Refutation of Presburger arithmetic via Axiom 2

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Abstract: In Presburger arithmetic, Axiom 2 as \( x+1 = y+1 \rightarrow x=y \) is not tautologous. Therefore Presburger arithmetic is a non tautologous fragment of the universal logic \( \mathbb{V}_4 \).

We assume the method and apparatus of Meth8/\( \mathbb{V}_4 \) with \( \text{Tautology} \) as the designated proof value, \( \text{F} \) as contradiction, \( \text{N} \) as truthity (non-contingency), and \( \text{C} \) as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

From: en.wikipedia.org/wiki/Presburger_arithmetic

The language of Presburger arithmetic contains constants 0 and 1 and a binary function +, interpreted as addition. In this language, the axioms of Presburger arithmetic are the universal closures of the following:

[Axiom] 2. \[ x+1 = y+1 \rightarrow x=y \] \hspace{1cm} (2.1)

\[
\text{LET} \quad p, q; \quad x, y; \quad (%r>r) \ 1; \ (r=r) \ \text{T}
\]

\[
((p+(%r>r))=(q+(%r>r)))->(p=q) ; \hspace{1cm} \text{TTCT} \hspace{0.5cm} \text{TCCT} \hspace{0.5cm} \text{TCCT} \hspace{0.5cm} \text{TCCT} \hspace{1cm} (2.2)
\]

Remark 2.2: If Eq. 2.1 takes ordinal constant 1 as \( \text{T} \), then:

\[
((p+(r=r))=(q+(r=r)))->(p=q) ; \hspace{1cm} \text{TTFT} \hspace{0.5cm} \text{TTFT} \hspace{0.5cm} \text{TTFT} \hspace{0.5cm} \text{TTFT} \hspace{1cm} (2.3)
\]

Remark 2.1: We attempt to resuscitate Eq. 2.1 by removing 1 from the antecedent:

\[
[(x+1 = y+1) -1] \rightarrow x=y \hspace{1cm} (3.1)
\]

\[
(((p+(%r>r))=(q+(%r>r)))-(%r>r))>(p=q) ; \hspace{1cm} \text{TNNT} \hspace{0.5cm} \text{TNNT} \hspace{0.5cm} \text{TNNT} \hspace{0.5cm} \text{TNNT} \hspace{1cm} (3.2)
\]

Eqs. 2.2, 2.3, and 3.2 are not tautologous, thereby refuting Presburger arithmetic by its own Axiom 2.