Science Front Publishers *Journal for Foundations and Applications of Physics*, **vol. 6, No. 2 (2019)**
(sciencefront.org) **ISSN 2394-3688**

"Spooky" interaction and non-classical interference interpreted by a product of classical electric field

Kazufumi Sakai

TPORD (Optical Science)
Hoei-Haitu 1-11, Yabu-machi 74-11
Tosu-city, SAGA 841-0055 JAPAN

**Corresponding author E-mail*: ksakai@phj.sakura.ne.jp

(Received 28 February 2019, Accepted 13 April 2019, Published 01 May 2019)

Abstract

Many experiments to verify nonlocal interaction and non-classical phenomena using entangled lights were conducted in the 1980s, and many physicists were interested in their unrecognizable correlation. These quantum mechanical effects were used in Aspect's experiments and Bell tests and had a great influence on the interpretation of quantum mechanics. However, their essence, including their "spooky" interaction, is unknown. In this study, we show that entangled light can be expressed by the product of electric fields and that the same result as quantum mechanics can be obtained using the product form.

Keywords**:** Entangled state, Nonlocal interaction, Simultaneous measurement, Classical theory, Product of electric field

1. Introduction

Hanbury Brown and Twiss proposed intensity interference as an alternative to Michelson's interferometer [1-4]. As a result, it became clear that intensity measurement measures the (second-order) interference of the electric field similar to Michelson's interferometer. The concept of coherence was then extended, and higher-order coherence and higher-order correlations became considered. If Young's interference experiment and Michelson's interferometer are called secondary interference of the electric field, the intensity interference of the Hanbury Brown–Twiss type can be considered the fourth-order interference of the electric field.

Experiments on non-classical quantum effects using two-photon fourth-order interference experiments and experiments on the verification of quantum mechanics have been actively conducted. In these experiments, lights in two-photon states that are correlated with each other similar to parametric fluorescence are used as incident light. Mandel et al. conducted a series of experiments, such as 100% visibility in a two-photon simultaneous measurement [5-8], vacuum effects on interference in a two-photon down conversion [9], and violation of Bell's inequality and classical probability [10] by a two-photon interference using parametric down-conversion photons. Many related studies subsequently began to be published [11-15]. The results of these experiments were established as phenomena peculiar to quantum mechanics that were not classically explained and suggested the intuitively unintelligible correlation.

We conducted experiments on the duality (particle and wave behavior) and the collapse of wave packet [16,17] but could not observe the collapse of the wave packet (nonlocal interaction) contrary to the experiment of Aspect and Mandel et al. [17]. We focused on the Bell test experiment (or Aspect's experiment) and examined whether it could be explained by classical theory and found that the same result could be obtained in classical theory by expressing entangled light by the product of the electric field (not sum) [18]. Moreover, the probability of simultaneous measurement treated as fourth-order interference could be expressed by the second-order interference of the pump light.

Using this product of the electric field, we reconsidered the experimental results in non-classical phenomena and nonlocal correlation using entangled light and showed that the same results as quantum theory could be obtained.

2. Classical interpretation of experiments claiming to be non-classical phenomena

Light waves in electromagnetism and quantum mechanics are given by the following formulas:

$$E(\boldsymbol{r},t) = iK\left[a\,e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t} - a^{*}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t}\right] \tag{1}$$

$$\hat{E}(\boldsymbol{r},t) = iK\left[\hat{a}\,e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t} - \hat{a}^{\dagger}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t}\right] \tag{2}$$

where K is the coefficient, a is the amplitude, $\boldsymbol{k}$ is the wave number vector, $\boldsymbol{r}$ is the position vector, and ω is the angular frequency. When Eq. (1) is expressed by the product of two electric fields, the following equation is obtained:

$$E(\boldsymbol{r},t) = iK\left[a_s a_i\, e^{i\boldsymbol{k_s}\cdot\boldsymbol{r}-i\omega_s t}e^{i\boldsymbol{k_i}\cdot\boldsymbol{r}-i\omega_i t} - a_s^{*}a_i^{*}e^{-i\boldsymbol{k_s}\cdot\boldsymbol{r}+i\omega_s t}e^{-i\boldsymbol{k}_i\cdot\boldsymbol{r}+i\omega_i t}\right] \tag{3}$$

where a_s, a_i, k_s, k_i, ω_s, and ω_i are the amplitudes, wave numbers, and angular frequencies, respectively. When Eq. (3) is represented by a q-number,

$$\hat{E}(\boldsymbol{r},t) = iK\left[\hat{a}_s\hat{a}_i\, e^{i\boldsymbol{k_s}\cdot\boldsymbol{r}-i\omega_s t}e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}-i\omega_i t} - \hat{a}_s^{\dagger}\hat{a}_i^{\dagger}e^{-i\boldsymbol{k_s}\cdot\boldsymbol{r}+i\omega_s t}e^{-i\boldsymbol{k}_i\cdot\boldsymbol{r}+i\omega_i t}\right]\ . \tag{4}$$

When Eq. (4) is applied to the vacuum field $|0_s,0_i\rangle_{vac}$,

$$\begin{aligned}\hat{E}(\boldsymbol{r},t)\left|0_s,0_i\right\rangle_{vac} &= iK\left[\hat{a}_s\hat{a}_i\, e^{i\boldsymbol{k_s}\cdot\boldsymbol{r}-i\omega_s t}e^{i\boldsymbol{k_i}\cdot\boldsymbol{r}-i\omega_i t} - \hat{a}_s^{\dagger}\hat{a}_i^{\dagger}e^{-i\boldsymbol{k_s}\cdot\boldsymbol{r}+i\omega_s t}e^{-i\boldsymbol{k}_i\cdot\boldsymbol{r}+i\omega_i t}\right]\left|0_s,0_i\right\rangle_{vac} \\ &= -iKe^{-i\boldsymbol{k_s}\cdot\boldsymbol{r}+i\omega_s t}e^{-i\boldsymbol{k}_i\cdot\boldsymbol{r}+i\omega_i t}\left|1_s,1_i\right\rangle\ .\end{aligned} \tag{5}$$

If the coefficients are omitted and r = 0 and t = 0, an expression of entangled light generation from the vacuum field is obtained.

$$\hat{a}_s^{\dagger}\hat{a}_i^{\dagger}\left|0_s,0_i\right\rangle_{vac}=\left|1_s,1_i\right\rangle \tag{6}$$

Therefore, the entangled light in electromagnetism can be considered to correspond to Eq. (3) and to the product of the electric field. The relationship between the positive frequency part of the pump light and the entangled light is

$$ae^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}=a_s a_i\, e^{i\mathbf{k_s}\cdot\mathbf{r}-i\omega_s t}e^{i\mathbf{k_i}\cdot\mathbf{r}-i\omega_i t} \tag{7}$$

from Eqs. (1) and (3). Subscripts *s* and *i* represent the signal and the idler, respectively. The conservation law of energy and momentum is satisfied. Using the product of the electric field, the probability of simultaneous measurement can be expressed by the following equation:

$$\left|E(\boldsymbol{r},t)\right|^2=K^2\left(a_s e^{i\mathbf{k_s}\cdot\boldsymbol{r}_s-i\omega_s t}a_i e^{i\mathbf{k}_i\cdot\boldsymbol{r}_i-\omega_i t}\right)^*\left(a_s e^{i\mathbf{k_s}\cdot\boldsymbol{r}_s-i\omega_s t}a_i e^{i\mathbf{k}_i\cdot\boldsymbol{r}_i-\omega_i t}\right) \tag{8}$$

Equation (8) represents the probability of the fourth-order interference of the signal and the idler light and the probability of the second-order interference of pump light. Simultaneous measurement is based on the assumption that the signal and the idler light are measured at the same time *t*. For example, when the idler light is measured after sufficient time has elapsed from the time when the signal light was measured, a_s becomes zero and the entangled state disappears. However, even in this case, the idler light does not disappear (collapse). The idler light exists even when a_s becomes zero and exists as a single wave packet; we consider this phenomenon to be related to the collapse of wave packets.

In the next section, the experimental results showing the quantum mechanical effects using entangled light emitted by spontaneous parametric down-conversion (SPDC) are interpreted using the product of the (classical) electric field.

2. 1 Nonlocal interference in separated photon channels

Ou, Zou, Wang, and Mandel [19] and Franson [11] showed a correlation between the signal and the idler light even when these lights do not overlap. Similar experiments were also reported by Kwiat and Varcka [14]. Ou et al. attempted to simultaneously detect the signal photon *s* and the idler photon *i* at different positions, as shown in Fig. 1 (a). Each photon can take two ways of a short optical path and a long optical path to each detector. The difference *L* between the two optical path lengths is equal in the optical path of *s* and the optical path of *i* and is much larger than the length of the wave packet $l = c / \Delta\omega$ ($\Delta\omega$ is the bandwidth). Simultaneous measurement interferes with the possibility of coming to the detector through a short optical path and a long optical path. When *L* is changed, the probability of simultaneous measurement increases and decreases sinusoidally, and 100% visibility is obtained in the calculation of quantum mechanics. Figure 1 (b) is an experimental apparatus based on Kwait and Varcka, and it shows that it has a visibility above 50% and is a quantum mechanical effect that cannot be explained by classical theory. Simultaneous measurement of photons is conducted at two spatially separated points (longer than the length of the wave packet), and thus it can be considered a typical example of breaking Bell's inequality.

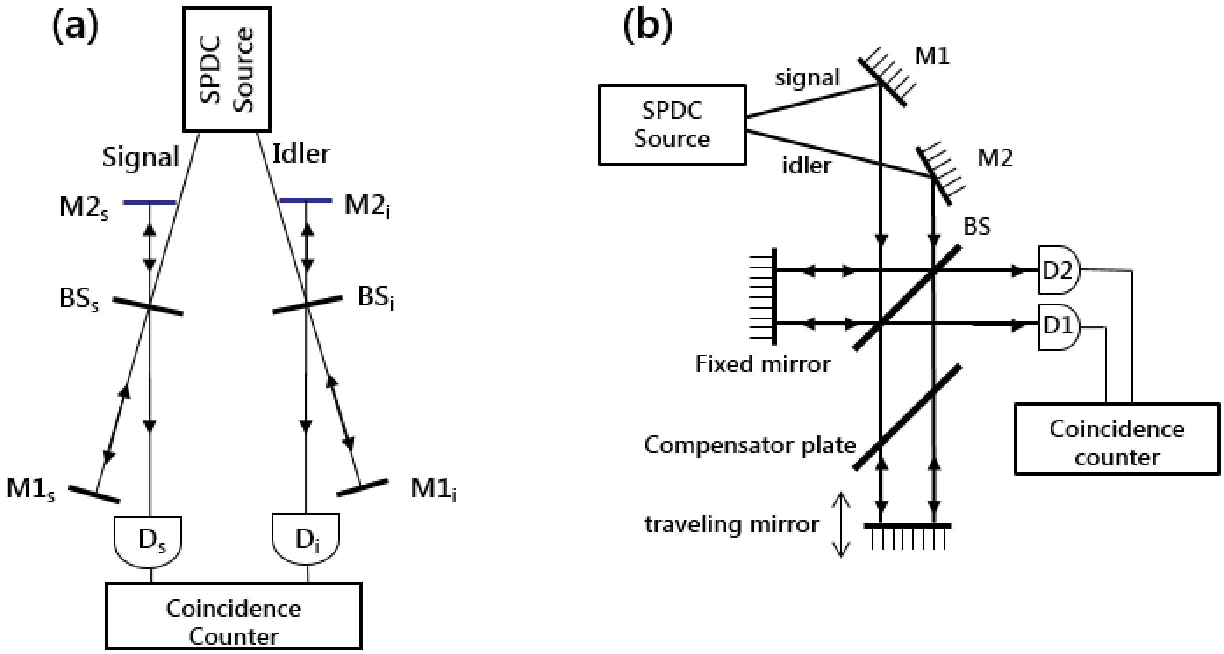

Fig. 1. Correlation of interference between the signal light and the idler light. Simultaneous measurement is performed by moving the mirrors M_{1i} and M_{1s} of (a) or the movable mirror of (b) to change the length of the long optical path. As the optical path length difference between the long optical path and the short optical path is 3 cm and the length of the wave packet is 5×10^{-5} m, the wave packet never overlaps.

This experiment can be explained by the same method [18] that interpreted the experiment of Aspect by the product of the electric field. When light is expressed by the product of the electric field (positive frequency part), the following equation is obtained:

$$\begin{aligned} E^{(+)}(\mathrm{D_s},\mathrm{D_i}) &\propto \left[a_s e^{ik_s r_1 - i\omega_s t} + a_s e^{ik_s r_2 - i\omega_s t}\right]\left[a_i e^{ik_i r_1 - i\omega_i t} + a_i e^{ik_i r_2 - i\omega_i t}\right] \\ &= \left[a_s e^{ik_s r_1 - i\omega_s t} + a_s e^{ik_s r_1 + ik_s L - i\omega_s t}\right]\left[a_i e^{ik_i r_1 - i\omega_i t} + a_i e^{ik_i r_1 + ik_i L - i\omega_i t}\right] \\ &= e^{-i(\omega_s+\omega_i)t}\left[a_s e^{ik_s r_1} + a_s e^{ik_s r_1 + ik_s L}\right]\left[a_i e^{ik_i r_1} + a_i e^{ik_i r_1 + ik_i L}\right] \end{aligned} \quad (9)$$

where r_1 and r_2 are the optical path lengths of the short and long path, and L is their optical path length difference. The reflectance and the transmittance by the beam splitter BS are assumed to be equal, and the phase change due to the reflection is omitted because it is the same number of times in the short and the long optical paths. The left parenthesis represents the signal light and is the sum of the two light waves with different optical path lengths. The right parenthesis similarly represents the idler light. As the difference between the optical path lengths of the short and long optical paths is longer than the length of the wave packet, the product of the term including r_1 and the term including r_2 is zero. Therefore, Eq. (9) becomes

$$E^{(+)}(\mathrm{D_s},\mathrm{D_i}) \propto a_s a_i e^{-i(\omega_s+\omega_i)t}\left(e^{ik_s r_1}e^{ik_i r_1} + e^{ik_s r_1 + ik_s L}e^{ik_i r_1 + ik_i L}\right) \; . \tag{10}$$

The probability of simultaneous measurement is given by the following equation:

$$P'_c(\mathrm{D_s},\mathrm{D_i}) \propto 2\left|a_s a_i\right|^2\left(1+\cos[(k_s+k_i)L]\right) \tag{11}$$

Therefore, 100% visibility is obtained. Ou and colleagues showed that the probability of simultaneous measurement appears as cos(kL_i) even if only L_i of the idler photon is changed, and it was verified by an experiment. When L in Eq. (10) is rewritten as L_i (idler light) and L_s (signal light), the following equation is obtained:

$$P'_c(\mathrm{D_s},\mathrm{D_i}) \propto 2\left|a_s a_i\right|^2\left(1+\cos[k_s L_s + k_i L_i]\right) \tag{12}$$

When $k_s L_s$ is constant, it varies with cos ($k_i L_i$+*const*).

2-2. Non-classical fourth-order interference

The experiments conducted by Ou et al. in 1990 were a clear contrast to classical theory [20]. Whereas interference does not occur classically, quantum theory argues that interference fringes with a visibility of 100% can be obtained. When the signal and the idler photons generated in the SPDC are incident on the Mach–Zehnder interferometer, as shown in Fig. 2, the probability P_q(4,5)

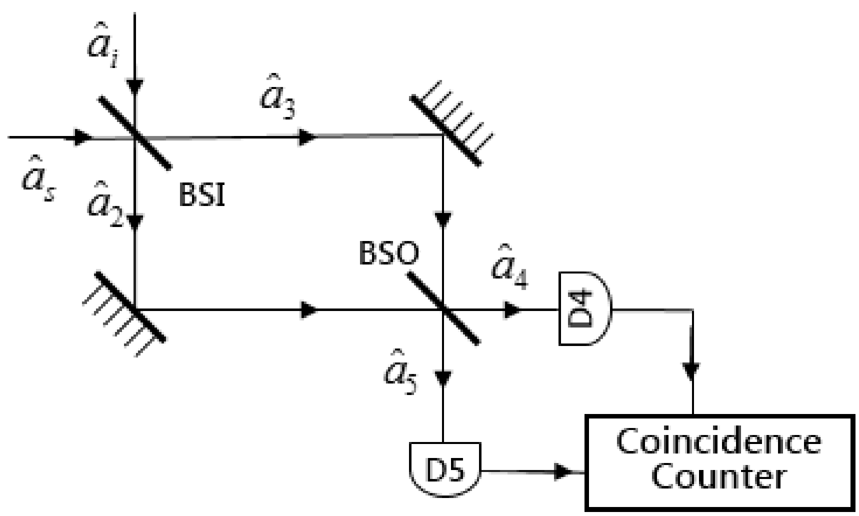

Fig. 2. Interference experiments caused by the entanglement of the two-photon state. When the signal light *s* and the idler light *i* are input from the upper left, the beam splitter BSI creates an entangled state. Simultaneously measurement is conducted by moving the beam splitter BSO and changing the optical path length of the upward and downward turns [20].

of the simultaneous measurement of the photons at the two exits (detectors D4 and D5) varies according to the difference between the two optical path lengths of the interferometer (adjustment of BSO position), as shown in Eq. (13) [20].

$$P_q(4,5) \propto 1 + \cos 2(\phi_2 - \phi_3) \tag{13}$$

ϕ_2 and ϕ_3 are the phase changes due to their respective optical paths.

When this system is expressed by the product of the electric field (assuming that the wave number and the angular frequency of the signal and the idler light are equal), the following equation is obtained:

$$\begin{aligned} E^{(+)}(r_4, r_5) \propto & \left(a_i \, \mathrm{e}^{ikr_i} \cdot a_s \, \mathrm{e}^{ikr_s}\right) \\ & \to \left(a_{i2} \, \mathrm{e}^{ikr_1 + i\phi_2} + ia_{i3} \, \mathrm{e}^{ikr_1 + i\phi_3}\right)\left(ia_{s2} \, \mathrm{e}^{ikr_1 + i\phi_2} + a_{s3} \, \mathrm{e}^{ikr_1 + i\phi_3}\right) \\ & \to \left[\left(a_{i4} \, \mathrm{e}^{ikr_2 + i\phi_2} + ia_{i5} \, \mathrm{e}^{ikr_2 + i\phi_2}\right) + i\left(ia_{i4} \, \mathrm{e}^{ikr_2 + i\phi_3} + a_{i5} \, \mathrm{e}^{ikr_2 + i\phi_3}\right)\right] \\ & \quad \times \left[i\left(a_{s4} \, \mathrm{e}^{ikr_2 + i\phi_2} + ia_{s5} \, \mathrm{e}^{ikr_2 + i\phi_2}\right) + \left(ia_{s4} \, \mathrm{e}^{ikr_2 + i\phi_3} + a_{s5} \, \mathrm{e}^{ikr_2 + i\phi_3}\right)\right] \end{aligned} \tag{14}$$

where k is the wave number of the signal and the idler light; r_i, r_s, r_1, and r_2 are the optical path lengths when phases ϕ_2 and ϕ_3 are ignored; and a, a_{i2}, a_{s2}, a_{i4}, and a_{s4} are the amplitude of the electric field. The term ωt is omitted for simultaneous measurement. The first row of Eq. (14) is the product of the incident electric field of SPDC light, the second row is the product of the electric field reflected and passed through the beam splitter BSI, and the third row is the product of the electric field reflected and passed through the beam splitter BSO. Here, assuming that $a_{i4} = a_{i5} = a_{s4} = a_{s5} = a$,

$$E^{(+)}(r_4, r_5) \propto -2a^2 \, \mathrm{e}^{i2kr_2} \cdot (\mathrm{e}^{2i\phi_2} + \mathrm{e}^{2i\phi_3}) \ . \tag{15}$$

Therefore, the probability of simultaneous measurement is given by the following equation:

$$\begin{aligned} P'_{\mathrm{c}}(r_4, r_5) &= \left|E^{(+)}(r_4, r_5)\right|^2 \\ &\propto a^4 (1 + \cos(2\phi_2 - 2\phi_3)) \end{aligned} \tag{16}$$

This result agrees with Eq. (13), and the visibility is 100%.

3. Conclusion

As explained above, nonlocal and non-classical interference experiments can be classically interpreted using the product form. Entangled light is considered a special classical electromagnetic wave that can be expressed by the product of electric fields. “Spooky” nonlocal correlation is a phenomenon that is intuitively difficult to understand, but this classical interpretation can be taken as a realistic phenomenon. As described in section 2, if a difference in the measurement time exists between the signal and the idler light, the entangled state disappears by one measurement but the other light does not disappear. We conducted an experiment to confirm the collapse of wave packets using the fourth-order interference and could not confirm the phenomenon (collapse) [18]. Therefore, we consider the collapse of the entangled state and the collapse of the wave packet to be related. To understand the meaning of the product of the electric field, we plan to continue this research in the future.

REFERENCES

[1] R. Hanburry Brown and R. Q. Twiss: "Correlation between Photons in two Coherent Beams of Light", *Nature*, **177**, 27 (1956).

[2] G. A. Rebka and R. V. Pound: "Time-Correlated Photons", *Nature*, **180**, 1035 (1957).

[3] R. J. Glauber: "Quantum Optics and Electronics", ed. C. DeWitt, A. Blandin and C. Cohen-Tannoudji (Gordon and Breach, New York, 1955) p. 63.

[4] R. Loudon: "The Quantum Theory of Light" (Oxford, 1973) p. 111.

[5] R. Ghosh and L. Mandel: "Observation of nonclassical effects in the interference of two photons", *Phys. Rev. Lett.* **59**, 1903(1987).

[6] C. K. Hong, Z. Y. Ou and L. Mandel: "Measurement of subpicosecond time intervals between two photons by interference", *Phys. Rev. Lett*., **59**, 2044 (1987).

[7] Z. Y. Ou and L. Mandel: "Observation of Spatial Quantum Beating with Separated Photodetectors", *Phys. Rev. Lett*. **61**, 54 (1988).

[8] Z. Y. Ou and L. Mandel: "Further evidence of nonclassical behavior in optical interference", *Phys. Rev. Lett.* **62**, 2941 (1989).

[9] Z. Y. Ou, L. J. Wang and L. Mandel: "Vacuum effects on interference in two-photon down conversion", *Phys. Rev. A* **40**, 1428 (1989).

[10] Z. Y. Ou and L. Mandel: "Violation of Bell's Inequality and Classical Probability in a Two-Photon Correlation Experiment", *Phys. Rev. Lett.* **61**, 50 (1988).

[11] J. D. Franson: "Bell inequality for position and time", *Phys. Rev. Lett.* **62**, 2205 (1989).

[12] P. Grangier, M. J. Potasck and B. Yurke: "Probing the phase coherence of parametrically generated photon pairs: A new test of Bell's inequalitie"s, *Phys. Rev. A* **38**, 3132 (1988) .

[13] M. A. Horne, A. Svhimony and A. Zeilinger: "Two-particle interferometry", *Phys. Rev. Lett.* **62**, 2209 (1989).

[14] P. G. Kwiat, W. A. Vareka, C. K. Hong, H. Nathel and R. Y. Chiao: "Correlated two-photon interference in a dual-beam Michelson interferometer", *Phys. Rev. A* **41**, 2910 (1990).

[15] J. G. Rarity and P. R. Tapster: "Experimental violation of Bell's inequality based on phase and momentum", *Phys. Rev. Lett*. **64**, 2495 (1990).

[16] K. Sakai, "Simultaneous measurement of wave and particle properties using modified Young's double-slit experiment", *Journal for Foundations and Applications of Physics* **5**, 49 (2018).

[17] K. Sakai, "Experimental verification of wave packet collapse using fourth-order interference", *Journal for Foundations and Applications of Physics* **5**, 216 (2018).

[18] K. Sakai, “Comment on Aspect's experiment: classical interpretation”, *Journal for Foundations and Applications of Physics* **6**, 11 (2019).

[19] Z. Y. Ou, X. Y. Zou, L. J. Wang and L. Mandel: “Observation of nonlocal interference in separated photon channels”, *Phys Rev Lett.* **65**, 321 (1990).

[20] Z. Y. Ou, X. Y. Zou, L. J. Wang and L. Mandel: “Experiment on nonclassical fourth-order interference”, *Phys. Rev. A* **42**, 2957 (1990)