Refutation of non Sahlqvist formulas by three counter examples

Abstract: We evaluate three equations as examples of non Sahlqvist formulas. None is tautologous. What follows is that Fine's theorem and monotonic modal logic are refuted. Therefore those conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Sahlqvist_formula

Examples of three non-Sahlqvist formulas:

1. The *McKinsey formula* does not have a first-order frame condition.

□◊p → ◊□p (1.1)

#%p>%#p ; `NNNN NNNN NNNN NNNN` (1.2)

2. The *Löb axiom* does not have a first-order frame condition.

□(□p → p) → □p (2.1)

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (2.2)

3. The conjunction of the McKinsey formula and the [modal] (4) axiom has a first-order frame condition … but is not equivalent to any Sahlqvist formula.

(□◊p → ◊□p) ∧ (◊◊q → ◊q) (3.1)

(#%p>%#p)&(%%q>%q) ; `NNNN NNNN NNNN NNNN` (3.2)

Eqs. 1.2-3.2 are *not* tautologous and refute the conjecture of *non* Sahlqvist formulas as tautologous. What follows is that Fine's theorem and monotonic modal logic are also refuted.