Modified spherical space-time gravity model

Tomasz Kobierzycki

December 20, 2019

Abstract

Understanding gravity at Planck scale is biggest goal of quantum gravity theory. In this paper i will present idea of quantum space-time that can be thought as gravity in quantum scale, this space-time is fixed it means this idea is not background independent it lives on specific modified spherical space-time. That spherical space-time does not break at inside of black hole and works in low energy very close to general relativity- it changes mostly after passing event horizon. Key idea is to use Planck energy units of energy and momentum as measure of curvature of spacetime.

Introduction

In this paper i will assume background dependent space-time, that is extension of 3-sphere metric with rotation in four dimension- that four dimension is 4-sphere dimension and rotation goes only from zero to π or from zero to $-\pi$. So it means there is only rotation by π angle in four dimension from this there is metric build. For each space-time direction x, y, z, ct there one of this metric, so there is 3-sphere and rotation in fourth direction of 4-sphere for each of that direction. There is a term that says how much rotation in fourth direction (4-sphere one) each coordinate has, that term is equal to:

$$dx^{\mu}dx^{\nu}\left(1-T_{\mu\nu}\right) \tag{0.0.1}$$

Where $T_{\mu\nu}$ is energy tensor, that term is in square form like in metric, for π rotation that term gives zero- so energy tensor is equal to one. For half rotation energy tensor is equal to one half and so on. I can write the rotation angle as: $\varphi = \pi T_{\mu\nu}$. Rotation to full π means that it goes from point A to point B where point B goes to point A and so on creating loop, that loop does not appear in lower energy scale. From it i can calculate relative change in length by using sec function of angle between two systems. I can write it as (where L' is distance change):

$$L' = \frac{\sec(\varphi)}{\sec(\varphi')} \tag{0.0.2}$$

Where $\sec(\varphi)$ comes from coordinate pointing in rotated angle to base angle of coordinate. If angle is half of π this means direction is perpendicular to normal angle and from point of any other direction it does not point in direction of that coordinate. This angle is rotation in four dimension that comes from 4-sphere , each coordinate moves forward in direction of 4-sphere and it moves in direction perpendicular to rotated 3-sphere angle.

Section 1: Field Equation

1.1 Metric tensor of spherical space-time

In this paper i will take into account model of modified spherical space-time. First i start by taking metric tensor that is equal to:

$$g_{\mu\nu} = \begin{pmatrix} 1 - T^0_{000} - d\Omega^2_{00} & -1 + T^0_{001} + d\Omega^2_{01} & -1 + T^0_{002} + d\Omega^2_{02} & -1 + T^0_{003} + d\Omega^2_{03} \\ -1 + T^0_{010} + d\Omega^2_{10} & -1 + T^0_{011} + d\Omega^2_{11} & -1 + T^0_{012} + d\Omega^2_{12} & -1 + T^0_{013} + d\Omega^2_{13} \\ -1 + T^0_{020} + d\Omega^2_{20} & -1 + T^0_{021} + d\Omega^2_{21} & -1 + T^0_{022} + d\Omega^2_{22} & -1 + T^0_{023} + d\Omega^2_{23} \\ -1 + T^0_{030} + d\Omega^2_{30} & -1 + T^0_{031} + d\Omega^2_{31} & -1 + T^0_{032} + d\Omega^2_{32} & -1 + T^0_{033} + d\Omega^2_{33} \end{pmatrix} \tag{1.1.1}$$

Where $d\Omega^2_{\mu\nu}$ is equal to 3-sphere metric that i can write as:

$$d\Omega^2_{\mu\nu} = \left(d\phi^2_{3\mu\nu} + \sin^2(\phi_{3\mu\nu}) \left(d\phi^2_{2\mu\nu} + \sin^2(\phi_{2\mu\nu}) d\phi^2_{1\mu\nu} \right) \right) \tag{1.1.2}$$

So for any given energy tensor metric is equal to $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ that is well understood formula from relativity. Coordinates are $dx^0 = cdt; dx^1 = dx; dx^2 = dy; dx^3 = dz$.

1.2 Energy tensor

In metric tensor term there is energy tensor $T^\gamma_{\gamma\mu\nu}$ that says how much energy is in the system. Tensor is summed over γ indexes. That tensor has to be normalized to Planck units of energy and momentum, when normalized it takes form of:

$$\begin{gathered} T^k_{k00} = \left(E^0_0\right)^2 - \left(E^1_1\right)^2 - \left(E^2_2\right)^2 - \left(E^3_3\right)^2 \\ T^0_{0ij} = -p_i p_j \\ T^k_{kij} = -p^0_{0i} p^0_{0j} + p^l_{li} p^l_{lj} \\ T^k_{k0j} = T^k_{kj0} = -E^0_0 p^0_{0j} + E^l_l p^l_{lj} \end{gathered} \tag{1.2.1}$$

Where E is energy and p is momentum. Each of those components need to be divided by Planck energy and momentum . Indexes k run from zero to three, indexes i, j run from one to three.

1.3 Relation between energy tensor and metric tensor

From energy tensor and metric tensor i can create a field equation that says how for given wave function curvature of space-time behaves. First part of equation says that wave function times wave operator is equal to energy i can write it as:

$$\Delta\psi_\mu - T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \tag{1.3.1}$$

Second equation connects energy tensor and wave function with metric tensor and it's equal to:

$$\psi_\mu - T^\gamma_{\gamma\mu\nu}\psi^\nu - g_{\mu\nu}\psi^\nu = 0 \tag{1.3.2}$$

I can add those two equations to get final one (they are both equal to zero) where wave operator for each coordinate of wave function of it acts as each of it is a scalar:

$$\boxed{\psi_\mu = 2T^\gamma_{\gamma\mu\nu}\psi^\nu - \Delta\psi_\mu + g_{\mu\nu}\psi^\nu} \tag{1.3.3}$$

1.4 General field equation

Wave equation can be extended to not only gravity systems by adding extra term to second equation (1.3.2), that extra term is tensor $K^{\gamma}_{\gamma\mu\nu}$, idea is that if there is gravity only system this part of equation gives zero if it's not there is another form of energy stored in wave function that does not come from gravity, so i can re write field equation to make it general as:

$$\boxed{\psi_\mu = 2T^{\gamma}_{\gamma\mu\nu}\psi^\nu - \Delta\psi_\mu + g_{\mu\nu}\psi^\nu + K^{\gamma}_{\gamma\mu\nu}\psi^\nu} \tag{1.4.1}$$

This tensor form does depend on how another field does act on wave function, it changes energy of wave function and it leads to more or less frequency (more if it's a positive value , less if it's negative).

1.5 Symmetries of general field equation

From general field equation there can be extracted two symmetries, first one is that system is massless so $\sum_\mu \Delta\psi_\mu = 0$ and second one says that tensor $K^{\gamma}_{\gamma\mu\nu} \geq 0$. From those symmetries i can create all possible pair of symmetry being fulfilled and not being fulfilled: $(+S_1,+S_2),(-S_1,+S_2),(+S_1,-S_2),(-S_1,-S_2)$, each of those pair has a specific wave function solution, so i need to modify field equation to match all possible symmetry states system does obey. I can think of symmetry pairs as an operator that does change wave function, matrix is a (1,1) tensor so i can write field equation as:

$$S^{\nu}_{\mu}\psi_\nu = 2\partial_\mu T^{\gamma}_{\gamma\mu\nu}\psi^\nu - \Delta S^{\nu}_{\mu}\psi_\nu + \partial_\mu g_{\mu\nu}\psi^\nu + \partial_\mu K^{\gamma}_{\gamma\mu\nu}\psi^\nu \tag{1.5.1}$$

Where ∂_μ is equal to $\frac{\partial}{\partial x_\mu}$, symmetry matrix has four components of summation (ν index) each of them represents one symmetry pair. So first one is first symmetry and so on, it gives four possible wave vector solutions that are summed. For equation to work i need to take derivative with respect to vector wave function with index μ, it just states that any change in of wave function vector due symmetry state is equal it's change in energy. It leads to change in metric and K tensor to rest part of equation. Idea behind symmetries is that sum of symmetries for any given system states constant- it does not change. I can write symmetries for any system as sum of vector components, that vector N is equal to:

$$N_\nu = \begin{pmatrix} S_1+S_2 & -S_1+S_2 & S_1-S_2 & -S_1-S_2 \end{pmatrix} \tag{1.5.2}$$

Where S can have value of one, minus one or zero. For each of this value there is assign matrix S^{ν}_{μ} components. So it means that if $\nu = 1$ o get first symmetry from N_ν matrix and it leads to first S^{ν}_{μ} component. Because symmetries stay constant for any system i can write that for any given wave function vector derivative with respect to ν index gives zero:

$$\partial_\nu\psi_\mu N^{\nu}_{\nu} = 0 \tag{1.5.3}$$

Where i use sum of vector N components that stays constant. First symmetry states that system does move with speed of light (it's massless), second symmetry states that energy of system is equal or greater than it's effect on curvature of space-time. Those two symmetries are natural ways to take basics properties of field equation and they lead to general conservation of those symmetries. Idea behind how system changes is that when it breaks symmetry it leads to energy change , for example electron emitting photon breaks symmetry and creates a photon (virtual one) then it absorbs it and gets extra energy that did come from symmetry breaking, that's general approach to how interaction of other than gravity system works, there is a symmetry breaking that generates force particle carrier and it has to give zero in symmetry change so it has to be absorbed or create another particle that is absorbed to match symmetry change equal to zero. Symmetry states are natural properties of field equation so extension to general field equation is just grouping field equation solutions.

1.6 Many systems equation

Field equation can be extended to many systems. First i use tensor product of wave function vectors, then extend sum of wave operator (Laplace operator) and add indexes to metric tensor and energy tensors. Field equation becomes for N body system:

$$\begin{aligned}\psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n} = 2T^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n \mu_1 \nu_1 ... \mu_n \nu_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n} - \left(\Delta_1 + \Delta_2 ... \Delta_n\right) \psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n} \\ + g_{\mu_1 \nu_1 ... \mu_n \nu_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n} + K^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n \mu_1 \nu_1 ... \mu_n \nu_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\end{aligned} \tag{1.6.1}$$

1.7 Measurement

First i write probability for one system, it's just sum of vector wave function components with it's complex conjugate:

$$P = \int_{x_1, t_1}^{x_2, t_2} \sum_{\mu=0}^{3} \psi_\mu \psi^*_\mu d^4 x \tag{1.7.1}$$

Probability tells what is change of particle being in position of spactime x_1, t_1 to x_2, t_2 where x has three components and $d^4 x$ means spacial and time components. Whole probability has to be equal to one so that integral for whole spacetime is one:

$$P = \int_X \sum_{\mu=0}^{3} \psi_\mu \psi^*_\mu d^4 x = 1 \tag{1.7.2}$$

Now i can write probability for n system state by just extending vector sum to many indexes:

$$P = \int_{x_1, t_1}^{x_2, t_2} ... \int_{x_{1n}, t_{1n}}^{x_{2n}, t_{2n}} \sum_{\mu=0}^{3} \psi_\mu \psi^*_\mu ... \sum_{\mu_n=0}^{3} \psi_{\mu_n} \psi^*_{\mu_n} d^4 x_1 ... dx^4_n \tag{1.7.3}$$

1.8 Plane wave solutions

Simplest solutions to this equations are plane waves. First i write down energy tensor, where zero component is changing with radius, i can write component $T^0_{000} = \frac{Ml_P}{m_P R}$, M means mass, R is radius and l_p, m_P is Planck length and mass. Rest of the components follow same rule just with momentum, so i can write energy tensor solutions as:

$$\begin{aligned}
T_{00} &= \left(\frac{Ml_P}{m_P R}\right)^2 - \left(\frac{p^0_{01} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{02} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{03} l_P}{p_P R}\right)^2 \\
T_{11} &= -\left(\frac{p^1_{10} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{11} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{12} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{13} l_P}{p_P R}\right)^2 \\
T_{22} &= -\left(\frac{p^2_{20} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{21} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{22} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{23} l_P}{p_P R}\right)^2 \\
T_{33} &= -\left(\frac{p^3_{30} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{31} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{32} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{33} l_P}{p_P R}\right)^2
\end{aligned} \tag{1.8.1}$$

From it follows plane wave solutions so i get wave function vector as (where j goes from one to three):

$$\psi_\mu = \begin{pmatrix} -\Psi_0 e^{ix^j k_{j0} - i\omega_0 t} & \Psi_1 e^{ix^j k_{j1} - i\omega_1 t} & \Psi_2 e^{ix^j k_{j2} - i\omega_2 t} & \Psi_3 e^{ix^j k_{j3} - i\omega_3 t} \end{pmatrix} \tag{1.8.2}$$

Section 2: Spherical model predictions

2.1 Cosmological model

Field equation solutions can be extended to universe as whole, to do it i just need to calculate balance between expansion and contraction of spacetime. Key idea is that universe can expand or contract or stay static it depends on energy of matter in universe and radius of visible universe. Universe is understood as all casual events that we can get information from so it means our light cone can reach it. Universe is understood only local, from our frame of reference. First i write Λ_D that is equal to:

$$\Lambda_D = 1 - \frac{2Ml_P}{Rm_P} \tag{2.1.1}$$

If Λ_D is equal to zero i get static universe that is equal to black hole event horizon, for positive value i get expanding universe because expansion has more energy than matter, for negative value i get contracting universe (energy of matter is bigger than expansion). Now i can write radius dependent on time as:

$$R(t) = R_0 + ct\left(1 - \frac{2Ml_P}{ctm_P}\right) \tag{2.1.2}$$

If mass stay constant i can write whole Λ_D as function of time:

$$\Lambda_D(t) = 1 - \frac{2Ml_P}{\left[R_0 + ct\left(1 - \frac{2Ml_P}{ctm_P}\right)\right] m_P} \tag{2.1.3}$$

If mass does not stay constant i need to make it a function of time. From those i can calculate energy density of expansion by:

$$\rho = \frac{Mc^2\left(1 - \frac{Ml_P}{Rm_P}\right)}{\frac{4}{3}\pi R^2 \frac{Ml_P}{m_P}} \tag{2.1.4}$$

And from it i can calculate content of dark energy in universe, where E_{DE} is dark energy and E_T is total energy:

$$\frac{E_{DE}}{E_T} = \left(1 - \frac{Ml_P}{Rm_P}\right) \tag{2.1.5}$$

From this model i can calculate universe as whole futures, this model focus only on energy tensor first component T_{00} it means it does not have momentum in it and rest of field equation components. But for low momentum universe it works as good approximation, for direct cosmological model all part of equation has to be taken into account like in this chapter.

2.2 Black holes

Black holes are most extreme test of gravity theory, i can use solutions from subsection (1.8) to create simplest model of black holes. First i write metric for event horizon without relative change in length that comes from introduction chapter:

$$\begin{aligned} ds^2 = \frac{1}{2}c^2dt^2 - c^2dt^2d\Omega_{00}^2 - dx^2\left(1 - T_{011}^0\right) + dx^2d\Omega_{11}^2 - dy^2\left(1 - T_{022}^0\right) + dy^2d\Omega_{22} \\ -dz^2\left(1 - T_{033}^0\right) + dz^2d\Omega_{33} \end{aligned} \tag{2.2.1}$$

Where there has to be sum of spacial components equal to one half i can write it as:

$$\frac{1}{2}c^2dt^2 = dx^2\left(1 - T^0_{011}\right) + dy^2\left(1 - T^0_{022}\right) + dz^2\left(1 - T^0_{033}\right) \tag{2.2.2}$$

Object at event horizon has to move at least half speed of light. Next step is inside of a black hole-singularity that in this model appears at half distance of event horizon first i write metric as:

$$ds^2 = -c^2dt^2d\Omega^2_{00} + dx^2d\Omega^2_{11} + dy^2d\Omega^2_{22} + dz^2d\Omega^2_{33} = 0 \tag{2.2.3}$$

Here 3-sphere part has to be equal to zero it can happen when there are two solutions to it first that all 3-sphere parts are equal to zero so there is only 2-sphere rotation left or space and time components are equal to zero but field equation limits that part with metric tensor is equal to zero so i get only solutions where all 3-sphere components are equal to zero, i can write it as:

$$-c^2dt^2d\Omega^2_{00} = dx^2d\Omega^2_{11} = dy^2d\Omega^2_{22} = dz^2d\Omega^2_{33} = 0 \tag{2.2.4}$$

It means inside a black hole there is only movement in 2-sphere parts that can move with any speed that is not faster than speed of light. I can write wave function vector in simplest case as:

$$\psi_\mu = \left(-\Psi_0 e^{\pm i\omega_0 t} \quad \Psi_1 e^{\pm i\omega_1 t} \quad \Psi_2 e^{\pm i\omega_2 t} \quad \Psi_3 e^{\pm i\omega_3 t}\right) \tag{2.2.5}$$

Where zero frequency is equal to one and sum of rest are equal to it. And those 2-sphere parts that stay are loops it comes from a fact that $\sin^2(\phi_2)$ or $\sin^2(\phi_3)$ has to be equal to zero, or both them have to equal to zero if there is movement in both $d\phi_1$ and $d\phi_2$ angle. So inside of a black hole is a loop of 2-sphere where movement speed depends on angle ϕ_1, ϕ_2 from metric there is need for it to be massless system it comes from fact that $ds^2 = 0$ it means inside a black hole there are massless object that move in 2-sphere loops.

2.3 Massive gravtion as extension of standard model

From general field equation there comes a possible solutions that represent forces and particles of standard model, only two particles that are not in standard model is graviton and massive graviton. Massless graviton has two pair of symmetries not equal to zero i can write it as:

$$N_\nu = \left(1+1 \quad 0 \quad 0 \quad -1-1\right) \tag{2.3.1}$$

It means it moves with speed of light and has energy equal to it's gravity effects. Second state of that particle is massive one that is the opposite of massless:

$$N_\nu = \left(-1-1 \quad 0 \quad 0 \quad 1+1\right) \tag{2.3.2}$$

This one has mass and it's energy is less than it's gravity effect. It means that for example in gravitation field it gets less energy from field than effects of gravity. It means tensor K is negative. It means it's wave function vector depends only on gravity field it is in. So if it moves at weak gravity field it's movement is slow- if it gets into strong gravitation field it moves faster, by but itself it's energy depends only on it's rest mass and it's effects to other body gravity. So when it interacts it's interaction depends only on rest mass for other body but it's movement depends only on gravity of that other body. It means that tensor K is equal to energy tensor but with minus sign i can write it as:

$$\begin{gathered}\psi_\mu = 2T^\gamma_{\gamma\mu\nu}\psi^\nu - \Delta\psi_\mu + g_{\mu\nu}\psi^\nu - K^\gamma_{\gamma\mu\nu}\psi^\nu \\ \psi_\mu = T^\gamma_{\gamma\mu\nu}\psi^\nu - \Delta\psi_\mu + g_{\mu\nu}\psi^\nu \rightarrow T^\gamma_{\gamma\mu\nu}\psi^\nu = \Delta\psi_\mu \\ \psi_\mu = g_{\mu\nu}\psi^\nu\end{gathered} \tag{2.3.3}$$

2.4 Massless particle space-time

Zero space-time interval for massless particles can be thought as some geometry of space-time. From space-time interval there comes two conditions that massless system has to obey, first one is that movement in space is equal to movement in time, same with 3-sphere part, i can right those both as:

$$\begin{gathered} c^2 dt^2 d\Omega_{00}^2 = dx^2 d\Omega_{11}^2 + dy^2 d\Omega_{22}^2 + dz^2 d\Omega_{33}^2 \\ c^2 dt^2 \left(1 - T_{000}^0\right) = dx^2 \left(1 - T_{011}^0\right) + dy^2 \left(1 - T_{022}^0\right) + dz^2 \left(1 - T_{033}^0\right) \end{gathered} \tag{2.4.1}$$

It states that for given differential of space there is equality of that differential in time and distance is equal to zero. First part means that all points in space-time are same point (for any given differential) second part says the same but for 3-sphere. To map massless particle space-time I must assume that there is a point that 3-sphere point is not pointing at. If 3-sphere gives zero it still points to some point on that 3-sphere, reasoning is that if i threat all 3-sphere parts as vectors there is a vector pointing at some part of 3-sphere part that is sum of all four 3-sphere parts. All other points are same points, i can write it using position vector, first i write X vector that is where currently particle is, second i write Y vector that is pointing where particle is not (i only use 3-sphere part it comes from a fact that differential of distance in normal coordinates makes always all points same):

$$\begin{gathered} dX^\mu dX_\mu = -c^2 dt^2 d\Omega_{00}^2 + dx^2 d\Omega_{11}^2 + dy^2 d\Omega_{22}^2 + dz^2 d\Omega_{33}^2 = 0 \\ dY^\mu dY_\mu = -c^2 dt^2 d\Omega_{00}^2 + dx^2 d\Omega_{11}^2 + dy^2 d\Omega_{22}^2 + dz^2 d\Omega_{33}^2 \neq 0 \end{gathered} \tag{2.4.2}$$

From it i can map space-time for massless particle, there is a distance in 3-sphere part that means that particle is pointing on one point of all 3-spheres summed and does not point at rest- that rest is needed to map geometry of that space-time. All points where particle is pointing at any given space-time interval are same points but those where it's not pointing are not zero distance apart.

2.5 Summary

In this paper i presented simple model of space-time based on background depended modified spherical space-time. Key is using Planck units of energy and momentum and field equation that can be derive from two equations, that equation for one system is equal to:

$$\boxed{\psi_\mu = 2T_{\gamma\mu\nu}^{\gamma} \psi^\nu - \Delta\psi_\mu + g_{\mu\nu}\psi^\nu + K_{\gamma\mu\nu}^{\gamma}\psi^\nu} \tag{2.5.1}$$

For many body system it has form of:

$$\boxed{\begin{gathered} \psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n} = 2T_{\gamma_1...\gamma_n\mu_1\nu_1...\mu_n\nu_n}^{\gamma_1...\gamma_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n} - \left(\Delta_1 + \Delta_2 ... \Delta_n\right) \psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n} \\ + g_{\mu_1\nu_1...\mu_n\nu_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n} + K_{\gamma_1...\gamma_n\mu_1\nu_1...\mu_n\nu_n}^{\gamma_1...\gamma_n} \psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n} \end{gathered}} \tag{2.5.2}$$

This model does not break at Planck scale of energy, it do not have any singularities when energy is limited to Planck energy same with momentum. At low energy scale it's very close to General Relativity, changes are present where energy is close to Planck energy. From general field equations symmetries there comes a particle that is massive graviton that would be addition to standard model as i presented in chapter (2.3), field equation can take into account effects of other than gravity forces by using tensor K, solutions to field equation is wave function vector.