# About the congruent number

Hajime Mashima

May 2, 2019

**Abstract**

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

**Theorem 1** *Pythagorean theorem*

$$\left(m^2+n^2\right)^2=(2mn)^2+\left(m^2-n^2\right)^2$$

**Definition 2**

$$ace(m_0^2+n_0^2)=ace\cdot\frac{f}{e}=acf$$
$$ace(2m_0n_0)=ace\cdot\frac{b}{a}=bce$$
$$ace(m_0^2-n_0^2)=ace\cdot\frac{d}{c}=ade$$

$$S'=\frac{bd}{2ac}\quad(bd=even)$$

**Definition 3** *S is a congruent number.(m , n* $=\mathbb{N}$*)*

$$(ace)^2S'=\frac{ace^2bd}{2}=mn\left(m^2-n^2\right)=k^2S\quad(k\geqq 1\quad,\quad m\neq n)$$

about $(k=1)$

$$m_1n_1\left(m_1^2-n_1^2\right)=A\quad\left(A\neq k''^2\mathbb{N}\right)$$
$$k'm_1k'n_1\left((k'm_1)^2-(k'n_1)^2\right)=k'^4A$$
$$mn\left(m^2-n^2\right)=k^2S$$

$$A=S$$

**Proposition 4** *The multiplication of the hypotenuse and one side of a right triangle is a congruent number.*

**Proof 5**

$$m = M^2 + N^2 \qquad n = 2MN$$
$$k^2 S = 2MN(M^2 + N^2)(M^2 - N^2)^2$$
$$S'' = 2MN(M^2 + N^2)$$

$$m = M' \qquad n = N'$$
$$k^2 S = M'^2 N'^2 (M'^4 - N'^4) \Rightarrow M'^4 - N'^4 = (M'^2 - N'^2)(M'^2 + N'^2)$$

□

**Corollary 6**

$$S'' = 2 \cdot 2m'^2 n'^2 (2^2 m'^4 + n'^4) \Rightarrow 2^2 m'^4 + n'^4$$
$$S'' = 2m'^2 n'^2 (m'^4 + n'^4) \Rightarrow 2m'^4 + 2n'^4$$

**Corollary 7** *If 1 is not a congruence number, Fermat's last theorem case 4 is equivalent.*

$$1 \cdot k^2 \neq z^4 - y^4$$