

# Proof of Twin Prime Conjecture that can be obtained by using Contradiction method in Mathematics

K.H.K. Geerasee Wijesuriya

Research Scientist in Physics and Astronomy

PhD student in Astrophysics, Belarusian National Technical University

BSc (Hons) in Physics and Mathematics University of Colombo, Sri Lanka

Doctorate Degree in Physics

geeraseew@gmail.com , geerasee1@gmail.com

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## **Author's Biography**

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Now she is a research scientist in Physics as her career. Furthermore she has achieved several other scientific achievements already.

## **List of Affiliations**

Faculty of Science, University of Colombo, Sri Lanka , Belarusian National Technical University Belarus

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I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

**Keywords:** prime; contradiction; greater than ; integer

## **Abstract**

Twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture.

## **Literature Review**

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes  $p$  such that  $p + 2$  is also prime. In 1849, de Polignac made the more general conjecture that for every natural number  $k$ , there are infinitely many primes  $p$  such that  $p + 2k$  is also prime. The case  $k = 1$  of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture, postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer  $N$  that is less than 70 million, there are infinitely many pairs of primes that differ by  $N$ . Zhang's paper was accepted by *Annals of Mathematics* in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

## Assumption

Let's assume that there are finitely many twin prime numbers.....(1.0)

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are  $P_{n-1}$  and  $(P_{n-1} + 2)$ . Then for all prime numbers  $P_N$  greater than  $(P_{n-1} + 2)$ ,  $(P_N + 2)$  is not a prime number.

## Methodology

With this mathematical proof, I use the contradiction method to prove that there are infinitely many twin prime numbers.

Let  $P_n$  is an odd number greater than 1. But let  $P_3$  is divisible by  $x_3$ . But  $x_3^2$  does not divide  $P_3$ . And let  $P_n$  is not divisible by  $x_3$ . We choose  $P_n$  such that  $P_n = (M + 4) - (D \cdot P_3 / x_3)$ ; for some integer  $D \neq 0$ . Where  $D$  is not divisible by  $x_3$ .

To see the meaning of  $P_3$ ,  $x_3$  and  $M$ , please refer the below content.

Let  $P_N$  is an arbitrary prime number greater than  $(P_{n-1} + 2)$ . Because there are infinitely many prime numbers. And here  $(P_N - 2) > (P_{n-1} + 2)$ . Thus  $(P_N - 2)$  is not a prime number.

And here  $(P_N + 2) > (P_{n-1} + 2)$ . Then according to our assumption,  $(P_N + 2)$  is also not a prime number. Here  $P_N$  is a prime number such that  $(P_N + 2)$  is dividing by prime number  $P_2$ .  
.....(1)

Thus  $(P_N + 2) = P_2 * x_2$  for some  $x_2$  natural number. Since  $P_N$  is a prime number, for some  $r_2$  (rational number which is not a natural number):  $P_N / r_2 = P_2$ .

Thus  $(P_N + 2) = P_2 * x_2$ .....(02) and  $P_N = r_2 * P_2$ .....(03)

$x_2$  is a natural number and  $P_2$  is a prime number. Since  $P_N$  is a prime number,  $(P_N - 2)$  is also not a prime number ( Since  $P_N - 2 > P_{n-1} + 2$ )

Then for some integer  $P_3$  greater than 1 such that  $(P_N - 2) / P_3 = x_3$ ; where  $x_3$  is an integer greater than 1. But here we considered that  $x_3 | P_3$ . **But we have  $P_L | (x_3 - 1)$  for some  $P_L$  an integer (not equals to 0). Then  $P_L | [P_L - (x_3 - 1)]$ .....(3.1)**

$$(P_N - 2) = P_3 * x_3 \dots\dots\dots(04)$$

But  $(P_N + 2)$ ,  $P_n$  both are odd numbers. Thus  $(P_N + 2) = P_n + 2.l$  ; for some  $l$  integer (where  $l \neq 0$ )  
 .....(05)

$$\text{Then } (P_N - 2) = P_n + 2.l - 4 = P_n + 2 . (l - 2) \dots\dots\dots(6.1)'$$

$$\text{And we know that } (P_N + 2) = P_n + 2.l \rightarrow P_N = P_n + 2.l - 2 \dots\dots\dots(*)$$

$$\text{Thus by } (*): P_n + 2.l - 2 = P_N. \text{ Thus by (04) and } (*): P_3 * x_3 + 2 = P_n + 2.l - 2$$

$$\text{Thus } P_3 * x_3 - 2.l + 4 = P_n \dots\dots\dots(6.1.0)$$

$$\text{Thus } P_3 * x_3 + 2. (l - 2) = P_n + 4. (l - 2) = P_n + 2.P_N - 4 - 2.P_n = 2.P_N - 4 - P_n \text{ ( by (6.1) )}'$$

$$\text{Thus } P_3 * x_3 + 2. (l - 2) = 2.P_N - 4 - P_n = P_n''$$

$$\text{Thus } P_3 * x_3 + 2. (l - 2) = P_n'' = 2. P_3 * x_3 - P_n \dots\dots\dots(7)$$

$$\text{Thus } P_3 * x_3 + 2.l = 4 + 2. P_3 * x_3 - P_n$$

$$P_3 * x_3 + ( 2.l + M) = (4 + M - P_n) + 2. P_3 * x_3 ; \text{ Where } M \text{ is an integer } (M \neq 0)$$

$$( 2.l + M ) = (4 + M - P_n) + P_3 * x_3 ; \text{ Where } M \text{ is an integer } \neq 0 \dots\dots\dots(8)$$

But we chose  $M$  such that  $(M + 4)$  is divisible by  $x_3$ . But let  $(M + 4)$  is not divisible by  $P_3$ .

But we know that  $P_3$  is divisible by  $x_3$ . But  $x_3^2$  does not divide  $P_3$ . And we know that  $(P_3 * x_3)$  is divisible by  $x_3$ . And we know that  $P_n$  is not divisible by  $x_3$ .....(8.1) .

Thus by (8):  $x_3$  does not divide  $( 2.l + M )$ . Since  $P_3$  is divisible by  $x_3$ ,  $P_3$  does not divide  $( 2.l + M )$  .....(i)

But  $P_N$  is an arbitrary prime greater than  $(P_{n-1} + 2)$ . Then let  $\{ (P_N + A_1) , P_N \}$  are two arbitrary consecutive primes set such that each primes are greater than  $(P_{n-1} + 2)$ .

Here since  $P_N > (P_{n-1} + 2)$  and since  $(P_N + A_1) > (P_{n-1} + 2)$ ,  $A_1 \neq (+/-) 2$ . Because for any two arbitrary consecutive primes greater than  $(P_{n-1} + 2)$ , the difference between those consecutive primes is greater than 2 (since the greatest twin primes are  $P_{n-1}$  and  $[P_{n-1} + 2]$  ).

But  $A_1 \neq 2.(x_3-1)$ . But now choose two particular two consecutive primes (greater than  $(P_{n-1} + 2)$  ) from the arbitrary prime number set  $\{ (P_N + A_1) , P_N \}$  such that those chosen two particular

consecutive primes obey  $[P_3 | (A_1 - 2)]$ . i.e. where particularly we choose  $A_1$  such that  $P_L - x_3 = B_2$ . Where  $(A_1 - 2) / P_3 = B_2$ . But by (3.1),  $P_L | [P_L - (x_3 - 1)]$ . But  $P_L - x_3 = B_2$ . Thus  $P_L | (B_2 + 1)$ . Then  $P_L = (P - 4) / P_3$ . Here  $P =$  chosen particular prime  $(P_N + A_1)$ . **Since  $A_1 \neq -2$ , there exists an odd number  $P_3$  greater than 1 such that  $[P_3 | (A_1 - 2)]$ . Refer the ‘Proof below to see the existence of two consecutive primes  $(P_N + A_1)$  and  $P_N$  such that  $[P_3 | (A_1 - 2)]$ . And refer ‘Proof 1’ to see the existence of an integer  $(P_N - 2)$  such that  $(P_N - 2) = P_3 \cdot x_3$  such that  $P_3$  is divisible by  $x_3$ . But  $x_3^2$  does not divide  $P_3$ .**

But we know that  $(P_N + A_1) > (P_{n-1} + 2)$ . Thus here  $A_1 \neq (+/-) 2$ , since there are finite number of twin primes according to our assumption. BUT REMEMBER THAT  $P_N$  AND  $(P_N + A_1)$  ARE CONSECUTIVE PRIMES greater than  $(P_{n-1} + 2)$ .

{ Here  $(P_N - 2) = P_3 \cdot x_3$  and  $(P_N + A_1) = P =$  Prime. That means  $P_3 \cdot x_3 + (A_1 + 2) = P$

But  $(A_1 - 2)$  is divisible by  $P_3$ . Thus  $(A_1 + 2)$  is not divisible by  $P_3$ . Because  $P_3$  does not divide 4.

But since  $P_3 * x_3$  is divisible by  $P_3$ ,  $P$  is not divisible by  $P_3$ .

But  $(A_1 - 2)$  is divisible by  $P_3$  and since  $(x_3 | P_3)$ ,  $x_3 | (A_1 - 2)$ . Thus  $(A_1 + 2)$  is not divisible by  $x_3$ . Because  $x_3$  does not divide 4 since  $x_3$  is an odd number (since  $(P_N - 2) = P_3 \cdot x_3$ ).

But since  $P_3 * x_3$  is divisible by  $x_3$ ,  $P$  is not divisible by  $x_3$ .

But  $P = P_3 \cdot x_3 + A_1 + 2 \neq P_3 \cdot x_3 + 2 \cdot (x_3 - 1) + 2 = P_3 \cdot x_3 + 2 \cdot x_3 = x_3 \cdot (P_3 + 2)$ . Thus  $P \neq x_3 \cdot (P_3 + 2)$ .

Therefore according to above steps, we can write  $P_3 \cdot x_3 + (A_1 + 2) = P$  as a prime }

But  $(2.l + M) = P_N - P_n + 2 + M = (P_N + A_1) + (M + 2 - A_1 - P_n) \dots \dots \dots (9)$

By (8.1):  $x_3 | (M + 4)$ . But  $[P_3 | (A_1 - 2)]$ .  $\dots \dots \dots (10)$

But since  $[P_3 | (P_N - 2)]$  and since  $P_3$  does not divide  $(A_1 + 2)$ ,  $\{ (A_1 + 2) + (P_N - 2) \}$  does not divide by  $P_3$ . i.e.  $P (= (P_N + A_1))$  does not divide by  $P_3$ . Thus our choice of  $A_1$  such that  $[P_3 | (A_1 - 2)]$  is okay.

But  $[P_3 | (P_N - 2)]$  and  $[P_3 | (A_1 - 2)]$ . Thus  $(P_N - 2) = P_3 \cdot x_3$  and  $(A_1 - 2) = P_3 \cdot B_2$ ; where  $x_3$  and  $B_2$  are integers and each of them not equals to 0.

Thus  $(P_N + A_1 - 4) = P_3 \cdot x_3 + P_3 \cdot B_2 = (P - 4)$

i.e  $P_3 | (P - 4) \dots \dots \dots (11)$

Let's consider M integer such that  $M = P - C$  ; for some integer 'C'  $\neq 0 \dots \dots \dots (12)$ .

But  $x_3 | (M + 4)$  and  $P_3 | (P - 4)$  by (8.1) and (11).

By (12):  $P = (M + C)$ . Thus  $[(M + C)] - 4 = P_3 \cdot P_L \dots \dots \dots (13)$

Where  $P_L = [(P - 4) / P_3] = \text{integer}$ , but not equals to 0.

Then  $(P_L / x_3) = [(P - 4) / (x_3 P_3)] = (P - 4) / (P_N - 2)$   
 $= [(P_N - 2) + (A_1 - 2)] / (P_N - 2) = 1 + [(A_1 - 2) / (P_N - 2)] = 1 + [(A_1 - P_N + P_N - 2) / (P_N - 2)]$   
 $= 2 + [(A_1 - P_N) / (P_N - 2)]$ . Since  $A_1 \neq (+ / -) 2$ ,  $[(A_1 - P_N) / (P_N - 2)]$  is not an integer.

Thus  $(P_L / x_3)$  is not an integer. Thus here  $x_3$  does not divide  $P_L \dots \dots \dots (13.1)$

But  $[(M + 4) / x_3] = P_Q = \text{integer}$ , but not equals to 0.

Thus by (13):  $[(x_3 \cdot P_Q - 4 + C)] - 4 = P_3 \cdot P_L$

Thus  $C - 4 = [(P_3 \cdot P_L + 4) - x_3 \cdot P_Q] \dots \dots \dots (14)$

By (09):  $(2I + M) = (P_N + A_1) + (M + 2 - A_1 - P_n) = P + (M + 2 - A_1 - P_n)$   
 $= P + P - C + 2 - A_1 - P_n = 2P - C - P_3 \cdot B_2 - P_n \dots \dots \dots (15)$

By (14):  $C = [(P_3 \cdot P_L + 8) - x_3 \cdot P_Q]$ . Then  $2P - C - P_n = 2P + x_3 \cdot P_Q - (P_3 \cdot P_L + 8) - P_n$   
 $= 2(P - 4) + x_3 \cdot P_Q - P_n - (P_3 \cdot P_L) = 2 \cdot P_3 \cdot P_L + x_3 \cdot P_Q - P_n - (P_3 \cdot P_L) = P_3 \cdot P_L + [x_3 \cdot P_Q - P_n]$   
 $= P_3 \cdot P_L + P_3 \cdot [x_3 \cdot (P_Q / P_3) - (P_n / P_3)] \dots \dots \dots (16)$

But we chose  $P_n$  such that  $(x_3 \cdot P_n) = x_3 \cdot (M + 4) - D \cdot P_3$  ; for some integer D (But we choose D such that  $x_3^2 | (P_L \cdot x_3 + D)$ ). Where D is not divisible by  $x_3$  and  $D \neq 0$ .

**To see the proof that proves that there exists an integer D ( $\neq 0$ ) such that  $x_3^2 | (P_L \cdot x_3 + D)$ , please refer 'Proof 2' below.**

Then  $P_n = (M + 4) - (D \cdot P_3 / x_3)$ . Then  $[x_3 \cdot (P_Q / P_3) - (P_n / P_3)] = (D / x_3)$

Then by (16):  $2P - C - P_n = P_3 \cdot (P_L + (D / x_3)) = P_3 \cdot x_3 [ (P_L / x_3) + (D / x_3^2) ]$  ; where  $(P_L / x_3)$  and  $(D / x_3^2)$  are not integers (by 13.1). But we choose D such that  $x_3^2 | (P_L \cdot x_3 + D)$ .

Then  $2.P - C - P_n = P_3 \cdot x_3 [ [ (P_L \cdot x_3) + D ] / x_3^2 ] = P_3 \cdot x_3 \cdot D'$  ; where  $D' = (P_L \cdot x_3 + D) / x_3^2$   
 = integer , but not equals to 0.

Then  $P_3 \mid (2.P - C - P_n)$ .....(17)

by (15), (17):  $P_3 \mid ( 2.l + M )$ .....(18)

Thus by (i):  $P_3$  does not divide  $( 2.l + M )$ .....(19)

Thus by (18) and (19): We have a contradiction.....(20)

Therefore the only possibility is: our assumption (1.0) is false. Therefore there are infinitely many Twin Prime Numbers.

**Proof**

Let's prove that there exists consecutive primes  $P_N$  and  $(P_N + A_1)$  such that  $[ P_3 \mid (A_1 - 2) ]$  for some odd integer  $P_3$  which is not equal to 1 (when there exist consecutive prime numbers  $P_N$  and  $(P_N + A_1)$  which both are greater than  $[P_{n-1} + 2]$  ) through this 'Proof' as below.

By 2<sup>nd</sup> reference:  $P_{N-1} = (P_N + A_1) = 2 + \sum_{j=1}^{N-2} h_j$  ,where  $h_j = P_{j+1} - P_j$  for all  $j \in \{ 1 , 2, \dots, (N-2) \}$   
 or  $P_{N+1} = (P_N + A_1) = 2 + \sum_{j=1}^N h_j$  when  $j \in \{ 1 , 2, \dots, N \}$ . Here  $(P_N + A_1) = P_{N+1}$  or  $P_{N-1}$  , depends on the sign of  $A_1$ .

$$P_{N-1} = P_N + A_1 = 2 + \sum_{j=1}^{N-2} h_j \text{ (when } A_1 < 0\text{)}. \text{ If } A_1 > 0, P_{N+1} = P_N + A_1 = 2 + \sum_{j=1}^N h_j$$

Consider the case that  $A_1 < 0$ .

$$\text{Then } (A_1 - 2) = - P_N + \sum_{j=1}^{N-2} h_j$$

$$\text{Then } (2 - A_1) = P_N - \sum_{j=1}^{N-2} h_j$$

Then  $(2 - A_1) = P_N - k' - \sum_{j=1}^{N-2} h_j$  .....(21) Because here inside  $\sum_{j=1}^{N-2} h_j$  , I have included  $(+ k')$  term.

But by 2<sup>nd</sup> reference: for all  $\epsilon > 0$ , there is a natural number 'm' such that for all  $(N- 2) > m$ ;

$$h_{N-2} < P_{N-2} \cdot \epsilon$$



Let  $C_s$  is a positive real number  $C_s = [ - B + C_s + k' + P_N + B_2. P_3 ] / P_s > 0$ , such that  $h_s < P_s * C_s$  for all  $s > (N - 3)$ . But here  $P_L | (B_2 + 1)$ . Let here the chosen  $C_s$  implies that  $m = (N - 3)$  (Here  $s$  is going from 1 to  $(N - 2)$ ). Then " for all  $s > (N - 3)$ " means  $s = (N - 2)$ . Where  $k'$  is an integer number which not equals to 0 and we choose  $k'$  such that  $k' / (N - 2)$  is an integer. Here the chosen  $k'$  integer number is responsible for  $h_s < P_s * C_s$  for all  $s > (N - 3)$  (i.e.  $s = N - 2$ ) and  $C_s > 0$ . That means here the value of  $k'$  is responsible to say "  $C_s$  is existing such that  $h_s < P_s * C_s$  , for  $s = (N - 2)$  ". Here  $h_j = b_j - [ k' / (N - 2) ]$  for all  $j < (N - 2) = s$ . And where  $\sum b_j = B$  for  $j < (N - 2) = s$ . Then for some  $C_s$  ,  $h_s = P_s * C_s - C_s$  ; here  $s \equiv (N - 2)$ . \*\*\* the meaning of 'j' is the order number and  $h_j$  is the prime gap between  $P_{j+1}$  and  $P_j$  . Please refer the below content and the 2<sup>nd</sup> reference. But here we chose  $C_{N-2}$  such that  $h_{N-2} = P_{N-2} * C_{N-2} - C_{N-2}$

But  $h_{N-2} = P_{N-2} * C_{N-2} - C_{N-2} = ( - B + k' + P_N + B_2. P_3)$ . Where  $k'$  is a natural number. Now let's use the 2<sup>nd</sup> reference to proceed further. By (21):

$$(A_1 - 2) = - P_N + \sum_{j=1}^{N-2} h_j = - P_N + ( - B + k' + P_N + B_2. P_3) + B - (N - 2). [ k' / (N - 2) ] = B_2. P_3 \dots\dots\dots(22)$$

Thus by (22):  $(A_1 - 2) = P_3.B_2$  . Thus there exist consecutive prime numbers  $P_N$  and  $(P_N + A_1)$  both greater than  $(P_{n-1} + 2)$  such that  $(A_1 - 2) = B_2.P_3$  ; for integer  $B_2 (\neq 0)$ .

And here we chose integer  $P_L$  such that  $P_L | (B_2 + 1)$ .

Similar to above, if  $A_1 > 0$ , we can proceed with the similar steps to prove that  $(A_1 - 2) = B_2.P_3$  ; for integer  $B_2 (\neq 0)$  when  $A_1 > 0$ .

**Proof 1**

Let's prove the existence of an integer  $(P_N - 2) (> P_{n-1} + 2)$  such that  $(P_N - 2) = P_3.x_3$  such that  $P_3$  is divisible by  $x_3$ . But  $x_3^2$  does not divide  $P_3$  as below.

By 2<sup>nd</sup> reference:  $P_N = 2 + \sum_{j=1}^{N-1} g_j$  , where  $g_j = P_{j+1} - P_j$  for all  $j \in \{1, 2, \dots, N-1\}$

Then  $(P_N - 2) = \sum_{j=1}^{N-1} g_j \dots\dots\dots(23)$

But by 2<sup>nd</sup> reference: for all  $\epsilon > 0$ , there is a natural number 'm<sub>0</sub>' such that for all  $N > m_0$ ;  $g_N < P_N \cdot \epsilon$ .

Let  $\epsilon_s$  is a positive real number  $\epsilon_s = [ - A + C_s + x_3^2 \cdot k_1 ] / P_s > 0$ , such that  $h_s < P_s * \epsilon_s$  for all  $s > (N - 2)$ . Let here the chosen  $\epsilon_s$  implies that  $m_0 = (N - 2)$  (Here  $s$  is going from 1 to  $N-1$ . Then " for all  $s > (N - 2)$ " means  $s = (N - 1)$  ). Where  $k_1$  is an integer number which is not divisible by  $x_3$ . Here the chosen  $k_1$  integer number ( $\neq 0$ ) is responsible for  $g_s < P_s * \epsilon_s$  for all  $s > (N - 2)$  (i.e.  $s = N - 1$  ) and  $\epsilon_s > 0$ . That means here the value of  $k_1$  is responsible to say "  $\epsilon_s$  is existing such that  $g_s < P_s * \epsilon_s$ , for  $s = N - 1$  ". Here  $g_j = a_j$  for all  $j < (N - 1) = s$ . And where  $\sum a_j = A$  for  $j < (N - 1) = s$ . Then for some  $C_s$ ,  $g_s = P_s * \epsilon_s - C_s$ ; here  $s \equiv (N - 1)$ . \*\*\* the meaning of 'j' is the order number and  $g_j$  is the prime gap between  $P_{j+1}$  and  $P_j$ . Please refer the below content and the 2<sup>nd</sup> reference. But here we chose  $C_{N-1}$  such that  $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ .

But  $g_{N-1} = (- A + x_3^2 \cdot k_1)$ . Now let's use the 2<sup>nd</sup> reference to proceed further. By (23):

$$(P_N - 2) = \sum_{j=1}^{N-1} g_j$$

$$\text{But } \sum_{j=1}^{N-1} g_j = A + (- A + x_3^2 \cdot k_1) = x_3^2 \cdot k_1 \dots\dots\dots(24)$$

Thus by (23) and (24):  $(P_N - 2) = x_3^2 \cdot k_1$ ; where  $k_1$  is not divisible by  $x_3$ .

Then  $(P_N - 2) = x_3 \cdot (x_3 \cdot k_1) = x_3 \cdot P_3$ ; where  $P_3$  is divisible by  $x_3$ . But since  $k_1$  is not divisible by  $x_3$ ,  $P_3$  is not divisible by  $x_3^2$ .

Thus  $(P_N - 2) = P_3 \cdot x_3$ ; where  $P_3$  is divisible by  $x_3$ . But  $P_3$  is not divisible by  $x_3^2$ .

**Proof 2**

Now let's prove that there exists an integer  $D (\neq 0)$  such that  $x_3^2 \mid (P_L \cdot x_3 + D)$ .

Let choose  $D' = (x_3 / G)$ ,  $D = G$  where  $D' \neq 1$  and  $G$  is an integer ( $\neq 0$ ).

Then  $(D' \cdot D) = x_3$ . Then  $[ (1/D') \cdot x_3 - D ] = 0$ .

Then  $[ (D')^2 \cdot x_3 + 1 ] \cdot [ (1/D') \cdot x_3 - D ] = 0 = D' \cdot x_3^2 - D + [ (x_3 / D') - D \cdot (D')^2 \cdot x_3 ]$

Then  $D' \cdot x_3^2 - D + [ (x_3 / D') - D \cdot (D')^2 \cdot x_3 ] = 0$ .

Then  $D' \cdot x_3^2 - D = D \cdot (D')^2 \cdot x_3 - (x_3 / D') \dots\dots\dots(24.1)$

Let's consider  $D.(D')^2 - (1/D')$ .

$$\begin{aligned} \text{Then } D.(D')^2 - (1/D') &= G. (x_3 / G)^2 - (G / x_3) = G. [ (x_3 / G)^2 - (1 / x_3) ] \\ &= ( G / (G^2 . x_3) ) . [x^3_3 - G^2] = (1/ G.x_3) . [x^3_3 - G^2] \dots\dots\dots(25) \end{aligned}$$

$$\begin{aligned} \text{By (25): } D.(D')^2 - (1/D') &= (1/ G.x_3) . [x^3_3 - G^2] = (1/ D.x_3) . [x^3_3 - D^2] \\ &= [ (x^2_3 / D) - (D / x_3) ] . \text{ But } (P_L - B_2) = x_3 . \text{ Because } (P_N - 2) = P_3.x_3 = P_3.P_L - P_3.B_2 \end{aligned}$$

$$\begin{aligned} \text{Thus } D.(D')^2 - (1/D') &= (x^2_3 / D) - (D / x_3) = (P_L - B_2)^2 / D - [D / (P_L - B_2) ] \\ &= [ (P_L - B_2)^3 - D^2 ] / [D.(P_L - B_2) ] \end{aligned}$$

But we chose D such that  $(P_L - B_2)^3 = k^3.D^3$  ; k is a real number ( $\neq 1$ ), but  $k \neq (1 / T)$  for all T integer other than 0.

$$\text{Then } D.(D')^2 - (1/D') = [ k^3.D^3 - D^2 ] / (D.k.D) = k^2.D - (1/k) = [k^3.D - 1] / k \dots\dots\dots(25.1)$$

But we chose D such that  $k = (- K / B_2)$ . Then  $(P_L - B_2) = - K.D / B_2$  ; K is a natural number.

$$\text{Then } [k^3.D - 1] / k = k^2.D + (B_2 / K) = (P_L - B_2)^2 / D + (B_2 / K) . \text{ Put } K = D .$$

But  $B_2 \neq D^2$  . Then  $k \neq (1 / T)$  for all T integer.

$$\text{Then } [k^3.D - 1] / k = [ (P_L - B_2)^2 + B_2 ] / D \dots\dots\dots(25.2)$$

We chose M' such that  $(P_L - B_2)^2 = M' - B_2$  ; where M' is a natural number.

$$\text{Then } [ (P_L - B_2)^2 + B_2 ] / D = ( M' - B_2 + B_2 ) / D = M' / D \dots\dots\dots(26)$$

But  $M' = (P_L - B_2)^2 + B_2$  . But  $(P_L - B_2)^2 = P_L - B_2 + K'$  ; K' is a natural number.

$$\text{Then } M' = P_L - B_2 + K' + B_2 . \text{ Then } M' = P_L + K' . \text{ Then } M' / D = (P_L + K') / D$$

But  $M' = P_L + P_L . (D-1) = P_L.D$  ; **whenever  $K' = P_L . (D-1)$ .**

{

Because  $K' = (P_L - B_2) . [P_L - B_2 - 1] = P_L . (D-1)$  . Because we can consider that

$P_L.N' = [P_L - B_2 - 1]$  and  $(P_L - B_2) = (D - 1) / N'$  for N' integer number not equals to 0.

Because:

$$(x_3 - 1) = \{ [ (P - 4) - P_3 . B_2 ] / P_3 \} - 1 . \text{ Then } [ (P - 4) / P_3 ] - B_2 - 1 = (x_3 - 1)$$

$$\text{Then } P_L - (B_2 + 1) = (x_3 - 1) . \text{ But } P_L - (B_2 + 1) = P_L . (1 - [B_2 + 1] / P_L) = P_L.N'$$

Thus  $P_L.N' = (x_3 - 1)$ . Where  $N' = (P_L - B_2 - 1) / P_L = 1 - (B_2 + 1) / P_L$  . But as in 'Proof' , we chose  $(P_N + A_1)$  and  $P_N$  such that  $(B_2 + 1)$  is divisible by  $P_L$ . Thus N' is an integer.

Let we choose integer  $D$  such that  $(x_3 \cdot N') + 1 = D = \text{integer}$ , for the integer number  $N'$ . Where  $D \neq 0$ .

Thus there exists  $N'$  an integer number ( $\neq 0$ ) such that  $P_L \cdot N' = [P_L - B_2 - 1]$  and

$$(P_L - B_2) = (D - 1) / N'. \text{ Where } D \text{ is an integer. Then } K' = (P_L - B_2). [P_L - B_2 - 1] = P_L \cdot (D-1)$$

}

Then  $(M' / D) = P_L$ . Then by (26):

$$[(P_L - B_2)^2 + B_2] / D = P_L = [k^3 \cdot D - 1] / k = D \cdot (D')^2 - (1/D')$$
 (by 25.1 and 25.2)

Then by (24.1):  $D' \cdot x_3^2 - D = P_L \cdot x_3$ . Then  $x_3^2 \mid (x_3 \cdot P_L + D)$  since  $D'$  is an integer.

## Discussion

We assumed initially that there are finitely many twin primes. After proceeding with that, I ended up with a contradiction. But to get the contradiction, I used that  $P_N$  as a prime number greater than  $(P_{n-1} + 2)$ . And we chose  $P_n$  odd integer ( $> 1$ ) and also we chose an integer  $A_1$  such that  $P_3 \mid (A_1 - 2)$ . Also to get the contradiction, I used the facts that  $(P_N + 2)$  and  $(P_N - 2)$  as non-primes since  $P_N - 2 > (P_{n-1} + 2)$ . And also I have used that  $x_2$  and  $x_3$  as natural numbers (since,  $(P_N + 2)$  and  $(P_N - 2)$  are not prime numbers). And also I have used the fact (to get the contradiction as in (20) ): The difference between any two consecutive prime numbers (which are greater than  $(P_{n-1} + 2)$  ) is greater than 2. Therefore to get the contradiction, I have used the facts got from our assumption (1.0). Then the only possibility is our assumption (1.0) is false.

## Results

Therefore I have used our assumption (1.0) to get the contradiction finally, as showed in (20). Therefore it is possible to conclude that our assumption (1.0) is false. Thus the negation of the assumption (1.0) is true.

**Thus there are infinitely many twin prime numbers.**

## Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote 'i' th prime gap  $g_i = P_{i+1} - P_i$

Then according to the 2<sup>nd</sup> reference; Prime number  $P_N = 2 + \sum_{j=1}^{N-1} g_j$

Also by 2<sup>nd</sup> reference: for all  $\epsilon > 0$ , there is a natural number 'n' such that for all  $N - 1 > n$ ;

$$g_{N-1} < P_{N-1} \cdot \epsilon$$

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