Vacuum in and around horizons

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Abstract

This paper puts forward the idea that vacuum is a collection of unlocalized quantum oscillators inside a horizon. The characteristics of such vacuum are studied inside the black hole and Hubble horizons. The results suggest that energy is quantized, with the minimal energy depending on the size of the horizon. This suggests that vacuum is more dense in black holes than in the surrounding universe, demanding by continuity the existence of a vacuum halo around every black hole. This vacuum halo could act exactly like dark matter if it exerted no pressure on matter, thus not interacting with matter in any way other than gravitationally. A suitable cosmological model presented here suggests that such vacuum could accelerate the universal expansion. This suggests that the vacuum studied in this paper could successfully stand instead of dark matter, cosmological vacuum and quantum vacuum of quantum theories.

Keywords: Cosmological vacuum, Quantum vacuum, Hubble horizon, Schwarzschild horizon, Dark matter, Dark energy MSC[2010]: 83-F05 81P16 81T25 85A15

1 Introduction

In the year 1998, two teams of astronomers, the Supernova Cosmology Project [1] and the High-Z Supernova Search Team [2], used supernovae of type Ia (SNeIa) as standard candles, and their data, gathered by observing SNeIa, confirmed that the universe is expanding in an accelerated fashion.

This result put forward the idea that the vacuum energy exists and must be included in calculations. Namely, no other type of matter could possibly produce an accelerated universal expansion.

If the vacuum is such that its pressure p and its energy density ε are related by an equation of state $p = w\varepsilon$ with $w = -1$, the vacuum is then called the cosmological constant, since with $w = -1$ the energy density in the adiabatic fluid equation is constant, $\varepsilon = \Lambda$.

If vacuum is such that $-1 < w < 1/3$, it's called a quintessence.

If vacuum is such that $w < -1$, it's called a phantom energy.

The cosmological constant Λ was first introduced by Einstein in an attempt to create a theory of a stable static universe.

At about the same time, Fridman[1] published his cosmological solution in the form of the most general spherically symmetric metric, that could describe an expanding or a contracting universe. Fridman's solution is the standard metric used in cosmology today, being simple and at the same time the most general spherically symmetric solution. The Fridman metric for flat space is

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\Omega^2\right] \tag{1}$$

with $a(t)$ being the scale factor and $d\Omega^2 = d\varphi^2 + \sin^2\vartheta d\vartheta^2$ being the purely angular part of the metric element.

Einstein didn't look upon Fridman's solution favourably, since Einstein was pursuing his idea of a stable static universe with the cosmological constant. And then Hubble analysed some astronomical observational data and realized that all the cosmological objects such as stars and galaxies are moving away from us, the recession velocity being larger the farther away the objects are. This Hubble law put an end to Einstein's idea of a static universe at once. In turn, this meant that the cosmological constant and the vacuum were no longer required. Einstein called his introduction of the cosmological constant "his greatest blunder". The Fridman solution was all that one required to describe an expanding universe in the most adequate way.

And then the SNeIa teams observed an accelerated expansion of the universe, thus re-introducing the vacuum energy with the negative equation of state parameter w.

The physical explanation of the cosmological vacuum energy is far from being clear. The notion of vacuum energy is not new though; vacuum was called ether before the introduction of the special theory of relativity, which demonstrated that the notion of vacuum was not necessary at all to describe all the laws of physics. Then, quantum theory was invented, and the most simple quantum object, a quantum harmonic oscillator, was shown to have a non-vanishing ground state energy that couldn't be just neglected. This ground state energy is called vacuum energy. Namely, if a quantum harmonic oscillator oscillates with frequency ω, then the energy levels of such quantum harmonic oscillator are given by $E_n = \omega(1/2 + n)$. The ground state has a non-vanishing energy $E_0 = \omega/2$, the so called zero-point energy. So this way the vacuum was introduced in physics once again as the ground state of a system.

Then the quantum field theory (QFT) was invented and is celebrated today as the most accurately experimentally confirmed theory. QFT represents a particle as an excitation of a vacuum field consisting of infinitely many harmonic oscillators that oscillate at all the possible frequencies, each at its own unique frequency. This way, each quantum oscillator of the particle field has ground energy $\omega/2$. Thus, the sum of ground energies of all the quantum oscillators of

[1] Fridman's surname is written Фридман in Russian cyrillic script, and transliterates directly to Fridman in Latin script.

the underlying quantum vacuum field is infinitely large, since this sum is the sum of all the positive real numbers. If one cuts the energy off at the Planck mass m_{Pl}, and cuts the distances off at the Planck length l_{Pl}, the density m_{Pl}/l_{Pl}^3 is still incredibly huge.

On the other hand, the vacuum energy density of a cosmological vacuum cannot be larger than the critical density. The critical density of the universe is a very tiny number, ensuring the universe is spatially flat, and so the vacuum energy density is very tiny in cosmology. This is one of the unsolved problems in physics: "how come the cosmological vacuum has so tiny energy density, whilst the QFT vacuum energy density is so large?"

In the early 20^{th} century, astronomers Zwicky and Smith noticed that in galactic clusters, objects move too quickly in their orbits around the galactic centre. This could be easily explained by the existence of more matter in galaxies. The problem was – there was not enough visible matter in and around galaxies, so the extra matter should be, well, invisible. Zwicky called this invisible matter "dark matter". No one paid attention to Zwicky's and Smith's observations, though.

In the 1960s Vera Rubin observed galaxies through the new and most powerful telescope to date, and she too observed the effect of dark matter in galaxies, with dark matter halo mass being comparable to the visible galactic matter mass. The community paid attention this time, and dark matter is the standard subject in both astronomy and cosmology today.

The problem with dark matter is – it is completely dark, which means it does not interact with the surrounding matter in any way other than gravitationally, and so it represents an exotic form of matter. No one knows how to explain what exactly dark matter is physically.

Rather recently, a diffuse galaxy has been discovered, with no black hole in its centre, that has no dark matter halo [3]. This only makes dark matter more exotic and more mysterious. Namely, some galaxies are surrounded by dark matter, some galaxies are not.

So, there is a cosmological vacuum of tiny energy density, then there is a QFT vacuum of large energy density, and there is dark matter. All three of these are mysterious and separate objects, it seems.

This paper addresses this issue, and puts forward a theory that all of these three mysterious objects, namely a cosmological vacuum, a QFT vacuum and the dark matter, are one and the same object we shall simply call vacuum. The theory presented here predicts the magnitude of the vacuum density, and is compatible with observations in astronomy, cosmology and QFT, when vacuum is modelled in accord with this new theory presented here in this paper.

We start by building a theoretical model, sometimes assuming some results, whenever we deal with objects not known yet to standard physics. We simply try by trial and error to build the working model of the structure of vacuum. So whenever we arrive at a problem that cannot be answered by standard physics, we assume a solution using intuition, carrying on with building the new theoretical model.After the model is constructed, we put it to the test. If the model fits experimental data well – it's a good model then. There are only three parameters guessed in this paper, though. We guess the answers to the questions "how small is the minimal wavelength", "how large is the maximal wavelength" and "how large is the equation of state parameter of vacuum".

With this in mind, we proceed by building the theoretical model of vacuum first, and then we put the finished model to the test.

This paper uses the natural system of units $\hbar = G = C = k_B = 1$. Also, let the subscript V stand for "vacuum", let the subscript S stand for "Schwarzschild", let the subscript M stand for "matter", and let the subscript H stand for "Hubble". So, for instance, ε_{VH} will stand for the energy density of vacuum inside the Hubble horizon, whilst ε_{VS} will stand for the energy density of vacuum inside the Schwarzschild horizon.

2 Black hole vacuum

We start by purely theoretically considering a Schwarzschild black hole. Black holes cannot be observed directly, since they too don't emit any light, similarly to how dark matter behaves. However, black holes have been detected indirectly by observing the movement of stars around invisible black objects. It is believed today that almost every galaxy has a black hole in its centre.

So, a Schwarzschild black hole is a static spherical object of mass m at the centre of a spherical horizon of radius r being equal to $2m$. The Schwarzschild horizon is such that no object from the horizon interior can communicate with its exterior. Not even light can cross the horizon. So no communication is possible between the interior and the exterior of a Schwarzschild horizon.

Consider any particle trapped inside a spherical horizon of radius r. Its wavelength cannot exceed some maximal length, since no particle from the interior of a black hole can be found at the exterior of a black hole. This is so because there is no communication between the interior and the exterior of a black hole. If a particle's wavelength was to be found in both regions about a horizon, this would mean that there is a probability of finding a particle in both regions. Therefore, a particle could interact with both the exterior and the interior of a horizon. However, this is impossible, because there is no communication between regions separated by a horizon. Thus, the probability of a particle trapped in the interior of a horizon being also outside a horizon is zero. In other words, such particle's probability amplitude vanishes identically in the exterior of a horizon.

Consider a particle inside a black hole circling around a horizon of radius r. Its wavelength is obviously $2r\pi$. Can a particle move inside a horizon along a closed orbit with a constant momentum whose length is larger than the length of this circular orbit around a horizon? It seems not. It's hard to imagine a particle moving in any closed orbit with constant impulse other than circulating inside a horizon. So, it seems that the maximal wavelength of a particle in a black hole is $2r\pi$.

So, this way, we arrive at the notion of a maximal wavelength for particles in a black hole. Denote it by λ_{MAX}. For a particle trapped in a Schwarzschild black hole with a horizon of radius r, the maximal wavelength is, it seems,

$$\lambda_{MAX} = 2r\pi \tag{2}$$

The associated minimal energy E_{min} is then calculated as a Compton energy of a particle with wavelength λ_{MAX} as follows.

$$E_{min} = \frac{2\pi}{\lambda_{MAX}} = \frac{1}{r} \tag{3}$$

Consider this now: if a particle was to release all of its energy in a single burst of radiation, the smallest energy a radiated photon could have would be E_{min}.

Can such a bursting particle have energy of, say, $0.5E_{min}$? Obviously not, since this energy is smaller than the minimal energy.

Can such a bursting particle have energy of, say, $1.5E_{min}$? Obviously not, since such a bursting particle could emit a photon of energy E_{min} and another photon of energy $0.5E_{min}$. However, a photon of energy $0.5E_{min}$ does not fit inside a horizon, and so a particle can only have energies E_{min} or $2E_{min}$, but not $0.5E_{min}$ or $1.5E_{min}$.

So, arguing this way, one finds that any particle energy must be a multiple of E_{min}. In other words, if a particle has energy E, then there exists some integer n such that

$$E = nE_{min} \tag{4}$$

Consider a quantum harmonic oscillator trapped inside a Schwarzschild horizon now. Let us assume that particles are described as excitations of such quantum oscillators, as is the case in QFT. Oscillator's energy is given by $E_n = \omega(1/2 + n)$. The measurable energy is the energy above the zero point, since vacuum energy is not measurable directly. Particle energy is the energy that exceeds the zero point energy. So, for a quantum harmonic oscillator, the overall energy E_n^O is given by $E_n^O = \omega(1/2 + n)$, the zero point energy is $\omega/2$, and thus the particle energy E_n is discrete and is given by $E_n = n\omega$. The energy of the first particle state is $E_1 = \omega$. This energy must be a multiple of the minimal energy E_{min}, and so there exists an integer k such that $\omega = kE_{min}$.

So, each oscillator has a ground state of vacuum energy E_{Vk} given by

$$E_{Vk} = \frac{\omega}{2} = \frac{kE_{min}}{2} \tag{5}$$

We notice here that E_{V1} equals $E_{min}/2$ and is smaller than the minimal energy E_{min}. This seems unsettling at first. However, vacuum particles are not experimentally measurable directly. The wavelength of a vacuum particle does not determine its probability of being detected in some region of space, since vacuum cannot be detected at all, at least directly. So, we argue that vacuum particles can have any energy, provided the excitations are quantized as in harmonic oscillator with energy jumps of E_{min}.

So, we continue with Equation (5). We are interested in the final results of this theoretical model.

Since vacuum consists of all of such oscillators, the vacuum energy E_V is therefore

$$E_V = \sum_{k=1}^{N} E_{Vk} = \frac{E_{min}}{2} \sum_{k=1}^{N} k \approx \frac{E_{min}}{4} N^2 \tag{6}$$

We have introduced a cut-off N here that says how large can the energy of a vacuum particle be. The energy E_N with $k = N$ is the largest possible energy E_{MAX}. So, the associated wavelength $\lambda_{min} = 2\pi/E_N$ is the minimal wavelength.

We argue about the cut-off N now. Let l_{Pl} be the Planck length. Set λ_{min} equal to the circumference of a circle of radius of the Planck length, $2l_{Pl}\pi$, since we expect physics to break at the Planck length,

$$\lambda_{min} = 2l_{Pl}\pi \tag{7}$$

This seems a natural choice, and since we do not know much about the physics at the Planck scale, we simply guess that the factor 2π should be included too in the definition $\lambda_{min} = 2l_{Pl}\pi$. We're still working blindly here, trying to construct a working model. However, it does seem plausible that the minimal radius should be the Planck length.

In natural units, one finds $l_{Pl} = 1$, and so one finds

$$E_{MAX} = E_N = NE_{min} = 2\pi/\lambda_{min} = 1 \tag{8}$$

and hence

$$N = 1/E_{min} \tag{9}$$

This way, the vacuum energy given by Eq. (6) becomes

$$E_V = \frac{1}{4E_{min}} = \frac{r}{4} \tag{10}$$

This is exactly one half of the energy of the entire black hole, whose energy is equal to $r/2$.

One can easily interpret this result as the vacuum energy of the underlying vacuum field. All the particles inside a horizon are excitations of this underlying vacuum field. There is no separate vacuum field for each particle; instead, all the particles are excitations of a single vacuum field. Otherwise the vacuum energy of all the particles would be too large and would exceed the energy of the entire black hole. And the vacuum consists of all the possible allowed quantum harmonic oscillators.

So this kind of reasoning puts forward the idea that all the particles are excitations of a single vacuum field, the vacuum being a collection of uncoupled oscillators inside a horizon.

Do notice that in this model vacuum inside a black hole has a mass, contributing to the size of the horizon radius.

3 Universal horizon

We next simply notice that Fridman universe has a horizon, called the Hubble horizon, and it is a sphere beyond which particles move faster than light. Hubble horizon is an apparent horizon; light that was emitted from us at the moment of the creation of the universe could have travelled only thus far.

Now, do notice that the black hole horizon of radius r is the surface which light cannot cross. This single fact sprung all the results of the previous section to life. Noticing that a black hole horizon acts as a wall that prevents any exchange of information was enough to conclude that there is maximal wavelength, which in turn produced all the other results.

For Fridman universes, the Hubble horizon is such a surface that blocks light from crossing it.

So, all the conclusions from the previous section apply here to Fridman universe with a Hubble horizon of radius r.

So, for instance, the minimal energy E_{min} is still given by $E_{min} = 1/r$ and the vacuum energy E_V is still $E_V = r/4$, exactly as with black holes.

Denote the Hubble radius by r_H and the volume enclosed by the Hubble horizon by V_H. We denote the Schwarzschild horizon radius by r_S and the volume enclosed by the Schwarzschild horizon by V_S.

Do notice that the vacuum energy density of a Fridman universe in this model is exactly one half of the critical density ε_{crit} defined by $\varepsilon_{crit} = 3/(8\pi r_h^2)$. This is so because all the conclusions about the Schwarzschild black hole vacuum apply to the Fridman universe inside a Hubble horizon, as argued earlier. This explains why the cosmological vacuum energy density is so small.

So, the vacuum energy density ε_{VH} of a Fridman universe is given by

$$\varepsilon_{VH} = \varepsilon_{crit} = \frac{E_{VH}}{V_H} = \frac{3}{16\pi r_H^2} \tag{11}$$

The vacuum energy density ε_{VS} of a Schwarzschild horizon is given by

$$\varepsilon_{VS} = \frac{E_{VS}}{V_S} = \frac{3}{16\pi r_S^2} \tag{12}$$

Do notice that the Schwarzschild radius r_S is much, much smaller than the Hubble radius r_H. Therefore, the vacuum energy density of a black hole is much, much larger than the vacuum energy density of the surrounding universe around the black hole.

As with black holes, one also finds that all the particles in the universe are persistent excitations of this single unique vacuum field of oscillators located entirely inside the Hubble horizon.

4 Dark matter halos

In the last section we've noticed that vacuum energy density is much larger in the black hole than outside the black hole.

An in-falling observer can cross the black hole horizon without noticing the horizon. There is no horizon as seen by an in-falling observer [4, p.35].

Hence, the vacuum energy density must be smooth in and around a black hole. Otherwise one could imagine that an in-falling observer would notice potential energy differences, and for such an observer vacuum would tend to smooth out if there were jumps in vacuum energy densities. This would be noticed by the distant observer as well. Since the black hole vacuum density cannot decrease, depending solely on a mass of the black hole, this implies a vacuum halo should surround a black hole.

So, arguing this way, one concludes that there must exist a smooth vacuum halo in and around any black hole.

This vacuum halo has energy, and so it produces gravitational effects, because energy density affects metric according to the general theory of relativity (GR). So, a vacuum halo acts gravitationally on the surrounding matter.

Additionally, vacuum does not interact with particles in any other way if the vacuum equation of state parameter w_V is zero.

So, with $w_V = 0$, a vacuum halo around a black hole acts exactly as if it was a dark matter halo: it only acts gravitationally and it only appears around black holes, it seems.

So we propose here that dark matter is nothing other than a vacuum halo around a black hole.

At the first glance, it is obvious that in the vicinity of a black hole, vacuum energy density is comparable to the energy density of the surrounding matter, since a black hole is at least as dense as matter around it. Hence, one expects the vacuum halo to be rather massive itself.

It is hard to guess the exact model for the vacuum halo around a black hole at this point, since currently there's only one boundary condition at hand, namely the continuity of energy density at the black hole horizon, whilst all the current theoretical dark matter halos in literature depend on at least two parameters. One can always adjust one of the parameters appropriately to match the data. This, however, seems to bring no theoretical insight into the formation and the shape of a vacuum halo at this point. However, it is very appealing to contemplate the idea that there is no mysterious dark matter at all, but that quantum vacuum lumps around black holes, forming massive halos.

One notices that a dark halo acts gravitationally, meaning it attracts matter. If the vacuum equation of state parameter w_V was negative, then vacuum halo would repel matter instead of attracting it. Hence, we conclude that it is possible to assume $w_V \geq 0$. Furthermore, if vacuum halo indeed acts as a dark matter halo, then it does not interact with matter in any way other than gravitationally. This suggests $w_V = 0$, so that vacuum exerts no pressure on matter. We test this hypothesis on the cosmological scale in the next section, assuming $w_V = 0$.

But before doing so, we notice that there exists a theoretical solution for a fluid surrounding a black hole [5]. The solution is found using the Rastall version of GR [6]. Rastall's modification of GR is equivalent to GR, so Rastall's theory is not a theory different than GR, as demonstrated in [7]. The solution for density of dust about a black hole in Rastall GR [5, Eqs.(20,21), pp.367-368] reads as follows.

$$\varepsilon_V(r) = \frac{A}{r^B} \tag{13}$$

Here, A and B are constants at our disposal. Since this halo density profile model has two parameters, it can approximately fit any halo. The Keplerian regime starts from the radius where $\varepsilon_V(r)$ is equal to one half of the critical density, which is the vacuum density of the rest of the universe. Hence the vacuum halo mass is naturally cut off and finite. And one expects from continuity that $\varepsilon_V(r_S)$ is equal to the vacuum density inside the central black hole at the horizon. Hence, the vacuum halo is not cuspy, since it begins only where black hole ends.

More research is required to test how this purely theoretical density profile would fare statistically against other, empirical fits.

5 A cosmological model

In the last section we found that vacuum may be described by $w_V = 0$. Hence, the following question springs to mind: "Can one build a cosmological model that describes a universe with accelerated expansion, with a vacuum with $w_V = 0$, that is compatible with the results of SNeIa observations?" The answer is "yes". Actually, a suitable theoretical cosmological mathematical model already exists.

Many authors entertained the idea of non-adiabatic universal expansion, because the energy transfer between the universe and its exterior can produce a universal accelerated expansion. We use some of the results of [8] here.

Authors of [8] considered the cosmological model based on the Fridman metric element. In the model presented in [8], authors assumed that the universe is filled with only dust with $w = 0$. They also assumed that the expansion is not adiabatic. So in their model the first Fridman equation becomes

$$\begin{aligned} \varepsilon_{crit} &= \varepsilon_{matter} + \varepsilon_{transfer} \\ \varepsilon_{matter} &= \varepsilon_{transfer} = \frac{\varepsilon_{crit}}{2} \end{aligned} \tag{14}$$

In their model, transfer energy is the energy that was transferred through the Hubble horizon during the expansion, with parameter $w = 0$.

Do notice that the transferred energy density in this cosmological model from [8] is exactly half of the critical density. This is so in our present model of vacuum as well, since the vacuum energy inside the horizon is exactly half of all of the energy contained inside the horizon, and we shall soon argue that all of it was transferred into the universe through a boundary.

So, the cosmological model presented in [8] seems suitable for the task at hand. It is suitable because, after all the physical assumptions were expressed mathematically, the resulting Fridman equations are exactly the same as in our model. Do notice that physical assumptions in our model are not the physical assumptions authors made in [8]. In [8] authors assumed that there's an entropic force acting on the Hubble horizon, producing non-adiabatic expansion, and thus introducing a surface term $\varepsilon_{transfer}$ in Fridman equations. Consequently, the matter fluid is treated as being expanded non-adiabatically. We shall not assume any of this. We shall assume that matter is the usual cold matter with the usual adiabatic fluid equation with $w_M = 0$. The vacuum fluid shall be treated as the one being expanded non-adiabatically. And we do not introduce a surface term at all. However, when everything is finally mathematically set up, the Fridman equations turn out exactly the same in both our cosmological model and in the cosmological model in [8].

The results predicted by the cosmological model in [8] are in an excellent agreement with the SNeIa observations up to $z = 2$. In other words, the model from [8] also predicts an accelerated expansion.

Hence, there is a cosmological model that is in agreement with both the observations as well as with the vacuum model presented in this paper.

The rest of this section describes in detail the cosmological model presented in [8], suitable for use with the vacuum as described in this paper.

We do adopt a different approach here, though. We know some details now, that are presented in previous chapters of this paper, so we have to assume less than the authors of [8] had to assume.

We start by noticing that the previous sections of this paper demonstrated that both matter and vacuum have the parameter w equal to zero.

Then, we know that the energy inside the Hubble horizon is of the form $E = nE_{min}$.

Then, there is the constant maximal energy E_{MAX} a particle can have, given by $E_{MAX} = 1$.

So, after the Big Bang, the universe starts with energy $E_{MAX} = E_{min}$. Namely, there can be no less energy in the universe than E_{min}, and the universal energy grows with the increasing universe, since, for instance, the universal energy E_H inside the Hubble horizon is given by $E_H = \varepsilon_{crit} V_H = r_H/2$. So, since the energy grows with the universal evolution, the universe started with zero energy, and then jumped to energy E_{min}. Then it jumped to energy $2E_{min}$. And so on.

Where did this energy come from?

As the universe grows, the Hubble sphere fills with energy in jumps. And the energy is such that $w = 0$. Hence, $p = w\varepsilon = 0$. So, with $p = 0$, the first law of thermodynamics $dQ = dE + pdV$ says

$$dQ = dE \tag{15}$$

Here, E is the energy contained within the control volume, and Q is the energy transferred to the control volume through the volume boundary.

In other words, all of the universal energy was transferred through the Hubble horizon, or simply through the entire volume inside the Hubble sphere.

Let us consider this a bit differently.

Consider a control volume filled with some particles of regular cold matter. Particles enclosed in the control volume have some energy E_M. Let the particle collisions be perfectly elastic.

Now enlarge the control volume. What happens with the particles? Well, the number of particles remains intact. And the energy inside the control volume remains the same. So, in this case, $dE_M = 0$, resulting in $dQ_M = 0$, and the cold matter therefore expands adiabatically.

Consider the control volume filled with matter with $w = 0$ now, but such that the energy of matter depends on the size of the control box. Say, let there be a minimal and a maximal wavelength in the control box.

What happens if the control volume is of the size smaller than the minimal wavelength? Well, there are no particles in the control volume then, since any particle should have a wavelength at least as large as the minimum allowed.

Enlarge the control volume now. What happens? A particle is created in the volume now with energy E_{min}. Where did the particle come from? Well, it came through some boundary of the control volume. It came either through the walls of the control box, or it came out of the empty volume of the control box itself. But do notice that the number of particles increased, and so did the energy.

Now enlarge the control volume a bit more. There are two particles in the control volume now and the overall energy is $2E_{min}$ and is larger than the energy with just one particle. Where did this energy come from? Obviously, $dE \neq 0$, so the energy had to come from outside the control volume.

Consider the following. Vacuum particles are created as the universe expands. At the very beginning, there were no vacuum at all. This means that in that epoch of vacuum, there were no particles either, since in this model particles are excitations of vacuum.

So, arguing in this manner, we notice the following.

$$Q = E \tag{16}$$

All of the universal energy entered the Hubble sphere through some boundary. In other words, the universal expansion is completely non-adiabatic.

One calculates energy Q by means of critical density, $Q = \varepsilon_{crit} V$. We notice here that this equation holds true for energy Q enclosed within volume V whatever the volume V may be, since ε_{crit} is assumed constant throughout the entire space, as is standard in cosmology.

So all this leads us to propose the cosmological model of the following form.

$$
\begin{aligned}
H^2 &= \frac{8\pi}{3}\varepsilon_M + \frac{8\pi}{3}\varepsilon_V \\
w_M &= w_V = 0 \\
\varepsilon_M = \varepsilon_V &= \frac{\varepsilon_{crit}}{2} = \frac{3H^2}{16\pi} \\
Q &= \varepsilon_{crit} V
\end{aligned}
\tag{17}
$$

Here, $H = \dot{a}/a$ is the time-dependent Hubble parameter, ε_M is the matter energy density, ε_V is the vacuum energy density, w_M is the matter equation of state parameter, w_V is the vacuum equation of state parameter, ε_{crit} is the critical density, and Q is all of the universal energy, being transferred inside the Hubble horizon.

All that remains to be deduced now is the fluid equation.

One way to produce the fluid equation is to simply group matter and vacuum together into a common energy density $\varepsilon = \varepsilon_M + \varepsilon_V = \varepsilon_{crit}$ and require $E = \varepsilon V$ in equation $Q = E$.

This, however, would produce a single fluid equation for a single fluid of density ε, which is not realistic, since there are two very different fluids present: the cold matter and the vacuum. Cold matter particles don't change their energy nor their particle number when a control volume is varied. On the other hand, vacuum does quite the opposite.

So we separate the fluid equations for matter and for vacuum.

Since matter is the usual cold matter with $w = 0$, we expect that its fluid equation is given by the usual fluid equations for cold baryons,

$$
\begin{aligned}
dE_M &= 0 \\
\dot{\varepsilon}_M + 3H\varepsilon_M &= 0
\end{aligned}
\tag{18}
$$

Hence, with the usual cold matter, the first law of thermodynamics demands

$$
dQ = dE_V \tag{19}
$$

with E_V being the energy of vacuum.

And this is exactly mathematically equivalent to the simple cosmological model presented in [8]. The matter density ρ in [8] should be identified with our vacuum density ε_V. Both ρ and ε_V are equal to $\varepsilon_{crit}/2$, and hence produce the same scale factor $a(t)$ in the Fridman metric, thus producing the same universal expansion.

We demonstrate this further here. Let any fluid of density ε with $w = 0$ be enclosed in a control volume V. The control volume is a sphere of radius r that expands according to the law $r = r_0 a(t)$. Here $a(t)$ is the scale factor from the Fridman metric, and r_0 is a constant co-moving radial distance. Hence,

$$
V = \frac{4\pi}{3} a^3 r_0^3
$$

and thus

$$\dot{V} = 4\pi a^2 \dot{a} r_0^3 = 3HV$$

Since $E = \varepsilon V$, one finds

$$\dot{E} = \dot{\varepsilon} V + \varepsilon 3HV$$

Let the control volume V be the Hubble volume $V_H = 4\pi/(3H^3)$ and let E be the vacuum energy E_V. The total energy Q in this control volume is $Q = \varepsilon_{crit} V_H = 1/(2H)$, and so one finds $\dot{Q} = -\dot{H}/(2H^2)$, and so the first law of thermodynamics $\dot{Q} = \dot{E_V}$ results in

$$\dot{\varepsilon}_V + 3\varepsilon_V H = -\frac{3H\dot{H}}{8\pi} \tag{20}$$

Thus, Eqs. (17) and (20) read

$$\begin{aligned} H^2 &= \frac{8\pi}{3}\varepsilon_V + \frac{1}{2}H^2 \\ \dot{\varepsilon}_V + 3\varepsilon_V H &= -\frac{3H\dot{H}}{8\pi} \end{aligned} \tag{21}$$

These are exactly the same as Eqs. (68) and (70) of the cosmological model [8] with $\gamma = 1/2$ and $w = 0$ as in their simple model that successfully reproduces the SNeIa results.

The acceleration equation is not independent of the last two equations, but is a simple consequence of the two equations in Eq. (21). It reads

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\varepsilon_V + \frac{1}{2}H^2$$

Hence, there is a cosmological model for vacuum with $w = 0$ that reproduces the SNeIa results. We notice [8, p.3, Eq.(19)] that in this model the scale factor a is given by

$$a = a_0 \left(\frac{3}{4} H_0 t\right)^{\frac{4}{3}}$$

and the luminosity distance d_L^V is given by [8, p.4, Eq.(30)]

$$\left(\frac{H_0}{c}\right) d_L^V = 4(1+z)\left[(1+z)^{\frac{1}{4}} - 1\right]$$

with the redshift z defined by $1 + z = a_0/a$. Parameters a_0 and H_0 are the scale factor today and the Hubble parameter today at the present moment.

The ΛCDM model luminosity distance d_L^Λ is given by the formula [8, p.10, Eq.(76)]

$$\left(\frac{H_0}{c}\right) d_L^\Lambda = (1+z)\int_0^z \frac{du}{\sqrt{(1+u)^2(1+\Omega_m u) - u(2+u)\Omega_\Lambda}}$$

with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$.

Figures 1 and 2 compare luminosity distances from the model presented in this paper to the ΛCDM model, over small and large redshifts z.

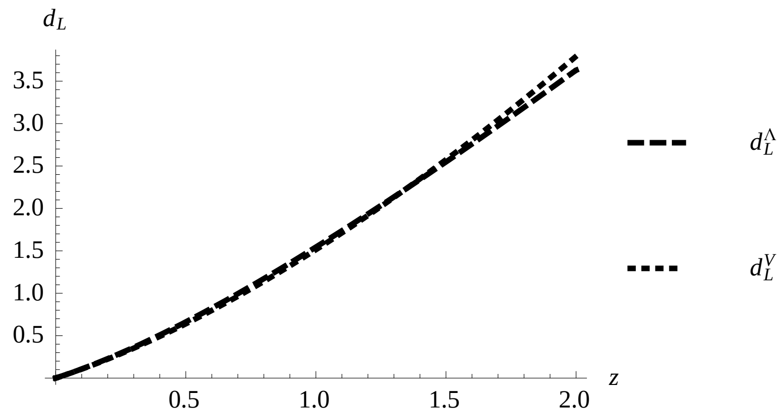

Figure 1: d_L^Λ *is the luminosity distance as calculated in the concordance* Λ*CDM model.* d_L^V *is the luminosity distance from the model presented in this paper.*

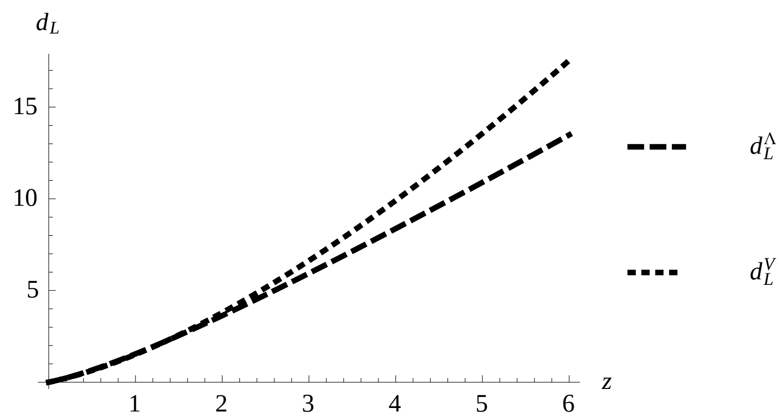

Figure 2: d_L^Λ *is the luminosity distance as calculated in the concordance* Λ*CDM model.* d_L^V *is the luminosity distance from the model presented in this paper.*

6 Higher z comparison

In paper [3] authors developed their very own novel method for measuring distances to distant quasars. They measured distances up to $z = 5.5$, which is a huge improvement with regard to the SNeIa measurements which measured distances up to $z = 2$.

The quasar results suggest that at higher redshifts z the distances should be larger than predicted by the concordance ΛCDM model. Authors of [3] suggest that the vacuum energy density ε_Λ should increase with time. Assuming adiabatic expansion, this means $\dot{\varepsilon}_\Lambda + 3H\varepsilon_\Lambda(1 + w_\Lambda) = 0$ and so, if $\dot{\varepsilon}_\Lambda > 0$, this implies $w_\Lambda < -1$. In other words, if the expansion was adiabatic, then the vacuum energy should be a phantom energy, if the universe was to expand adiabatically.

The ratio of the ΛCDM luminosity distance d_L^Λ and the best fit quasar luminosity distance is printed in Supplementary Figure 5 in [3, Fig.5, p.20]. The figure is reproduced here in Figure 3.

The quasar team used polynomials of the form $P_n = \sum_{k=1}^{n} a_k \log^k(1 + z)$ to model the fitting curve for the luminosity distance to their data.

We notice here that the cosmological model presented in this paper does follow the trend of the fitting model of the quasar team.

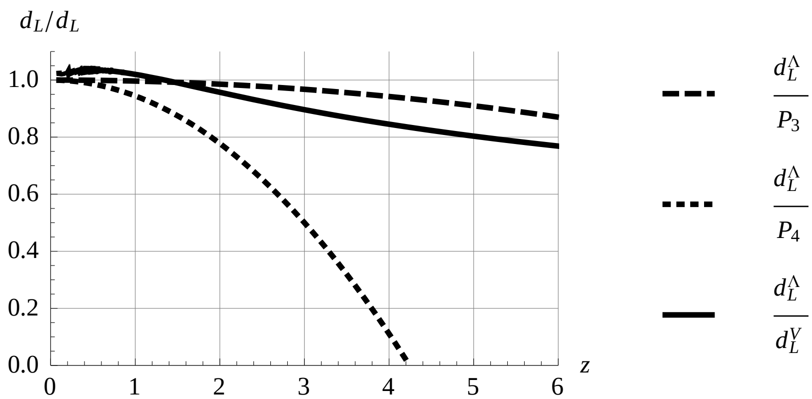

Figure 3: *Ratios* $\frac{d_L^\Lambda}{P_3}$, $\frac{d_L^\Lambda}{P_4}$ *and* $\frac{d_L^\Lambda}{d_L^V}$. d_L^Λ *is the luminosity distance as calculated in the concordance* Λ*CDM model.* P_3 *is the third order polynomial logarithmic fit to the quasar data for the luminosity distance.* P_4 *is the fourth order polynomial logarithmic fit to the quasar data for the luminosity distance.* d_L^V *is the luminosity distance from the model presented in this paper.*

7 Discrete QFT

We simply notice here that, in summary, vacuum itself has quantized energies in a universe with a horizon, which in turn means that any particle has quantized energies. More precisely, if a particle has energy E, then there exists an integer n such that $1 \leq n \leq N$, so that the allowed particle energies are given by $E_n = nE_{min}$. This in turn quantizes impulses p as well, since the on-shell relation for a particle of rest mass m is given by $E_n^2 = p_n^2 + m^2$, and so if there are N energies allowed, then there are only $2N$ impulses allowed, N positive ones and N negative ones. Also, do notice that there exist both the minimal and the maximal wavelength, and so the energy is cut off both above and below. Finally, all particles are completely described by a field enclosed by a horizon.

This means one could build a quantum theory on a finite lattice.

8 Discussion

We notice here briefly that the theory presented here depends on three parameters: the minimal wavelength λ_{min}, the maximal wavelength λ_{MAX}, and the vacuum equation of state parameter w_V. The theory of discrete vacuum presented here is still new and young, and so only one set of values λ_{min}, λ_{MAX} and w_v has been studied here in this paper. Different values of these three parameters may predict different results in different models. We notice that the magnitudes of all three of these parameters are un-known to the present day physics, so your guess and your model is as good as anyone's thus far. It is interesting to notice though that in general a discrete vacuum inside a horizon could explain the smallness of cosmological vacuum, could make quantum vacuum be exactly the same as the cosmological vacuum, and could even be the dark matter. More research is required, obviously. However, these three possibilities are fairly obvious even at this early stage of this vacuum model.

9 Conclusions

This paper presents a model of a quantum vacuum.

Vacuum is assumed to be a collection of un-coupled quantum harmonic oscillators, all located within a horizon, whose position is otherwise completely unknown.

These vacuum oscillators are confined within a horizon. Indeed, our universe itself is inside the Hubble horizon, for instance. Also, black holes have horizons. So, this paper puts forward a proposition that vacuum is realistically confined inside a horizon.

Assuming a horizon is such that no communication is possible between the regions inside and outside a horizon, one could argue that there must exist both maximal and minimal wavelength any particle can have inside a horizon. The existence of a maximal wavelength then suggests that all energy is discrete and comes in lumps of this minimal energy. The maximal wavelength associated with the minimal energy depends on the size of the horizon.

When applied to a black hole interior, the theory presented in this paper predicts that the vacuum density inside a black hole is much larger than the vacuum density outside a black hole. From continuity one then concludes that there could exist a halo of vacuum particles around any black hole. This immediately suggests that dark matter may not exist at all, but is instead a vacuum halo around black holes. This conclusion is further strengthened by the fact that there exist diffuse galaxies without the central black hole that are also without any dark matter.

Then, since our universe has an apparent horizon, the Hubble horizon, we inspect the effect of such vacuum on universal expansion. The result calculated here agrees with both observations from supernovae and observations of distant quasars. Hence, the model of vacuum presented here could explain the cosmological vacuum properties as well.

Since this paper assumed that particles are discrete excitations of this discrete vacuum inside a horizon, one is also led to conclude that the vacuum presented here can explain the properties of the vacuum of the quantum field theories, without any singularities, if one builds a quantum theory on a suitable finite lattice.

Hence, the model of vacuum presented here may describe dark matter, cosmological vacuum as well as quantum vacuum, suggesting all of these are just one object we simply call vacuum.

Both galactic and cosmological observations are very hard to perform, with the results with large deviations. This is why this paper cannot claim that the presented model of vacuum predicts all the phenomena about dark matter and universal expansion precisely and correctly. The experimental data does not allow a high-precision fitting. Hence it is hoped that this paper may inspire researchers to test this model of vacuum over large data sets and through new experiments, thus improving the precision and improving the theoretical models used to describe such discrete vacuum.

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