Refutation of remainder sets for paraconsistent revisions

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Abstract: Two definitions for expansion, remainder, and selection of K functions are *not* tautologous. Two definitions implication and paraconsistent/weak negation operators are *not* tautologous. These refute remainder sets and paraconsistent valuations of logic **mbC**, an extension of **CPL+**. Therefore these conjectures are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Resta, R.; Fermé, E.; Garapa, M.; Reis M. (2018).
How to construct remainder sets for paraconsistent revisions: preliminary report.
no emails proffered.
academia.edu/attachments/58978670/download_file?st=MTU1NjM2NDU2OCw3NS43MS4xNjEuMTQ2LDc2MDk1MzU4&s=swp-toolbar&ct=MTU1NjM2NDU2OSwxNTU2MzY0NTg2LDc2MDk1MzU4

Remark 0: The AGM model of belief systems is named for Alchourrón, Gärdenfors, and Makinson (1985). Axioms are supposed to be a subset of classical logic CPL+.

Formally we have the following:
Definition 1. *The expansion of K by α (K + α) is given by K + α = Cn*(*K*∪{*α*}) (D1.1)

LET p, q, r, s: *α* , *C*, *K*, *n*.
(r+p)=((q&s)&(r&p)) ; `T`**F**`T`**F** **FFFF** `T`**F**`T`**F** **FFF**`T` (D1.2)

Definition 2 (Remainder)**.** *The set of all the maximal subsets of K that do not entail α is called the* remainder set *of K by α and is denoted by K⊥α, that is, K′ ∈ K⊥α iff:*
(i) *K′ ⊆ K.*
(ii) *α ∉ Cn*(*K′*).
(iii) *If K′ ⊂ K″ ⊆ K then α ∈ Cn*(*K″*). (D2.1)

LET p, q, r, s, t, u: α ,C, K, n, K' , K'' .
((~(u<t)&~(p<((q&r)&t)))&(~(r<(t<u))>(p<((q&s)&u))))>(t<(r@p)) ;

TTTT T**F**T**F** TTTT T**F**T**F**(1), TTTT TTTT TTTT TTTT(2),
TTTT T**F**T**F** TTTT T**F**T**F**(2), TTTT TTTT TTTT TTTT(2),
TTTT T**F**T**F** TTTT T**F**T**F**(1) (D2.2)

Definition 3 (selection function)**.** *A selection function for K is a function γ such that, for every α:*
1. $\gamma(K\perp\alpha) \subseteq K\perp\alpha$ *if* $K\perp \neq \varnothing$.
2. $\gamma(K\perp\alpha) = \{K\}$ *otherwise*. (D3.1)

LET p, q, r: α, γ, K.
(((r@#p)@(s&s))>~((r@#p)<(q&(r@#p))))+((q&(r@#p))=r) ;
TTTT **F**NTT TTTT TTTT (D3.2)

Definition 13 (Valuations for **mbC** (Carnielli and Coniglio 2016))**.** *A function* $v : \mathbb{L} \rightarrow \{0,1\}$ *is a* valuation for **mbC** *if it satisfies the following clauses*:

($v \rightarrow$) $v(\alpha \rightarrow \beta) = 1 \Longleftrightarrow v(\alpha) = 0$ *or* $v(\beta) = 1$ (Implication) (D13.3.1)

LET p, q, r: α , β , v .
((r&(p>q))=(s=s))=(((r&p)=(s@s))+((r&q)=(s=s))) ;
FFFF TTTT **FFFF** TTTT (D13.3.2)

($v\neg$) $v(\neg\alpha) = 0 \Rightarrow v(\alpha) = 1$ (Paraconsistent/Weak negation) (D13.4.1)

((r&~p)=(s@s))>((r&p)=(s=s)) ; **FFFF** TTTT **FFFF** TTTT (D13.4.2)

Remark 13: Defs. 13.3.2 and 13.4.2 are equivalent.

The Defs. D1.2-3.2 as rendered are *not* tautologous for expansion, remainder, and selection of K functions. The Defs. 13.3.2-13.4.2 are *not* tautologous for implication and paraconsistent/weak negation operators. These refute remainder sets and paraconsistent valuations of logic **mbC**, an extension of **CPL+**.