Disproof of Twin Prime Conjecture

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Author’s Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this disproof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's disproof.

Geerasee is now 30 years old and she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Furthermore she has achieved several other scientific achievements already.

List of abbreviations

Faculty of Science, University of Colombo, Sri Lanka , Belarusian National Technical University Belarus

Acknowledgement

I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

Abstract

A twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid disproof for twin prime conjecture.

Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture (see below), postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

Assumption

Let's assume that there are infinitely many twin prime numbers.

Then we know that when we consider that there are finitely many Twin Prime Numbers, then we should get a contradiction.

Therefore we proceed by considering that there are finitely many twin prime numbers then we should get a contradiction according to our initial assumption. Then let the highest twin prime numbers are P_{n-1} and $(P_{n-1} +2)$. Then for all prime numbers P_n greater than P_{n-1} , $(P_n + 2)$ is not a prime number.

Methodology

With this mathematical proof, I use the contradiction method to disprove the twin prime conjecture.

Let P_n is an arbitrary prime number greater than P_{n-1} (because there are infinite number of prime numbers). Then according to our assumption, $(P_n +2)$ is not a prime number. Since $P_n > 2$ and since P_n is a prime number and since P_n is an odd number, for all prime numbers P_i :

$P_i \ (< P_n / 2): \ P_n / P_i = r_1$

Thus $P_n = P_i * r_1$..................(01)

Where r_1 is a rational number (which is not a natural number)

But according to our assumption, $P_n +2$ is not a prime number. Also since P_n is a prime number greater than 2, $(P_n + 2)$ is an odd number.

Thus for some prime number P_1 (< [$(P_n + 2) / 2$]) ; $(P_n + 2) / P_1 = x_1$. Where we choose P_1 such that x_1 is a natural number. But since previously chose P_i is any arbitrary prime number less than $(P_n / 2)$; now we consider $P_1 = P_i$.

Then $(P_n + 2) = P_1 * x_1$ ....(02) and $P_n = P_1 * r_1$..................(01)

Let P_N is a prime number (greater than P_n). Then according to our initial assumption, (P_N + 2) is not a prime number. Here P_N is a prime number such that <u>(P_N + 2) is dividing by prime number P_2.</u> …………………(1.1)

Thus (P_N + 2) = P_2 * x_2 for some x_2 natural number. Because there are infinitely many prime numbers.

Since P_N is a prime number, for some r_2 (rational number which is not a natural number):

$P_N / r_2 = P_2$.

Thus (P_N + 2) = $P_2 * x_2$ ………………(03) and $P_N = r_2 * P_2$ ……………….(04)

x, x' are natural numbers and P_1 and P_2 are prime numbers.

Since P_N is a prime number , (P_N – 2) is also not a prime number (Since $P_N - 2 > P_{n-1}$)

Then for some prime P_3 , (P_N – 2) / P_3 = x_3

(P_N – 2) = $P_3 * x_3$ ……………………..(05)

By (03) and (05): $P_3 * x_3 = (P_2 * x_2) - 4$…………………….(5.1)

By (04) and (05): $P_3 * x_3 = P_2 * r_2 - 2$ …………………..(06)

Since P_N and P_n are two prime numbers greater than 2, P_N and P_n both are odd numbers.

Thus $P_N - P_n = 2*l$ for some natural number l . Thus $P_N = P_n + 2*l$ ………………….(*)

By (*): $P_1 * r_1 + 2l = P_2 * r_2$. Thus by (06): $P_3 * x_3 = P_1 * r_1 + 2(l - 1)$……………(07)

By (5.1) and (07): $P_1 * r_1 + 2(l - 1) = (P_2 * x_2) - 4$

Thus $(P_2 * x_2) - 2 * (l + 1) = P_1 * r_1$ ……………………….(08)

But we can put l as a natural number greater than 1. We can’t put l = 1. Because $P_N - P_n \neq 2$

Choose $l = (m.P_2 - 1)$ where m is not a natural number. But $(m.P_2 - 1)$ is a natural number.

Please <u>refer “proof “ below to see the proof of the existence of ‘*l*’ natural number</u> such that <u>$P_N = P_n + 2*l$</u> . i.e. there exists P_N (> P_n) such that for P_n prime; $P_N = P_n + 2*l$. where we consider that $l = m.P_2 - 1$ …………..(9.0.0)

But we choose m positive real number such that $2*(m.P_2 – 1) = A_0. P_1$; A_0 even natural number.

NOW BEFORE READING THE NEXT PART BELOW, PLEASE REFER 'PROOF'.

And for $l = m.P_2 - 1$, by (08): $(P_2 * x_2) - [2* m.P_2] = P_1 * r_1$ ………………………………(09) where m is not a natural number . But $(m.P_2 – 1)$ is a natural number.

But for the already chosen $P_{N\,(}$ greater than P_{n-1}), there exists P_1 prime number such that: and $(P_N + 2) / P_1 = x_2$ …………………………(10) (According to the equation (15) in the "proof" mentioned below)

Because we chose P_N (i.e. $(P_N + 2)$) in that manner. Where P_N is a prime number.

Thus in (03) and in (09): $P_2 \equiv P_1$.

Thus by (09): $(P_1 * x_2) - [2.m.P_1] = P_1 * (x_2 - 2.m) = P_1 * r_1$

Thus $x_2 - [2* m] = x_2 - [P_1. A_0 + 2] / P_1 = [x_2 . P_1 – (P_1. A_0 + 2)] / P_1 = r_1 = P_n / P_1$

Then $x_2 . P_1 – (P_1. A_0 + 2) = (P_N + 2) - (P_1. A_0 + 2) = (P_N + 2) – 2.m.P_1 = P_n$

But $(P_N + 2) - 2.m.P_1 = P_n = (P_n + 2) + 2. (m.P_{2\,–}1) - 2.m.P_1 =$

$(P_n + 2) + 2. (m.P_{1\,–}1) - 2.m.P_1 \rightarrow P_n = P_n$ ……..(11)

{ Above $(P_N + 2) - 2.m.P_1 = (P_n + 2) + 2. (m.P_{2\,–}1) - 2.m.P_1$; because by (15.0.0.) $\rightarrow$

$P_N = P_n + 2. (m.P_{2\,–}1)$ }

Therefore by (11), we didn't get a contradiction. But we know according to our initial assumption, we should get a contradiction. But as in our assumption, we can't get a contradiction……(12)

Therefore our initial assumption is false.

Therefore there are finitely many Twin Prime Numbers.

Proof

We know the equation related to "prime gap" as written below.

$P_N = 2 + \sum_{j=1}^{N-1} gj$ ……………..(i)   *** refer the 2nd reference.

Let $g_j = a_j$ ……..(ii) for all $j < (N – 1)$. Where a_j is a natural number. Let $\Sigma\, a_j = A$ for $j < N – 1$.

But for all $\epsilon_{N-1} > 0$, there exists 'N - 2' natural number such that for all $N -1 > N - 2$,

$g_{N-1} < P_{N-1} * \epsilon_{N-1}$

*** refer the 2nd reference below.

Then for some C_{N-1} positive number, $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $\epsilon_{N-1} > 0$

But $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $N – 1 > N - 2$

Choose $\epsilon_{N-1} = [(P_n + 2.m.P_2 - 4 - A) + C_{N-1}] / P_{N-1} > 0$. Then $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1} =$ $(P_n + 2.m.P_2 - A - 4)$.

Thus by (i): $P_N = 2 + P_n + 2.\ m.P_2 - A - 4 + A = P_n + 2.m.P_2 - 2 = P_n + 2.\ (m.P_2 - 1)$

Therefore there exists a natural number l (= $m.P_2 - 1$) such that $P_N = P_n + 2.\ (m.P_2 - 1)$…(15.0.0)

Where we chose $m = [P_1 .\ A_0 + 2] / 2.P_2$

But $(P_N + 2) = (P_n + 2) + 2.\ (m.P_2 - 1)$. But, $2*(m.P_2 - 1) = P_1 * A_0$; A_0 is an even natural number. Thus $(P_N + 2) = P_1.\ x_1 + P_1 * A_0 = P_1 *A_1$; A_1 is a natural number.

Thus $(P_N + 2)$ is divisible by P_1

That means we have the capability to consider $P_2 \equiv P_1$ in the equations in the methodology…..........................(15)

Discussion

We know that according to our initial assumption, there are infinitely many twin prime numbers. Then once we consider that there are finitely many twin prime numbers, we should get a contradiction. When we consider that there are finitely many twin prime numbers: for all P_n primes (> P_{n-1}) : (P_n + 2) is not prime. Therefore there exist prime number P_1 such that (P_n + 2) / P_1 = x_1 ; x_1 is a natural number. Thus for that fact, I have used the fact of (P_n + 2) is not a prime number. Also I have used that P_n and P_1 are as prime numbers. And the same argument for (P_N + 2), P_2 , P_3 and (P_N – 2). Where P_N is a prime number greater than P_n.

Results

Therefore I have used our initial assumption by expecting to get a contradiction finally. But as showed in (12), we don't get a contradiction. But according to our main assumption, we should get a contradiction. Therefore it is possible to conclude that our main assumption is false.

Thus there are finitely many twin prime numbers.

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote 'i' th prime gap $g_i = P_{i+1} - P_i$

Then according to the 2nd reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} gj$

Also by 2nd reference: for all Є > 0, there is a natural number 'n' such that for all N -1 > n;

$g_{N-1} < P_{N-1} . Є$

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