Refutation of overlap algebras constructively to prove complete Boolean algebras

Abstract: We evaluate four equations which are *not* tautologous, but in fact produce the equivalent logic table values result. This means that the stated problem of applying singletons to the powerset is equivalent to proving singletons are atoms and that every subset satisfying a singleton is also an atom. Hence, overlap algebras do not constructively prove complete Boolean algebras. Therefore that conjecture for intuitionistic logic forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ciraulo, F.; Contente, M. (2019) .
Overlap algebras: a constructive look at complete Boolean algebras.
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The problem of finding a constructive characterization of powersets is related to the problem of finding a suitable algebraization of the notion of a singleton. Apparently, none of the first-order (in the sense of the language of lattices) attempts to define
is an atom is satisfactory from an intuitionistic point of view; consider, for instance, the following.

$$a \neq 0 \wedge (\forall x \in L)(x \neq 0 \wedge x \leq a \Rightarrow x = a) \quad (1.1.1)$$

(p<q)>((p@(p=p))&((#r<q)&(((r@(r=r))&~(p<r))>(r=p)))) ;
TFTT TFTT TFTT TFTT (1.1.2)

$$a \neq 0 \wedge (\forall x \in L)(x \leq a \Rightarrow x = 0 \vee x = a) \quad (1.2.1)$$

(p<q)>((p@(p=p))&((#r<q)&(~(p<r)>(r=((r@r)+(r=p)))))) ;
TFTT TFTT TFTT TFTT (1.2.2)

$$a \neq 0 \wedge (\forall x \in L)(x < a \Rightarrow x = 0) \quad (1.3.1)$$

(p<q)>((p@(p=p))&((#r<q)&((r<p)>(r=(r@r))))) ;
TFTT TFTT TFTT TFTT (1.3.2)

$$a \neq 0 \wedge \neg (\exists x \in L)(x \neq 0 \wedge x < a) \quad (1.4.1)$$

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(p<q)>((p@(p=p))&(~(%r<q)&((r@(r=r))&(r<p)))) ;
                                     TFTT TFTT TFTT TFTT            (1.4.2)
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> Indeed, when applied to the case L = Pow(X), singletons cannot be proven to be atoms in the sense of (1.1) or (1.2), and it is impossible to prove that every subset satisfying (1.3) or (1.4) is a singleton, although a singleton satisfies (1.3) and (1.4).

Eqs. 1.1.2-1.4.2 are *not* tautologous, but in fact produce the equivalent logic table values result. This means that the stated problem of applying singletons to the powerset is equivalent to proving singletons are atoms and that every subset satisfying a singleton is also an atom. Hence, overlap algebras do not constructively prove complete Boolean algebras.