as just interaction between charges. Unlike existing idea, a magnet or current carrying conductor can exert some force even on charges at rest. Magnetic field due to a current carrying conductor is nothing but ratio of approximate relativistic electric field to the velocity of reference frame. Lorentz force can be calculated as 'Just Electric Force' rather as 'Sum of Electric and Magnetic Force' by using Relativity. Magnetic force is just an electric force in Relativistic Electrodynamics that could not be explained by Classical Electrodynamics. The values of ε_0 and c are fundamental and the formula $\mu_0 = 1/\varepsilon_0 c^2$ is their relation with permeability of free space . It is claimed that "As Maxwell's Equations are based upon existing equations of Magnetism which are only approximation for lower velocities and fail in higher velocities, they are likely to be useless now". It is suggested that, "As Magnetic Force is nothing but Electric Force in Relativistic Electrodynamics, the name of one of the four fundamental forces ; 'Electromagnetic Force' should be changed into just 'Electric Force'." It is claimed that radiations are just non-mechanical waves that can propagate at velocity of light and are not necessarily electromagnetic in nature. Moreover, the writer suggests that electromagnetic waves may not exist at all.

KEYWORDS : *Coulomb's Law ,Special Relativity, Classical Electrodynamics, Relativistic Electrodynamics, Lorentz's Factor , Electromagnetic Force ,Electric Force, Magnetic Force ,Electric Field , Magnetic Field , Biot- Savart Law , Maxwell's Equations , Fleming's Left/Right Hand Rules, Length Contraction, Einstein's Velocity Addition*, *Drift Velocity, Permeability of free space, Permittivity of vacuum, Velocity of light , Lorentz's Force Law* .

INTRODUCTION : Shortly after first batteries were invented in early nineteenth century, it was found that a current carrying solenoid behaves as a magnet and two current carrying conductors exert force upon each other. Thus it was confirmed that electricity and magnetism are related.

But according to Classical Electrodynamics, "There is exactly equal distribution of positive and negative charges in a current carrying conductor. A moving charged particle near the wire hence should not experience any electric field and any electric force." The force experienced by the charged particle was named as magnetic force because current carrying conductors were found exerting force on magnets.

Later, Faraday found that a moving conductor near a magnet gets an emf produced in it . Many formulas, laws and hand rules were introduced to explain magnetism in nineteenth century. They were very successful in those times.

But still the causes of origin of magnetism and hand rules remained mystery until Einstein came with his famous "Theory of Special Relativity"[1] in 1905.

According to relativistic electrodynamics," The charged particle near a current carrying conductor gets different length contractions[2] for positive and negative charge distribution along the wire. Hence the charged particle should experience some relativistic electric force."

The 'Magnetic force' according to classical electrodynamics and 'relativistic electric force ' were found to have same values. Moreover, classical electrodynamics needs hands rules to explain the directions of magnetic forces but relativistic electrodynamics can explain the same force without any hand rules ,it can easily explain the force as "Just Relativistic Interaction between Charges."

Many Scientists like Leigh Page[3], Edward Mills Purcell in his book Electricity and Magnetism[4], Richard Feynman[5] etc have successfully worked in the field of "Relativistic Origin of Magnetism."

The writer of this paper was actually influenced by a youtube video having title "How Special Relativity make Magnets work" by Veritasium[6] to work on this research . The writer has put some of the 'already done works by others' just to discuss about some 'more general' formulas and many other phenomena. It is not based upon any science experiments in lab but this research is entirely based upon pre existing ideas and laws plus some thought experiments and mathematical logics of the writer.

. As many things are discussed in this paper, it had to be divided into different parts.

PART 1 : TO SHOW THAT MAGNETISM IS ENTIRELY RELATIVISTIC EFFECT AND EXISTING FORMULAS OF MAGNETISM FAIL IN HIGHER VELOCITIES

Imagine a long, thin and straight conducting wire. Generally, the wire has exactly equal number of electrons and protons. Let their number be 'n'. When there is no flow of current, a charged particle moving near the wire experiences exactly equal positive and negative electric fields. They cancel each other and that charged particle experiences no electric force . All electrons in the wire in this condition when there is no flow of electricity, remain in same atom and exactly cancel the electric fields caused by protons. We can say that protons and 'electrons remaining in same atom' , if are equal in number then they exactly cancel each other.

Now let current starts flowing in that wire. Some of the electrons actually move with very little average drift velocity and most of the atoms remain in the same atom . Let the average drift velocity of electrons be 'u' . Let number of moving electrons in conductor be 'x'. The number of electrons remaining in atoms of wire now become 'n-x".

The 'n-x' unmoving electrons exactly cancel the effect of 'n-x' protons of wire. Remaining x electrons (which were moving) and x protons cannot cancel each other when observed from a reference frame moving with certain velocity 'v' . It is because the length contractions of linear charge distribution of those x electrons and x protons are different due to their slightly different velocities. The linear charge density of positive and negative charge of wire becomes different for a reference frame and an electric field appears from there. A charged particle in that reference frame experiences some electric force.

Let the charge contained by 'x' moving electrons in that wire be ' negative charge' and charge of 'x' protons be 'positive charge' of that wire. It is because remaining protons and remaining unmoving electrons exactly cancel each other for all reference frames.

(it is supposed that value of electric charge remains same in all frames of references.)

Now study the figure give below .

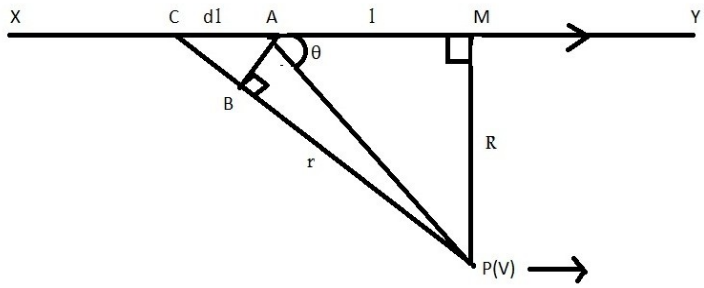

figure 1

Let XY be a long , thin and straight current carrying conducting wire of length 'L' in which current is flowing from X to Y. It means electrons are moving from Y to X in the wire with average drift velocity 'u' . AC is a tiny segment of wire of length dl . P is a reference point which makes angle θ with AC and is moving with velocity V exactly parallel to the direction of current. M is a point in the wire in a position such that PM is perpendicular to the wire. Let MA= l , AP=r and PM=R.

Let the velocity from left to right be positive and from left to right to be negative.

Now, for reference frame of P,

Velocity of positive charges of wire = **-v** ….. (1.0)

Velocity of negative charges of wire = **-** $\frac{v+u}{1+\frac{uv}{c^2}}$ …….(1.1) (by using Einstein's velocity addition[7] where c is speed of light.)

Suppose the charge carried by those x electrons be –Q and charge of x protons be +Q .By using formulas of length contraction, we can go to the following conclusions:

P observes +Q charge in **L** $\sqrt{1-\frac{v^2}{c^2}}$ length of positive charge distribution.

OR, P observes that dl length has **Q / L** $\sqrt{1-\frac{v^2}{c^2}dl}$ positive charge.

Let $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ = $\boldsymbol{\beta_v}$ ……..(1.2) where $\boldsymbol{\beta_v}$ is Lorentz factor [8] for v.

So , P observes that there is + $\frac{Q}{L}$ ***β_v dl** charge in dl* length.

Similarly P observes –Q charge in **L** $\sqrt{1-\left[\frac{v+u}{1+\frac{uv}{c^2}}\right]^2/c^2}$ length of negative charges .

OR, P observes that dl length has **-Q** / $L\sqrt{1-\left[\frac{v+u}{1+\frac{uv}{c^2}}\right]^2/c^2}$ **dl** negative charge.

let $\frac{1}{\sqrt{1-\left[\frac{v+u}{1+\frac{uv}{c^2}}\right]^2/c^2}} = \beta_{uv}$ **......** 1.3

Hence 'P' Observes that dl length of wire has – $\frac{Q}{L}$ ***β_{uv}.dl*** negative charge.

If the point P has a positive electric charge '+q' then we calculate force on 'P' due to dl segment of wire in this way;

Now using Coulomb's law[9] ,

Force due to positive charge of dl, **dF_1=** $\frac{1}{4\pi\varepsilon_0 r^2} \beta_v \frac{Qq}{L}$ ***dl*** …..(1.4)

and Force due to negative charge of dl, **dF_2** = $\frac{-1}{4\pi\varepsilon_0 r^2} \beta uv \frac{Qq}{L}$ ***dl*** *.....(1.5)* (negative sign indicates attractive force)

∴ Total electric force = $dF_1 + dF_2$

$$= \frac{1}{4\pi\varepsilon_0 r^2} (\beta_v - \beta_{uv}) \frac{Qq}{L} \, \mathbf{dl} \text{….(1.6)}$$

and force along PM **(dF)**= $\frac{1}{4\pi\varepsilon_0 r^2} (\beta_v - \beta_{uv}) \frac{Qq}{L}$ **dl sinθ**….(1.7)

Similarly, we can get,

Electric field along PM due to dl **(dE)=** $\frac{1}{4\pi\varepsilon_0 r^2}$ **(** β_v **-** β_{uv} **)** $\frac{Q}{L}$ **dl sinθ**……1.8

Since β_v and β_{uv} are not equal,we can say that a charge +q at P, experiences some 'relativistic electric force' due to elementary length 'dl' of that conductor.We are now going to find approximate formula for this force along PM for u<<v<<c .For this,

$$\beta_v - \beta_{uv} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{1}{\sqrt{1-\left[\frac{v+u}{1+\frac{uv}{c^2}}\right]^2 / c^2}}$$

$$\approx (1+\frac{v^2}{2c^2}) - [1+\frac{1}{2c^2}(v+u)^2] \quad (\text{since } 1+\frac{uv}{c^2} \approx 1)$$

$$Hence\ \beta_v - \beta_{uv} \approx -\frac{uv}{c^2} \quad ……1.9$$

From 1.7,1.8 and 1.9 we get ,

$$\mathbf{dF} = -\frac{1}{4\pi\varepsilon_0 r^2} \cdot \frac{uv}{c^2} \cdot \frac{Qq}{L}\ \mathbf{dl\ sin\theta} \quad …..2.0$$

and $$\mathbf{dE} = -\frac{1}{4\pi\varepsilon_0 r^2} \cdot \frac{uv}{c^2} \cdot \frac{Q}{L}\ \mathbf{dl\ sin\theta} \quad ….2.1$$

We know that ,for an observer at rest for wire,

L length has Q charge

So dl length has $\frac{Q}{L}$ dl charge.

Let charge in dl length be dQ.

Since $\mathbf{I} = \frac{dQ}{dT}$ **' I =** $\frac{Q}{L} \frac{dl}{dt} = \frac{Qu}{L}$ …..2.2

(Here we have neglected Lorentz factor for u because since u is microscopic the Lorentz factor due to u is 1.)

Putting **I** = $\frac{Qu}{L}$ in 2.0 ,2.1 and $1/\varepsilon_0 c^2$ **=** $\boldsymbol{\mu_0}$, we get ,

dF = - $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ** …..2.3a

and **dE = -** $\frac{\mu 0}{4\pi r^2}$ **I dl v sinθ** ….2.4

The value of dF in 2.3 is actually the value of electric force (where negative sign denotes attractive force) but we have exactly same formula for magnetic force given by classical electrodynamics too.

magnetic force along PM due to **dl =** $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ** ….2.5

But the problem with equation 2.5 is , we need hand rules in order to find the direction of the magnetic force. In case of 2.3, direction of force is known without using hand rules.

Also, magnetic force at P due to dl according to classical electrodynamics is, **dF = - dBqv** ….2.6

(negative sign indicates that the force is attractive)

Comparing 2.3 and 2.6 , we get

dB = $\frac{\mu 0}{4\pi r^2}$ **I dl sinθ….**2.7

This is equation for magnetic field according to Biot- Savart law[10].

Similarly, from 2.4 and 2.7 , we get

dE = dB.v , Hence **dB = dE/v ……**2.8

Equation 2.8 gives us definition of magnetic field at P according to relativistic electrodynamics . We can say that " Magnetic field at a reference frame due to a current carrying conductor is the ratio of relativistic electric field due to a conductor at a reference frame to the velocity of that reference frame with respect to the conductor."

Similarly, if P was moving with velocity '-v' , i.e. from right to left then

$$\beta_{uv} = \frac{1}{\sqrt{1-\left[\frac{v-u}{1-\frac{uv}{c^2}}\right]^2/c^2}}$$

$$\therefore \beta_v - \beta_{uv} \approx \frac{uv}{c^2}$$

then **dF =** $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ** ….2.9

and if there was charge –q at P then , **dF = -** $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ**….2.9a

It means if the positively charged particle moves parallel in direction opposite to current then the force would be repulsive.

We can get same formula for magnetic force according to classical electrodynamics too .By using hand rules, we can conclude that the force is repulsive.

<u>If the wire is very long then relativistic electric force at P is as follows</u> Let the wire starts from $-L_1$ from left and ends at L_2 at right then force on the charged particle due to whole conductor is(Integrating equation 1.7) ;

$$\mathbf{F} = \int_{-L1}^{L2} \frac{1}{4\pi\varepsilon_0 r^2}(\beta_v - \beta_{uv})\frac{Qq}{L} \sin\theta dl$$

put $\sin\theta$ = R/r and we get

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}(\beta_v - \beta_{uv})\frac{QqR}{L}\int_{-L1}^{L2}\frac{1}{r^3}\,\mathbf{dl}$$

$$\text{or, } \mathbf{F} = \frac{1}{4\pi\varepsilon_0}(\beta_v - \beta_{uv})\frac{QqR}{L}\int_{-L_1}^{L_2}\frac{1}{(l^2+R^2)^{3/2}}\,\mathbf{dl}$$

Since L_1 and L_2 are very large we get the solution of integrable part only as

$$\int_{-L_1}^{L_2}\frac{1}{(l^2+R^2)^{3/2}}\,\mathbf{dl} = \frac{2}{R^2}$$

finally we get ,

$$\mathbf{F} = \frac{1}{2\pi RL\varepsilon_0}\,\mathbf{Qq}\,(\beta_v - \beta_{uv}) \quad \text{….. 3.0}$$

For u<<v<<c , $\beta_v - \beta_{uv} \approx \frac{-uv}{c^2}$ So,

$$\mathbf{F} = -\frac{1}{2\pi R\varepsilon_0 c^2}\left(\frac{Qu}{L}\right)\mathbf{qv} \ldots 3.1$$

put $\frac{1}{\varepsilon_0 c^2} = \mu_0$ and $\frac{Qu}{L} = \mathbf{I}$ so ,

$$\mathbf{F} = -\frac{\mu_0 I}{2\pi R}\mathbf{qv} \ldots 3.2$$

Similarly, we can get equation for electric field for reference frame of P due to whole length of that conductor as ,

$$\mathbf{E} = -\frac{\mu_0 I}{2\pi R}\mathbf{v} \ldots 3.3$$

Equation 3.2 is the equation of electric force . We can see that it is the same formula of magnetic force in this condition.

The formula of magnetic force due to a long wire with current I and a point charge +q moving with velocity 'v' parallel to the conductor is, $\mathbf{F_M} = \frac{\mu_0 I}{2\pi R}qv \ldots 3.4$

The problem with equation 3.4 is that it cannot say whether the force is attractive or repulsive . to know this, we need to use right hand grip rule and right hand slap rule. After using them , we found the force to be attractive so ,

$$, \quad \mathbf{F_M} = -\frac{\mu_0 I}{2\pi R}\mathbf{qv} \ldots 3.5$$

also, $\mathbf{F_M} = -\mathbf{Bqv} \ldots \mathbf{3.6}$ (negative sign indicates attractive force)

from equations **3.3**,**3.5** and **3.6** we can write ;

E =Bv or B= E/v …..3.7

Here we got the same conclusion again that " Magnetic Field is just the approximate ratio of ' Electric Field due to a current carrying conductor on a reference frame' to ' velocity of that reference frame with respect to the conductor'."

For higher values or microscopic values of V , we cannot use approximation by binomial expansion for Lorentz Factors. In such condition, the existing equations of magnetism will surely not be valid . This paper claims that ," If magnetism can be explained by relativity then the existing formulas of magnetism should fail for higher or microscopic velocities ."

If P was moving in direction opposite to the Current

In such condition, the velocity of P for the wire becomes '-v' . An observer at 'P ' finds that positive charges on the wire have velocity '**v**' and moving electrons in that wire have velocity ' $\frac{v-u}{1-\frac{uv}{c^2}}$ ' .

In this condition, $$\beta_{uv} = \frac{1}{\sqrt{1-\left[\frac{v-u}{1-\frac{uv}{c^2}}\right]^2/c^2}}$$ ….. 3.8

and for **u<<v<<c ,** β_v - β_{uv} ≈ **(1+ $v^2/2c^2$)- [1+ $(v-u)^2/2c^2$]**

or, β_v - β_{uv} $\approx \frac{uv}{c^2}$ **…. 3.9**

Here the electric force along PM due to elementary length dl to the point charge at P becomes,

dF = $\frac{1}{4\pi\varepsilon_0 r^2} \cdot \frac{uv}{c^2} \cdot \frac{Qq}{L}$ **dl sinθ** (from 1.7 and 3.9)

Since $1/\varepsilon_0 c^2$ **=** μ_0 and **Qu/L = I** so,

dF = $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ ….** 4.0

Positive sign indicates that the force is repulsive.

If there was negative charge –q at P then ,

dF = - $\frac{\mu 0}{4\pi r^2}$ **I dl qv sinθ** …4.1

Similarly, we can get electric force due to whole wire as ,

F = $\frac{\mu_0 I}{2\pi R}$ **qv** ….. 4.2

and if there was –q charge at P then,

$$F = -\frac{\mu_0 I}{2\pi R} qv \;\ldots\ldots\; 4.3$$

The equations of electric force again in 4.0 and 4.2 are exactly same as equation of magnetic force in those condition. Moreover, the direction of the forces are also consistent with the direction of magnetic force explained by right hand rules.

Thus we can safely conclude that magnetic force explained by classical electrodynamics is nothing but electric force in relativistic electrodynamics which exists only due to length contraction and Einstein's velocity addition.

Classical Magnetism Vs Relativistic Magnetism

_There is a long , thin and straight conductor of length L in which a steady current 'I' is flowing from left to right with drift velocity 'u' . There is a charged particle with charge +q at distance R from the conductor which is moving parallel to conductor with velocity 'v' with respect to the conductor . In this condition we have formula for magnetic force according to classical magnetism as **,**

$$F_1 = -\frac{\mu_0 I}{2\pi R} qv \;\ldots\text{(a)}$$ (negative sign indicates attractive force)

and the formula derived by us by using relativity is (from 3.0) ,

$$F_0 = \frac{1}{2\pi R L \varepsilon_0} Qq(\beta_u\beta_v - \beta_{uv}) \;\ldots\text{(b)}$$

using $1/\varepsilon_0 c^2 = \mu_0$ **and** $Qu/L = I$ we get ,

$$F_0 = \frac{\mu_0 I}{2\pi R} \frac{c^2}{u} q(\beta_u\beta_v - \beta_{uv}) \;\ldots\ldots\text{(c)}$$

Comparing equations a and c we get,

$$F_0 = -F_1 \frac{c^2}{uv} (\beta_u\beta_v - \beta_{uv}) \;\ldots\ldots\text{(d)}$$

Equation 'd' is the relationship between magnetic force explained by classical electrodynamics and magnetic force explained by relativistic electrodynamics (we can say it be relativistic electric force too).

For general velocities , $(\beta_u\beta_v - \beta_{uv}) = -\frac{uv}{c^2}$. In such condition, F_1 and F_0 become the same. But they don't get equal in microscopic velocities and very high value of v (when v approaches to velocity of light.)

Now, lets compare the values of magnetic force measured by classical electrodynamics and relativistic electrodynamics separately :

use **I =Qu/L** and $1/\varepsilon_0 c^2$ **=** $\boldsymbol{\mu_0}$ in (a) ,we get ,

$$F_1 = \frac{Quqv}{2\pi RL\varepsilon_0 c^2} \quad \text{.....(e)}$$

Use **u = 10⁻⁵m/s , q = e = - 1.60219E-19 C , Q =10000C , R =0.1m and L = 1000m** in equations (b) and (e).

$$\left(\mathbf{F_0} = \frac{1}{2\pi RL\varepsilon_0} \mathbf{Qq(\beta_u\beta_v - \beta_{uv})} \quad \text{....(b)} \right.$$

$$\text{and} \quad \mathbf{F_1} = \frac{Quqv}{2\pi RL\varepsilon_0 c^2} \quad \text{.....(e)} \quad \Big)$$

For different values of v, we can get the following values of F_1 and F_0 (in Newtons) which are filled in the table below:

SN	V(m/s)	F_1	F_0
1	0	0	8.91338E-52
2	-u/2	-1.60219E-34	0
3	u	3.204378E-34	4.806576E-34
4	100	3.204378E-27	3.204378E-27
5	1000	3.204378E-26	3.204378E-26
6	10000	3.204378E-25	3.204378E-25
7	100000	3.204378E-24	3.204378E-24
8	1000000	3.204378E-23	3.204396E-23
9	10000000	3.204378E-22	3.20616E-22
10	c/2	4.80324E-21	5.5463E-21
11	0.9c	8.645836E-21	1.983491E-20
12	0.99c	9.510412E-21	6.741758E-20
13	0.999c	9.596878E-21	2.14646E-19
14	c	9.606485E-21	∞

By observing the above table , we can conclude that the values of magnetic force measured by classical electrodynamics and relativistic electrodynamics remain same for normal velocities but they get very different when v approaches u or when v approaches c .We can say that classical electrodynamics fails to measure magnetic force in microscopic velocities and higher velocities.

Other Conditions

When Force is Equal to Zero

Previously , it was said that magnetic force between a charged particle and a straight conductor becomes zero if :

a; Current flowing through the conductor is zero i.e. if drift velocity 'u' is zero

b; Charged particle is at rest with respect to the conductor i.e. 'v' is zero.

But if we think by using relativity then it is obvious that there is some length contraction for distribution of moving electrons in the conductor from the reference frame of charge at P .In such condition, the rest charge should experience some negative electric field due to the current carrying conductor. hence the case 'b' will not be true in relativity.

We are now going to find the value of velocity of charged particle if the net force between a positive 'point charge' and current carrying conductor is zero.

from 1.7 , **dF =** $\frac{1}{4\pi\varepsilon_0 r^2}$ **($\beta_u\beta_v - \beta_{uv}$)Q/L.dlsinθ** (force due to dl part of conductor)

When dF=0 then as other constants are non-zero , we can write,

$$\beta_v - \beta_{uv} = 0 \text{ ,or, } \beta_v = \beta_{uv} \text{ or, } \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left[\frac{v+u}{1+\frac{uv}{c^2}}\right]^2/c^2}}$$

(We can also use the other value of β_{uv} like in 3.8 but we will get same result)

Solving the above relations we get , a; v =+-c b; v= u/2

We can assume that v cannot be equal to c since charged particles have mass and cannot move at c.

Condition 'b' is logically possible. W can say that at v=u/2 ,the length contraction of both charges' distribution in conductor become exactly equal for particle at P. In this condition ,the charge experiences no force at all.

When Charged Particle is at Rest (i.e. v=0)

We know that, when v=0 then, β_v =1 and $\beta_{uv} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

So, $\beta_v - \beta_{uv} = 1 - \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \approx 1-(1+\frac{u^2}{2c^2}) = -\frac{u^2}{2c^2}$ ….4.4

again from 1.7, **dF=** $\frac{1}{4\pi\varepsilon_0 r^2}$ **($\beta_u\beta_v - \beta_{uv}$)Qq/L.dlsinθ** (force due to dl part of conductor)

$= -\frac{1}{8\pi\varepsilon_0 r^2}\frac{u^2}{c^2}\frac{Qq}{L}$ **dlsinθ** (from 1.7 and 4.4)

Since Qu/L =I and $1/\varepsilon_0 c^2 = \mu_0$ so,

F = $-\frac{\mu 0}{8\pi r^2}$ **I dlqusinθ ……**4.5

and if the point charge was negative then we get ,

F = $+\frac{\mu 0}{8\pi r^2}$ **I dlqu sinθ** ….4.6

Similarly, due to the whole length of wire, we get,

F = $-\frac{\mu_0 I}{2\pi R}$ **qu** ……4.7(for positive charge +q)

and **F =** $+\frac{\mu_0 I}{2\pi R}$ **qu** …..4.8(for negative charge –q)

It means, a positively charged particle at rest experiences very little attraction and a negatively charged particle experiences very little repulsion near a current carrying conductor.

We can say that a moving charge perpendicular to conductor experiences force in similar way that a charge at rest experiences force due to a current carrying conductor.

Force due to a Circular Loop of Current Carrying Conductor

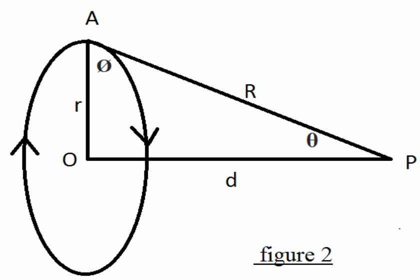

figure 2

Suppose there is a circular loop of wire in which current is flowing with drift velocity 'u' . Point 'A' has dl part of circumference of the loop. There is a rest charge +q at P which is exactly perpendicular to centre of loop 'O' at distance 'd' .

From equation 1.7, force on 'P' due to dl part of wire is ,

$$\mathbf{dF=} \frac{1}{4\pi\varepsilon_0 R^2} \left(\beta_v - \beta_{uv} \right) \frac{Qq}{L} \ \mathbf{dl\ sin}\text{Ø} \text{ (force along PO)}$$

but v=0 ,so β_v **=1** and $\beta_{uv} = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

so for u<<c , $\beta_v - \beta_{uv} \approx \dfrac{-u^2}{2c^2}$

$$\text{Hence ,}\mathbf{dF} = - \frac{1}{8\pi\varepsilon_0 R^2} \cdot \frac{u^2}{c^2} \cdot \frac{Qu}{L} \ \mathbf{.qu\ dl\ sin}\text{Ø}$$

$$\mathbf{or,\ dF} = - \frac{\mu 0}{8\pi R^2} \ \mathbf{I\ dl\ qu\ cos\theta} \quad \ldots 4.9$$

And the force due to whole loop of wire is ,

$$\text{F} = 2\pi \ \mathbf{r * (-} \frac{\mu 0}{8\pi R^2} \ \mathbf{I\ dl\ qu\ cos\theta)} \qquad \text{or , } \mathbf{F} = \frac{-\mu 0\, Irqud}{4\left(d^2+r^2\right)^3} \quad \mathbf{....5.0}$$

$$\text{If there is a-q charge at P then , F} = \frac{\mu 0\, Irqud}{4\left(d^2+r^2\right)^3} \quad \ldots\ldots 5.1$$

It means a circular loop with current flowing, attracts a positive charge at rest and repels a negative charge . The force is very tiny as it is directly proportional to drift velocity of current in the loop.

We can consider a magnet as group of large number of such loops of currents. It means we can say that a magnet can exert force even on charge at rest. A positive charge gets very slightly attracted and a negative charge gets very slightly repelled by magnets from any pole.

Part 2 : Explanations of Fleming's Right and Left Hand Rules

We got the following conclusions in equations above in part 1 :

1. When a negative charge -q is moving parallel to direction of current then it experiences some repulsive relativistic electric force towards the wire . (by putting –q in place of +q in equation 2.3a)

2. When a negative charge -q is moving antiparallel to direction of current then it experiences some repulsive relativistic electric force towards the wire .(equation 2.9a)

3. When a negative charge –q is moving perpendicular to direction of current carrying straight conductor conductor then it experience negligible force in comparison to conditions 1 and 2 above(it is because there will be exactly equal length contraction on linear distribution of both type of charges in the conductor for a moving charge near the conductor) .

Now we are going to explain Fleming's right and Left hand rules based on above three facts.

Explanation of Fleming's Right Hand Rule

We are considering here a single circular loop of current carrying conductor as a magnet (we can also take a rectangular loop and make the explanation even better). The current is flowing in the circular loop of conductor in clockwise direction with drift velocity 'u' .
Let a vertical, straight conductor moves from left to right in front of the loop with velocity 'v' . Let M ,N ,O, X and Y be points on conductor as shown in figure below.

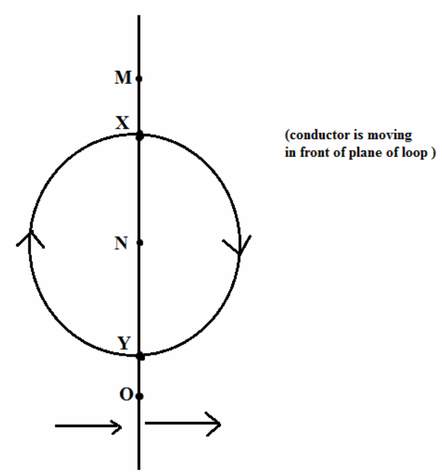

figure 3

Take only five points ; M ,N ,O ,X and Y of that vertical conductor as shown in figure below.

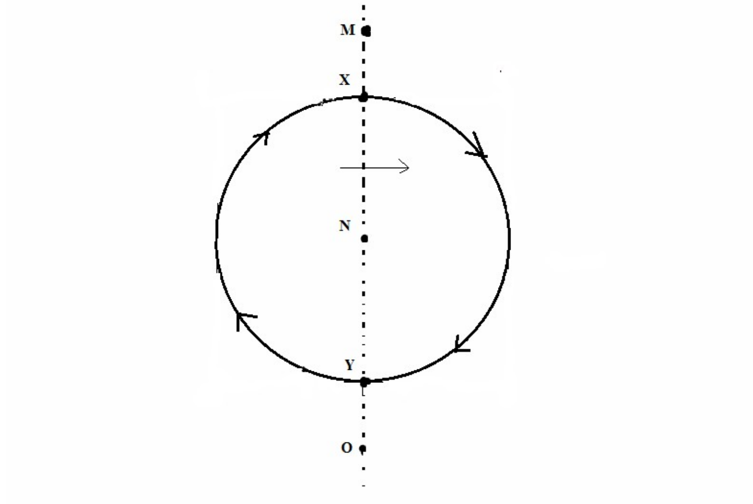

Figure 4

Divide the loop into two equal halves ; upper and lower half .

Each part of upper half of loop has current with some of directional component parallel to direction of electrons at N. These components repel the electrons . The electrons experience force downward away from the plane of the loop. The force has two components ; downward (vertical) and away from the plane of loop (horizontal).

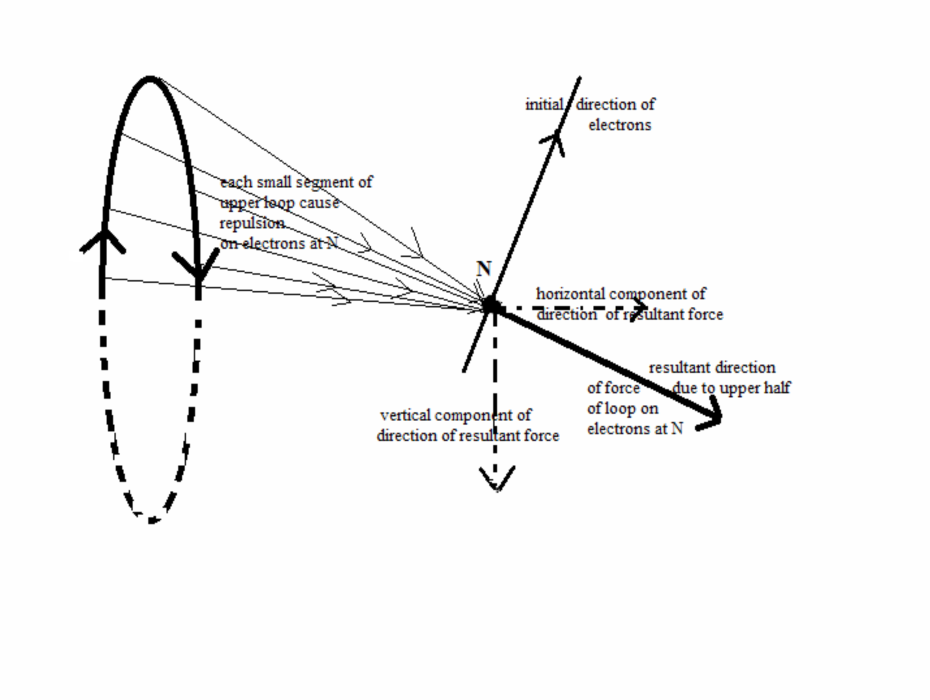

Figure 5

Each part of lower half of loop has current with some of directional component exactly opposite to direction of electrons at N . These components attract the electrons. The electrons experience force downwards towards the plane of loop . The force has two components ; downward (vertical) and towards the plane of loop (horizontal).

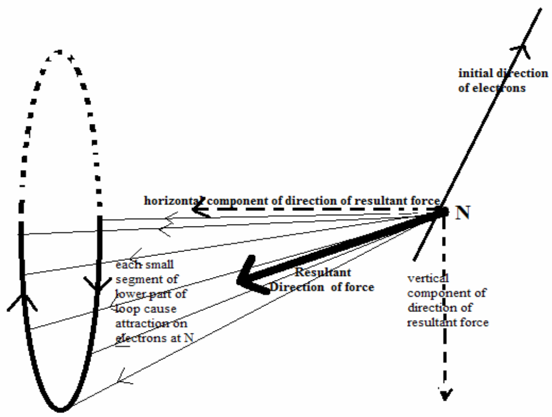

figure 6

The horizontal components of force on electrons (at N) due to these two halves of loops act on opposite directions and almost cancel each other.

But the vertical components of force on electrons (at N) due to these two halves of loops act on same direction and get added.

As a result , electrons at N experience a net force downward . The free electrons at N move/ try to move downwards in that conductor.

[* Note :There are some directional component of current vertical (both upward and downward) to direction of electrons at N too on both halves of loops but force on electrons due to vertical component of current is totally negligible.]

If we think about force on M (or points near M) on same way then we get that the electrons near M get repulsed upward because the repulsion of upward half of loop is more than attraction of lower half of loop(M is above the upper half). But due to downward movement of electrons below X, a partial positively ionized zone is created at that part of conductor. The positively ionized zone pulls the electron near M downwards which otherwise would go up due to repulsion.

Again, if we think about force on electrons near O then we find that electrons there try to move upward because the attraction of lower half of loop is more than repulsion of upper half of loop of current (O is below the lower half). But these electrons get repelled downward again due to the stream of electrons coming downward from the part of conductor above Y.

Thus electrons move downward in the wire . i.e. induced current moves upward.

In other hand ,positive charges in the conductor try to move exactly in opposite direction (i.e. upwards) but they are not motile . In such condition , the whole wire experiences some force upward till the wire is in motion.

We can consider a magnet as large number of such loops aligned parallel to each other . When a conductor moves near a magnet , all the loops of magnet affect the electrons of the conductor in the same way above .

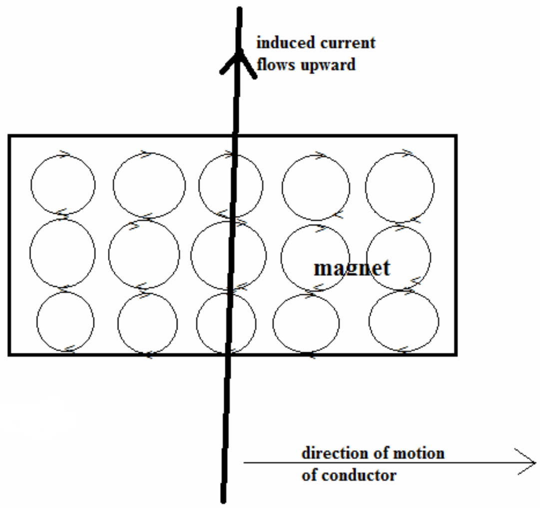

figure 7

As the left –right motion of electrons in the slowly get changed into downward vertical motion, the horizontal motion of the conductor gets decreased gradually. This is mechanism of Lenz's Law.

On comparing the above condition with Fleming's Right Hand Rule ,loops of current flowing in clockwise direction produce magnetic field lines going inside the plane of the loop. A conductor moving from left to right near the loop from the front should produce current flowing up(inside it) as shown in the figure :

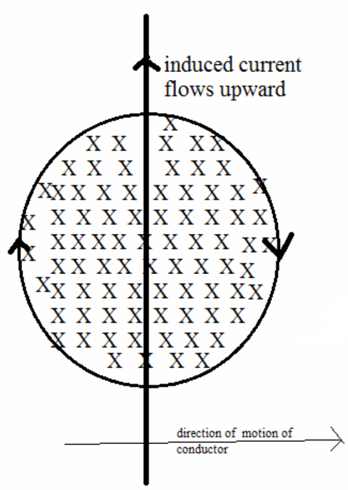

figure 8

('X' sign indicates that magnetic lines of force are going inside the plane of loop)

Thus our explanation of magnetic force as just ' a condition of Electric force with relativity applied' is able to explain Fleming's Right Hand Rule . We are able to show that the mechanism of Fleming's Right Hand Rule can be explained by ' relativistic interaction of charged particles'.

Explanation of Fleming's Left Hand Rule for Motors :

Consider an elementary electromagnet consisting of only one circular loop of current (we can also consider a rectangular loop for easier explanation but we generally consider a loop as circular one.) in which uniform current is flowing in clockwise direction.

A straight horizontal conductor is put in front of the loop in which uniform current is flowing from right to left. It means ,electrons are moving from left to right in this horizontal conductor.

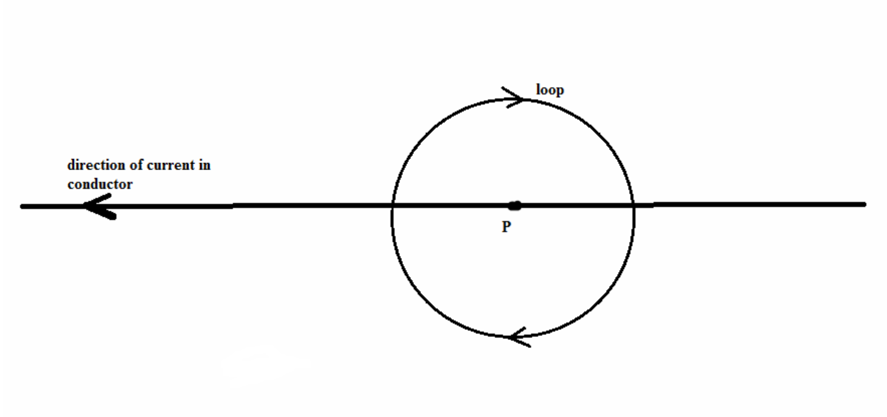

figure 9

Divide the loop into two equal halves ; upper and lower half in comparison to that straight conductor.

Take an electron at point P of that conductor . Since current in flowing from right to left in that conductor ,the electron at P moves from left to right as shown in figure below. Lets observe force on that electron due to those two halves of loop of current.

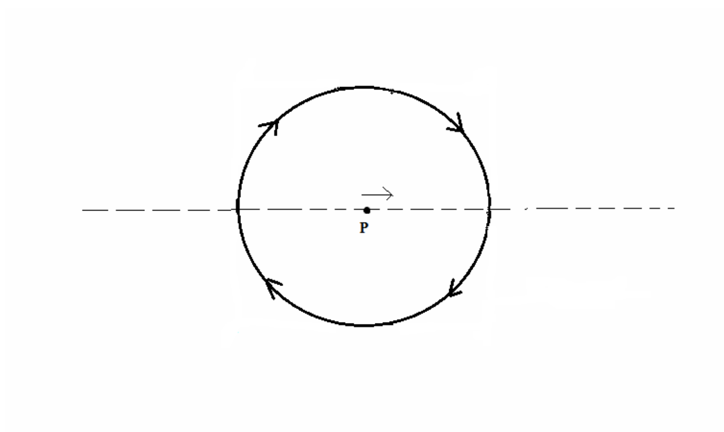

figure 10

The condition is exactly similar to that of figure 4 . By following the similar logics there we can come to conclusion that the electron at P experiences some force downwards.

Similarly, all other moving electrons in the straight conductor also experience force downwards. They push the whole conductor downward with some force . Hence as a result , the wire moves/ tend to move downward.

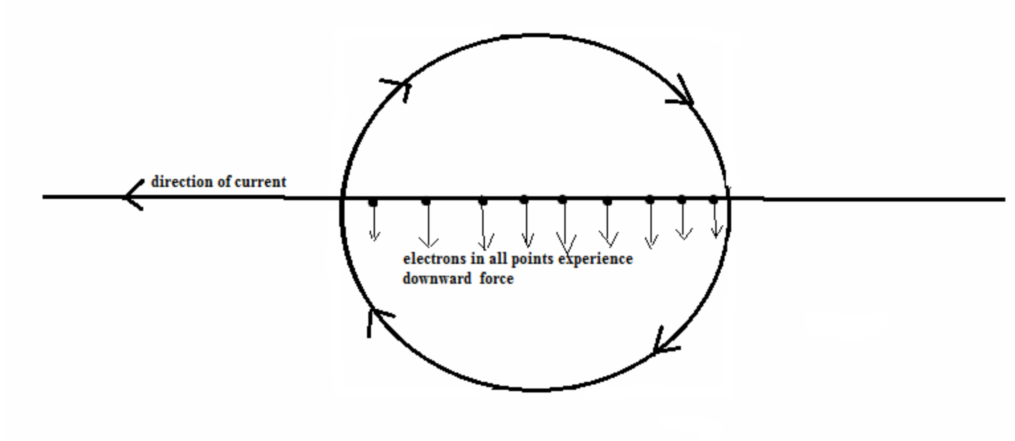

figure 11

If we study the figure 9 then after applying right hand rule we will find that the loop produces magnetic lines of forces that go inside the plane of loop (inside the loop) .When a straight horizontal conductor with current flowing from right to left is kept in such magnetic field then it experiences force according to Fleming's Left Hand Rule as shown in the figure below :

('X' sign indicates that magnetic lines of force are going inside the plane of loop)

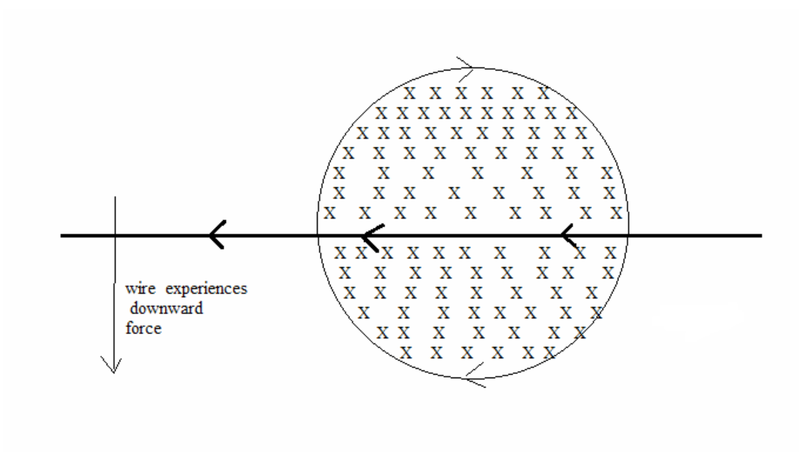

Figure 12

The result in figure 12 is exactly same as result in figure 119. Hence we can say that our model of 'Magnetism as relativistic interaction between moving charges' is totally successful to explain Fleming's Left Hand rule for motors too.

We can consider a permanent magnet as group of many such tiny loops arranged parallel. All the loops of current of the magnet affect a current carrying conductor in similar way discussed above .

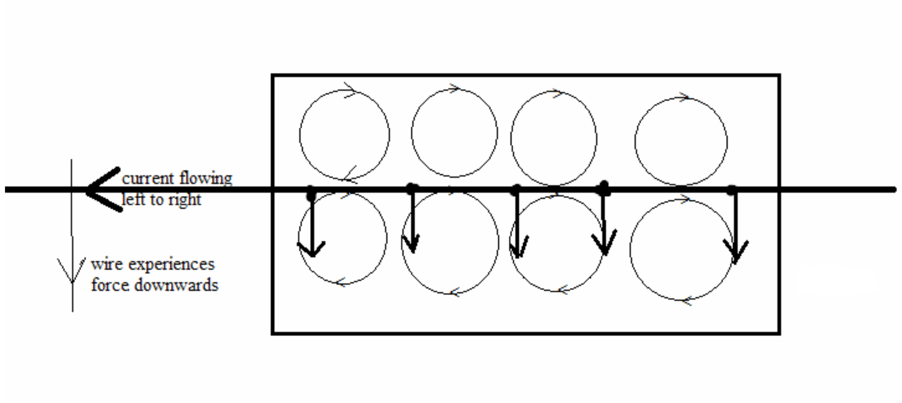

figure 13

Result

Thus we are able to show Fleming's Right/Left hand rules as effects of ' Relativistic Interaction between charges ' rather as 'unknown causes' while studying electromagnetism.The mechanism of Fleming's hand rules are not mysterious at all .

Part 3: Explanation of Lorentz's Force as only 'Electrical Force' but not as ' Sum of Electric and Magnetic

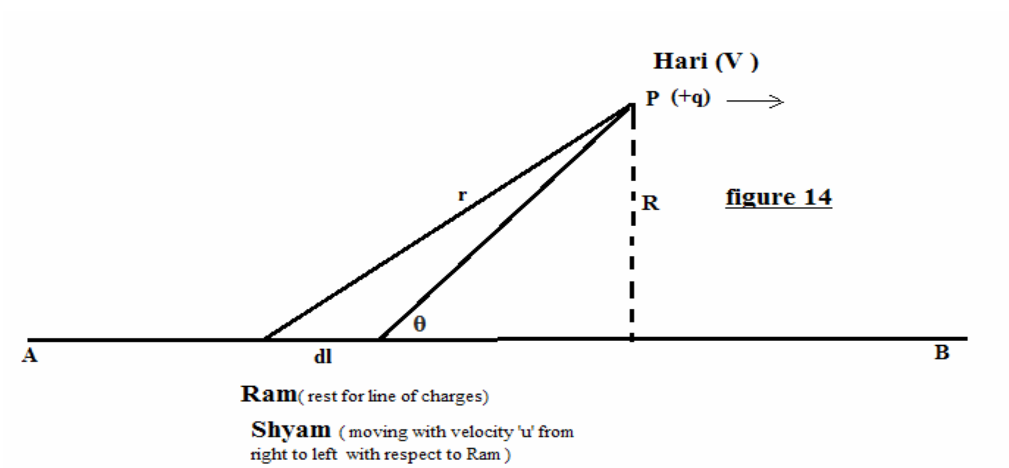

figure 14

Suppose there is an extremely thin straight wire-like line of positively charged particles between A and B which are at rest for reference frame of ' Ram' . Another man 'Shyam' is moving with velocity 'u' from right ot left. There is another person called 'Hari' at point P, carrying charge +q who is moving parallel to AB with velocity 'v' with respect to Shyam.

Now we get the following results :

i)For Ram, there is only some electric force between the line of charges and the charge with Hari.

ii)For Shyam, the charged particles in the line are moving with velocity 'u' from left to right ,they produce some electric current for him (the current not due to flow of electrons but due to flow of positive charges.) . The current produces some magnetic force too on charge +q with Hari .This force is repulsive too. Let the magnetic force be F_B . Hence the total force experienced by +q with Hari according to Shyam is ,

Total Force = Electric Force + Magnetic Force.

or, $\mathbf{F_T} = \mathbf{F_E} + \mathbf{F_B}$ …….(6.1)

The above formula is given by Lorentz Force Law[14].

Now we are going to explain the above condition by using Coulomb's law and Special Relativity:

Velocity of Hari according to Ram = $\dfrac{v-u}{1-\dfrac{uv}{c^2}}$

Let the length of AB be 'L' for Ram. Let there is +Q charge in that line of charges. Hence,

For Ram, dl length has + $\dfrac{Q}{L}$ **.dl** charge

According to Ram, Hari finds $\dfrac{+Q}{L\sqrt{1-\left[\dfrac{v-u}{1-\dfrac{uv}{c^2}}\right]^2/c^2}}$ **dl** charge in 'dl' length.

= + $\beta_{uv}\dfrac{Q}{L}$ **dl** charge in 'dl' length .

(We have supposed $\dfrac{1}{\sqrt{1-\left[\dfrac{v-u}{1-\dfrac{uv}{c^2}}\right]^2/c^2}} = \beta_{uv}$)

According to Ram,the electric force and electric field experienced by charge with Hari due to dl segment of AB are as follows :

$$\mathbf{E_1} = \frac{1}{4\pi\varepsilon_0 r^2}\beta_{uv}\frac{Q}{L}\,dl\sin\theta \quad ….6.2$$

and $\mathbf{F_1} = \dfrac{1}{4\pi\varepsilon_0 r^2}\beta_{uv}\dfrac{Qq}{L}$ ***dl*****sinθ …. 6.3**

(As $\mathbf{F_1}$ is electric force so $\mathbf{F_1} = \mathbf{F_E}$)
Similarly, for Shyam ,

There is **+Q** charge in **L** $\sqrt{1-\dfrac{u^2}{c^2}}$ length

So there is $\dfrac{+Q}{L\sqrt{1-\dfrac{u^2}{c^2}}}$ **dl** charge in dl length

= + β_u**Q/L.dl** charge in dl length.

(we have supposed $\dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}} = \beta_u$)

according to Shyam, Hari finds $+\beta_u \dfrac{Q}{L\sqrt{1-\dfrac{v^2}{c^2}}}$ dl charge in dl length

= + $\beta_u\beta_v$**Q/L.dl** charge in dl length.

(We have supposed $\beta_v = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$)

According to Ram ,the electric field and electric force experienced by Hari is as follows (for dl part)

$\mathbf{E_2} = + \dfrac{1}{4\pi\varepsilon_0 r^2}$ $\beta_u\beta_v$**Q/L.dl**sin**θ** …6.4 and

$\mathbf{F_2} = + \dfrac{1}{4\pi\varepsilon_0 r^2}$ $\beta_u\beta_v$**Qq/L.dl**sin**θ** ….6.5

(The value of F_2 in 6.5 is equal to total force F_T in 6.1)

By observing above equations we find that, Hari experiences more electric force /field in Shyam's reference frame than Ram's frame . The difference in electric field/ force is ,

$\Delta\mathbf{E} = \mathbf{E_2} - \mathbf{E_1}$

or, $\Delta E = + \frac{1}{4\pi\varepsilon_0 r^2} (\beta_u\beta_v - \beta_{uv}) \frac{Q}{L}$ ***dl* sinθ ….6.6**

and $\Delta F = F_2 - F_1$ **…..6.7**

or , $\Delta F = +(\beta_u\beta_v - \beta_{uv}) \frac{1}{4\pi\varepsilon_0 r^2} \frac{Qq}{L}$ ***dl* sinθ …..6.8**

For u<<v<<c , $\beta_u \quad \beta_v - \beta_{uv} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \quad \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{1}{\sqrt{1-\left[\frac{v-u}{1-\frac{uv}{c^2}}\right]^2 / c^2}}$

on solving it for above condition we get $\beta_u \quad \beta_v - \beta_{uv} \approx \frac{uv}{c^2}$ **….6.9**

From 6.6 and 6.9 , $\Delta E = + \frac{uv}{c^2} \frac{1}{4\pi\varepsilon_0 r^2} \frac{Q}{L}$ ***dl*sinθ … 7.0**

put Qu/L = I , and $1/\varepsilon_0 c^2 = \mu_0$ **in 7.0 , we get**

$\Delta E = \frac{\mu 0}{4\pi r^2}$ **Ivdl**sin**θ …..7.1**

Similarly , $\Delta F = \frac{\mu 0}{4\pi r^2}$ **Iqvdl**sin**θ ….. 7.2**

Actually the force Δ F in 7.2 is difference of electric forces according to two different reference frames but the formula of that electric force is exactly same to the formula of magnetic force given by classical electrodynamics.

It means magnetic force defined by classical electrodynamics is nothing but difference in electric force for two different reference frames according to relativistic electrodynamics. We can conclude that there is no such force called magnetic force in relativistic electrodynamics.

From 6.7 , $F_2 = F_1 + \Delta F$ **…..7.3**

The value of F_2 in 7.3 is equal to F_T (total force) . F1 is equal to electric force and Δ F is equal to magnetic force according to classical electrodynamics but difference in electric forces for two different observers according to relativistic electrodynamics. the 'Total Force' given by Lorentz's Force Law is not sum of electric and magnetic forces but only ' total electric force ' which cannot be explained by classical electrodynamics.

On studying 7.0, we can find that ,Hari finds slightly more electric field according to Shyam than according to Ram . The increased electric field is velocity dependent i.e. dependent upon drift velocity of virtual current and velocity of reference of moving charge.

According to Biot- Savart law, $\Delta\mathbf{B} = \frac{\mu 0}{4\pi r^2}$ **I dl sinθ ….7.4**

Comparing 7.1 and 7.4 we get , $\Delta \mathbf{E} = \mathbf{v}\,\Delta \mathbf{B}$

or $\Delta B = \Delta E/v$ ….. 7.5

It means, the magnetic field experienced by Hari according to Shyam is only the ratio of difference of electric fields for him (according to Ram and Shyam) and his velocity according to Shyam. This field can be defined as 'difference in electric fields according to two different observers per unit velocity of that reference frame(according to relativistic electrodynamics).

Moreover, this field is equal to magnetic field explained by Biot-Savart Law only for normal velocities. The formulas of magnetism totally fail for a reference frame of moving charge with very high velocity.

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Conclusions and Opinions

Conclusions

1.Magnetism exists only due to Special relativity .The existence of magnetism is strong proof of truth of special relativity.

2.According to relativity, a reference frame near a current carrying conductor experiences a velocity dependent electric field which is directly proportional to drift velocity of electrons in the conductor and velocity of that reference frame . Magnetic field is the ratio of that electric field and velocity of that reference frame.

3.The force explained as ' Magnetic Force ' by classical electrodynamics is just 'a type of velocity dependent electric force' in relativity.

4.Classical electrodynamics says that," An electric field for a reference frame is found to be mixture of electric and magnetic fields by another reference frame.' But according to Relativistic electrodynamics, the field observed by another reference frame too is just ' electric field of different strength.'

5.There is existing of only electric field/force in relativistic electrodynamics but not magnetic field/force.

6.Currently existing formulas of magnetism are valid for only normal velocity range. They fail in velocities comparable to speed of light and also very low velocities.

7.Even a charged particle at REST experiences some force near a current carrying conductor or a magnet. Negatively charged particles are slightly repelled and positively charged particles are slightly attracted .

8. The force between a current carrying conductor and a charged particle moving parallel to it is zero only when velocity of particle is half of drift velocity of electrons in that conductor .

9. The values of ε_0 and c are fundamental and the formula $\mu_0 = 1/\varepsilon_0 c^2$ is their relation with permeability of free space . In other words , c is fundamental and does not depend upon ε_0 and μ_0 but μ_0 depends upon c and ε_0 given by formula $\mu_0 = 1/\varepsilon_0 c^2$.

Opinions :

1 Maxwell's equations are based upon equations of classical electrodynamics which can be proved just approximations (not general) by relativistic electrodynamics . The mystery of hand rules also can be explained just by relativity and interaction of charges. Maxwell's equations are likely to be useless now.

2. Since it is proved that magnetic force is just electric force under special relativity and does not exist separately ,we should call the force 'Electromagnetic Force' as just 'Electric Force' .

3. It is possible that radiations are just non mechanical waves that propagate at highest possible speed © in vacuum and are not necessarily 'electromagnetic' . It is also possible that there is no such wave called electromagnetic wave.

4.Since the existing formulas of magnetism are valid for only 'normal velocities' and fail in higher velocity , the results of many high speed particle physics experiments where magnetism is used (like particle accelerators) may be proved wrong.

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