Refutation of Lusin’s separation theorem

Abstract: “In descriptive set theory and mathematical logic, Lusin's separation theorem states that if $A$ and $B$ are disjoint analytic subsets of Polish space, then there is a Borel set $C$ in the space such that $A \subseteq C$ and $B \cap C = \emptyset$.” We evaluate two renditions of that equation, both non tautologous, refuting it. Therefore, the separation theorem of Lusin forms a non tautologous fragment of the universal logic $\text{VL}_4$.

We assume the method and apparatus of Meth8/$\text{VL}_4$ with Tautology as the designated proof value, $\text{F}$ as contradiction, $\text{N}$ as truthity (non-contingency), and $\text{C}$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

\text{LET } \neg \text{ Not, } \neg ; \; + \text{ Or, } \lor, \lor ; \; - \text{ Not Or; } \& \text{ And, } \land, \land, \land, \land ; \; \setminus \text{ Not And;}
> \text{ Imply, greater than, } \rightarrow, \Rightarrow, \supset, \supset ; \; < \text{ Not Imply, less than, } \in, \prec, \subset, \vartriangleleft, \vartriangleleft, \vartriangleleft ; \; \neq \text{ Not Equivalent, } \neq ;
\% \text{ possibility, for one or some, } \exists, \emptyset, \diamond; \; \# \text{ necessity, for every or all, } \forall, \Box, L;
(z=z) \text{ T as tautology, } T, \text{ ordinal } 3; \; (z@z) \text{ F as contradiction, } \emptyset, \text{ Null, } \perp , \text{ zero;}
(\%z>\#z) \text{ N as non-contingency, } \Delta, \text{ ordinal } 1; \; (\%z<\#z) \text{ C as contingency, } \nabla, \text{ ordinal } 2;
\neg(y<x) \text{ ( } x \leq y \text{), } (x \subseteq y), (x \sqsubseteq y); \; (A=B) \text{ ( } A\sim B \text{).}

Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Lusin%27s_separation_theorem

In descriptive set theory and mathematical logic, Lusin's separation theorem states that if $A$ and $B$ are disjoint analytic subsets of Polish space, then there is a Borel set $C$ in the space such that

\[ A \subseteq C \text{ and } B \cap C = \emptyset \ldots \] \hspace{1cm} (1.1)

\text{LET } p, q, r, s: \; A, B, C, D

\[ (\neg(r<p) \& (q\&r)) = (s@s); \quad \text{T TTTT TTTF TTTT TTTF} \] \hspace{1cm} (1.2)

\textbf{Remark 1.1: } If Eq. 1.1 is rendered in theorem variables, then

\[ (\neg(C<A) \& (B\&C)) = (D@D); \quad \text{T TTTT TTTT TTTT, TTTT TTTT TTTT TTTT, TTTT TTTT TTCC TTCC, TTTT TTTT TTCC TTCC, TTTT TTCC TTCC TNCF} \] \hspace{1cm} (1.3)

Eqs. 1.2 and 1.3 are not tautologous, thereby refuting Lusin’s separation theorem.