

Refutation of domain theory and the Scott model of language PCF in univalent type theory

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Abstract: The definitions of directed, complete posets for antisymmetry and transitivity are *not* tautologous, thereby refuting basic domain theory. By extension, the Scott model of language PCF in univalent type theory is also refuted and another *non* tautologous fragment of the universal logic $\mathbb{V}\mathbb{L}4$.

We assume the method and apparatus of Meth8/ $\mathbb{V}\mathbb{L}4$ with Tautology as the designated proof value, \mathbf{F} as contradiction, \mathbf{N} as truthity (non-contingency), and \mathbf{C} as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \sqcap , $;$; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \rightsquigarrow ;
 $<$ Not Imply, less than, \in , $<$, \subset , \prec , $\#$, \ll , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , \triangleq , \approx , \cong ; @ Not Equivalent, \neq ;
 $\%$ possibility, for one or some, \exists , \diamond , \mathbf{M} ; # necessity, for every or all, \forall , \square , \mathbf{L} ;
 $(z=z)$ \mathbf{T} as tautology, \mathbf{T} , ordinal 3; $(z@z)$ \mathbf{F} as contradiction, \emptyset , Null, \perp , zero;
 $(\%z\#z)$ \mathbf{N} as non-contingency, Δ , ordinal 1;
 $(\%z\#z)$ \mathbf{C} as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); $(A=B)$ ($A\sim B$); $(B>A)$ ($A\vdash B$); $(B>A)$ ($A\#B$).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: de Jong, T. (2019). The Scott model of PCF in univalent type theory.
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2 Basic domain theory

We introduce basic domain theory in the setting of constructive univalent mathematics.

2.1 Directed complete posets

Definition 2.1. A *poset* (X, \leq) is a set X together with a proposition valued binary relation $\leq: X \rightarrow X \rightarrow \Omega$ satisfying:

$$(i) \text{ reflexivity: } \mathbf{Q}x:X \quad x \leq x; \tag{2.1.i.1}$$

$$\text{LET } p, q, r: x, y, z. \tag{2.1.i.2}$$

$$\sim(p < p) = (p = p); \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T}$$

$$(ii) \text{ antisymmetry: } \mathbf{Q}x,y:X \quad x \leq y \rightarrow y \leq x \rightarrow x = y; \tag{2.1.ii.1}$$

$$(\sim(q < p) \sim (p < q)) > (p = q); \quad \mathbf{T}\mathbf{F}\mathbf{F}\mathbf{T} \quad \mathbf{T}\mathbf{F}\mathbf{F}\mathbf{T} \quad \mathbf{T}\mathbf{F}\mathbf{F}\mathbf{T} \quad \mathbf{T}\mathbf{F}\mathbf{F}\mathbf{T} \tag{2.1.ii.2}$$

$$(iii) \text{ transitivity: } \mathbf{Q}x,y,z:X \quad x \leq y \rightarrow y \leq z \rightarrow x \leq z. \tag{2.1.iii.1}$$

$$(\sim(q < p) \sim (p > r)) \sim (r < p); \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{F}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{T}\mathbf{T} \quad \mathbf{T}\mathbf{T}\mathbf{F}\mathbf{T} \tag{2.1.iii.2}$$

Eqs. 2.1.ii.2 and 2.1.iii.2 as rendered are *not* tautologous. The definitions of directed, complete posets for antisymmetry and transitivity are *not* tautologous, thereby refuting basic domain theory. By extension the Scott model of programming language PCF in univalent type theory is also refuted.