# Proof of Twin Prime Conjecture that can be obtained by using Contradiction method in Mathematics

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## Author’s Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Now she is a research scientist in Physics as her career. Furthermore she has achieved several other scientific achievements already.

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**Keywords:** prime; contradiction; greater than ; integer

## Abstract

Twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture.

## Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture, postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

## Assumption

Let's assume that there are finitely many twin prime numbers……………(1.0)

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are $P_{n-1}$ and $(P_{n-1} + 2)$. Then for all prime numbers $P_N$ greater than $(P_{n-1} + 2)$, $(P_N + 2)$ is not a prime number. Where N and n denote two distinct natural numbers.

## Methodology

With this mathematical proof, I use the contradiction method to prove that there are infinitely many twin prime numbers.

Let $P_n$ is an odd number greater than 1. But let $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$. And let $P_n$ is not divisible by $x_3$. We choose $P_n$ such that $P_n = [ (M + 4) / x_3 ] – (D_0.P_3 / x_3)$ ; for some integer $D_0 \neq 0$. Where $(D_0 + 2t)$ is divisible by $x_3$. Here $D_0$ is not divisible by $x_3$. But $(D_0 + 2t)$ is not divisible by $x^2_3$. Where t is an integer $\neq 0$. Although $D_0$ is not divisible by $x_3$ and although $[P_3 / x_3]$ is also not divisible by $x_3$, let here $(D_0.P_3 / x_3)$ is divisible by $x_3$ ……………….(0.0)

**Now please refer facts in (16.1) below, before reading the next facts.**

Then $P_n$ is not divisible by $x_3$ (Since $[ (M + 4) / x_3 ]$ is not divisible by $x_3$, according to the definition of M and etc).

To see the meaning of $P_3$ , $x_3$ and M, please refer the below content.

Let $P_N$ is an arbitrary prime number greater than $(P_{n-1} + 2)$. Because there are infinitely many prime numbers. And here $(P_N – 2) > (P_{n-1} + 2)$. Thus $(P_N – 2)$ is not a prime number.

Where each arbitrary $P_N$ prime number obey (that means we choose a set of $P_N$ ( > $(P_{n-1} + 2)$ ) arbitrarily such that each arbitrarily chosen $P_N$ give us :

“ $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$; whenever $(P_N – 2) = P_3.x_3$ ”

And here $(P_N + 2) > (P_{n-1} + 2)$. Then according to our assumption, $(P_N + 2)$ is also not a prime number. Here $P_N$ is a prime number such that $(P_N + 2)$ is dividing by prime number $P_2$. …………………(1)

Thus $(P_N + 2) = P_2 * x_2$ for some $x_2$ natural number. Since $P_N$ is a prime number, for some $r_2$ (rational number which is not a natural number): $P_N / r_2 = P_2$ . Thus $(P_N + 2) = P_2 * x_2$ ………………(02) and $P_N = r_2 * P_2$ ……………….(03)

$x_2$ is a natural number and $P_2$ is a prime number. Since $P_N$ is a prime number , $(P_N – 2)$ is also not a prime number ( Since $P_N – 2 > P_{n-1} + 2$). Then for some integer $P_3$ greater than 1 such that $(P_N – 2) / P_3 = x_3$ ; where $x_3$ is an integer greater than 1. But here we considered that $x_3 | P_3$.

$(P_N – 2) = P_3 * x_3$ ……………………..(04)

But $(P_N + 2)$, $P_n$ both are odd numbers. Thus $(P_N + 2) = P_n + 2.l$ ; for some $l$ integer (where $l \neq 0$) ……………………...(05)

Then $(P_N - 2) = P_n + 2.l - 4 = P_n + 2 . (l - 2)$ ..........(6.1)'

And we know that $(P_N + 2) = P_n + 2.l \rightarrow P_N = P_n + 2.l - 2$ ……………………(*)

Thus by (*): $P_n + 2.l - 2 = P_N$ . Thus by (04) and (*): $P_3 * x_3 + 2 = P_n + 2.l - 2$

Thus $P_3 * x_3 - 2.l + 4 = P_n$ ......................(6.1.0)

Thus $P_3 * x_3 + 2. (l - 2) = P_n + 4. (l - 2) = P_n + 2.P_N - 4 - 2.P_n = 2.P_N – 4 – P_n$ ( by (6.1)' )

Thus $P_3 * x_3 + 2. (l - 2) = 2.P_N – 4 – P_n = P_{n''}$

Thus $P_3 * x_3 + 2. (l - 2) = P_{n''} = 2. P_3 * x_3 - P_n$ ..........................(7)

Thus $P_3 * x_3 +2.l = 4 + 2. P_3 * x_3 - P_n$

$P_3 * x_3 + ( 2.l + M) = (4 + M - P_n) + 2. P_3 * x_3$ ; Where M is an integer ($M \neq 0$)

$( 2.l + M ) = (4 + M - P_n) + P_3 * x_3$ ; Where M is an integer $\neq 0$………(8)

But we chose M such that $(M + 4)$ is divisible by $x_3$. But let $(M + 4)$ is not divisible by $x^2_3$. And let $(M + 4)$ is not divisible by $P_3$.

But we know that $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$. And we know that $(P_3 * x_3)$ is divisible by $x_3$. And we know that $P_n$ is not divisible by $x_3$...............(8.1) .

Thus by (8): $x_3$ does not divide $(2.l + M)$. Since $P_3$ is divisible by $x_3$, $P_3$ does not divide $(2.l + M)$ …………….(i)

But $P_N$ is an arbitrary prime greater than $(P_{n-1} + 2)$. <u>Then let $\{(P_N + A_1), P_N\}$ are two arbitrary consecutive primes set such that each primes are greater than $(P_{n-1} + 2)$. Where each arbitrary $P_N$ prime number obey:</u>

" $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$ ; whenever $(P_N - 2) = P_3.x_3$ "

Here since $P_N > (P_{n-1} + 2)$ and since $(P_N + A_1) > (P_{n-1} + 2)$, $A_1 \neq (+/-) 2$. Because for any two arbitrary consecutive primes greater than $(P_{n-1} + 2)$, the difference between those consecutive primes is greater than 2 (since the greatest twin primes are $P_{n-1}$ and $[P_{n-1} + 2]$ ).

But $A_1 \neq 2.(x_3-1)$. But now choose two particular two consecutive primes (greater than $(P_{n-1} + 2)$ ) from the arbitrary prime number set $\{(P_N + A_1), P_N\}$ such that those chosen two particular consecutive primes obey $[P_3 | (A_1 - 2)]$. i.e. where particularly we choose $A_1$ such that $P_L - x_3 = B_2$. Where $(A_1 - 2) / P_3 = B_2$. Where $P_L = (P - 4) / P_3$. Here P = chosen particular prime $(P_N + A_1)$. **Since $A_1 \neq -2$, there exists an odd number $P_3$ greater than 1 such that $[P_3 | (A_1 - 2)]$. Refer the 'Proof' below to see the existence of two consecutive primes $(P_N + A_1)$ and $P_N$ such that $[P_3 | (A_1 - 2)]$. And refer 'Proof 1' to see the existence of an integer $(P_N - 2)$ such that $(P_N - 2) = P_3.x_3$ such that $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$.**

But we know that $(P_N + A_1) > (P_{n-1} + 2)$. Thus here $A_1 \neq (+/-) 2$ , since there are finite number of twin primes according to our assumption. BUT REMEMBER THAT $P_N$ AND $(P_N + A_1)$ ARE CONSECUTIVE PRIMES greater than $(P_{n-1} + 2)$.

{ Here $(P_N - 2) = P_3.x_3$ and $(P_N + A_1) = P$ = Prime. That means $P_3.x_3 + (A_1 + 2) = P$

But $(A_1 - 2)$ is divisible by $P_3$ . Thus $(A_1 + 2)$ is not divisible by $P_3$. Because $P_3$ does not divide 4.

But since $P_3 * x_3$ is divisible by $P_3$ , P is not divisible by $P_3$.

But $(A_1 - 2)$ is divisible by $P_3$ and since $(x_3 | P_3)$ , $x_3 | (A_1 - 2)$. Thus $(A_1 + 2)$ is not divisible by $x_3$. Because $x_3$ does not divide 4 since $x_3$ is an odd number (since $(P_N - 2) = P_3.x_3$ ).

But since $P_3 * x_3$ is divisible by $x_3$ , P is not divisible by $x_3$.

But $P = P_3.x_3 + A_1 + 2 \neq P_3.x_3 + 2.(x_3 - 1) + 2 = P_3.x_3 + 2.x_3 = x_3.(P_3 + 2)$. Thus $P \neq x_3.(P_3 + 2)$.

Therefore according to above steps, we can write $P_3.x_3 + (A_1 + 2) = P$ as a prime }

But $( 2.l + M ) = P_N - P_n + 2 + M = (P_N + A_1) + (M + 2 - A_1 - P_n)$…………..(9)

By (8.1): $x_3 | (M + 4)$. But $[ P_3 | (A_1 – 2) ]$. …….(10)

But since $[ P_3 | (P_N - 2) ]$ and since $P_3$ does not divide $(A_1 + 2)$, $\{ (A_1 + 2) + (P_N - 2) \}$ does not divide by $P_3$ . i.e. P (= $(P_N + A_1 )$ ) does not divide by $P_3$. Thus our choice of $A_1$ such that $[ P_3 | (A_1 – 2)]$ is okay.

But $[ P_3 | (P_N - 2) ]$ and $[ P_3 | (A_1 – 2)]$. Thus $(P_N - 2) = P_3.x_3$ and $(A_1 – 2) = P_3.B_2$ ; where $x_3$ and $B_2$ are integers and each of them not equals to 0.

Thus $( P_N + A_1 – 4 ) = P_3.x_3 + P_3.B_2 = ( P – 4 )$

i.e $P_3 | (P - 4)$………………(11)

Let's consider M integer such that $M = P – C$ ; for some integer 'C' $\neq 0$ ……….……….(12).

But $x_3 | (M + 4)$ and $P_3 | (P - 4)$ by (8.1) and (11).

By (12): $P = (M + C)$. Thus $[ (M + C) ] – 4 = P_3 . P_L$ …………(13)

Where $P_L = [ (P - 4) / P_3 ]$ = integer, but not equals to 0.

Then $( P_L / x_3 ) = [ (P - 4) / (x_3 P_3) ] = (P – 4 ) / (P_N - 2)$

$= [ (P_N - 2) + (A_1 - 2) ] / (P_N - 2) = 1 + [ (A_1 - 2) / (P_N - 2) ]$

If $A_1 < 0$, then either $1 + [ (A_1 - 2) / (P_N - 2) ] < 0$ or $0 < 1 + [ (A_1 - 2) / (P_N - 2) ] < 1$.

But $P_L = [ (P - 4) / P_3 ]$. Since $(P - 4) > 0$ and since $x_3 > 1$, $P_3 > 1$ ; $P_L > 0$. Thus it is impossible to have $1 + [ (A_1 - 2) / (P_N - 2) ] < 0$. Thus if $A_1 < 0$, we have $0 < 1 + [ (A_1 - 2) / (P_N - 2) ] < 1$. Then $( P_L / x_3 )$ is not an integer. If $A_1 > 0$, then $A_1 > 2$. Then $( P_N – A_1 ) < ( P_N – 2 )$……….(13)'

Then $( P_L / x_3 ) = 1 + [ (A_1 - 2) / (P_N - 2) ] = 1 + [ (A_1 - P_N + P_N - 2 ) / (P_N - 2)]$

$= 2 + [ (A_1 - P_N) / (P_N - 2)] = 2 - [ ( P_N - A_1) / (P_N - 2) ]$.

Since $A_1 \neq (+ / -)\ 2$ and by (13)' : $[ ( P_N – A_1) / (P_N - 2)]$ is not an integer.

Then $2 - [ ( P_N - A_1) / (P_N - 2) ] = ( P_L / x_3 )$ is not an integer.

Then for $A_1 > 0$ and for $A_1 < 0$ : $( P_L / x_3 )$ is not an integer.

Thus here $x_3$ does not divide $P_L$…………………(13.1)

But $[ (M + 4) / x_3 ] = P_Q =$ integer, but not equals to 0. Where the chosen $P_Q$ obeys conditions in (16.1). Thus by (13): $[ ( x_3 .P_Q – 4 + C ) ] – 4 = P_3. P_L$

Thus $C - 4 = [ ( P_3 .P_L + 4 ) – x_3 .P_Q ]$ ……………………….(14)

By (09): $( 2.l + M ) = (P_N + A_1) + (M + 2 - A_1 - P_n ) = P + (M + 2 - A_1 - P_n )$

$= P + P – C + 2 - A_1 - P_n = 2.P – C – P_3.B_2 - P_n$ ………..(15)

By (14): $C = [ ( P_3 .P_L + 8 ) – x_3 .P_Q ]$. Then $2.P – C – P_n = 2.P + x_3 .P_Q - ( P_3 .P_L + 8 ) - P_n$

$= 2.(P - 4) + x_3 .P_Q - P_n - ( P_3 .P_L) = 2. P_3 .P_L + x_3 .P_Q - P_n - ( P_3 .P_L) = P_3.P_L + [ x_3 .P_Q - P_n ]$

$= x_3. P_3.\{ ( P_L / x_3) + [(P_Q / P_3) - (P_n / (x_3. P_3) ) ] \}$ ……………………..(16)

But we chose $P_n$ such that $( x_3. P_n ) = (M + 4) – D_0.P_3$ ; for some integer $D_0$.

But we choose $D_0$ integer such that $x^2_3 \mid (P_L.x_3 + (D_0 + 2t) )$. Where $D_0 \neq 0$ and $(D_0 + 2t)$ is divisible by $x_3$. But $(D_0 + 2t)$ is not divisible by $x^2_3$. But here $(P_L / x_3) + [(D_0 + 2t) / x^2_3] = D'$ for some integer D'.

Since $P_L$ is not divisible by $x_3$ , there exists such an integer number $(D_0 + 2t)$ (not equals to zero), such that $x^2_3 \mid (P_L.x_3 + (D_0 + 2t) )$ ; whenever $(2t + D_0)$ is not divisible by $x^2_3$.

Then $P_n = [ (M + 4) / x_3 ] - (D_0.P_3 / x_3)$. Then $[ (P_Q / P_3) - (P_n / P_3 ) ] = (D_0 / x_3)$

$= [ x_3. (P_Q / P_3) - (P_n / P_3 ) ] – (x_3 – 1). (P_Q / P_3)$ .

Then $[ x_3. (P_Q / P_3) - (P_n / P_3 ) ] = (D_0 / x_3) + (x_3 – 1). (P_Q / P_3)$

Then $[ (P_Q / P_3) - (P_n / (x_3. P_3) ) ] = ( D_0 / x^2_3) + (x_3 – 1).(P_Q / (x_3. P_3) ) =$

$( D_0 / x^2_3) + (x_3 – 1).(P_Q / (x^2_3. u) ) = [ D_0.u + (x_3 – 1).P_Q ] / ( u. x^2_3 )$……..(16.0)

where $u.x_3 = P_3$ , then u is not divisible by $x_3$.

But we particularly chose $(P_3 / x_3) = u = [\ P_Q.\ (x_3 - 1) / 2t\ ]$ ; where t is an integer $\neq 0$ which is not divisible by $x_3$.

Although $[\ P_Q.\ (x_3 - 1) / 2t\ ]$ and $D_0$ are not divisible by $x_3$, $x_3 \mid [D_0. P_Q.(x_3 - 1) / 2t\ ]$.

**i.e. we have chosen integer $D_0, P_Q$ and t such that they obey condition $x_3 \mid [D_0. P_Q.(x_3 - 1)/ 2t\ ]$ and $(D_0 + 2t)$ is divisible by $x_3$. But $(D_0 + 2t)$ is not divisible by $x^2_3$** …………………..(16.1)

{

Let's show that we can have ( $x_3 \mid [D_0. P_Q.(x_3 - 1)/ 2t\ ]$ ) and $x_3 \mid (D_0 + 2t)$ both as below.

Consider $[D_0. P_Q.(x_3 - 1)/ 2t\ ] = x_3.\ v\ \neq 0$.

$(D_0 + 2t) = x_3.v'$ .

Now let's show that we can have v and v' are as integers $\neq 0$.

Then $D_0 + [\ (\ D_0. P_Q.(x_3 - 1)\ ) / x_3.\ v\ ] = x_3.v'$

Then $D_0. [x_3.v + P_Q.(x_3 - 1)] = x^2_3 .v.v'$

Then $D_0. [x_3.v + 2.t.u] = x^2_3 .v.v'$

Although $[x_3.v + 2.t.u]$ is not divisible by $x_3$ (since t and u both are not divisible by $x_3$), and although $D_0$ is not divisible by $x_3$ , it is possible to have $\{\ D_0. [x_3.v + 2.t.u]\ \}$ is divisible by $x_3$. Because $(x_3 \mid u.D_0)$ according to (0.0). Then $D_0. [x_3.v + 2.t.u] = [D_0. x_3.v + 2.t.(D_0. u)] = x^2_3 .v.v'$ $= [D_0. x_3.v + 2.t.(x_3.v'')] = x_3. v'''$ ; where v'' and v''' both are integers ($\neq 0$)……………..(16.2)

But we choose $D_0$ and t such that the chosen $D_0$, t and were responsible for $(x_3 \mid [D_0. v + 2.t.v'']$ ).

Then $[D_0. x_3.v + 2.t.(x_3.v'')] = x^2_3.v_0$ ; where $v_0$ is an integer ($\neq 0$).

Then by (16.2): $(v.v') = v_0$. Then it is possible to have as v and v' both are integers.

Thus it is possible to have ($x_3 \mid [D_0. P_Q.(x_3 - 1) / 2t\ ]$ ) and $x_3 \mid (D_0 + 2t)$.

}

Thus by (16.0): $[\ (P_Q / P_3) - (P_n / (x_3. P_3)\ )\ ] = [\ D_0.u + 2.t.u\ ] / (\ u.\ x^2_3\ ) = (D_0 + 2.t) / x^2_3$ ; where $P_Q$ is not divisible by $x_3$. Then according to the formula for u, still 'u' is not divisible by $x_3$.

Then $\{\ (\ P_L / x_3) + [(P_Q / P_3) - (P_n / (x_3. P_3)\ )\ ]\ \} = (\ P_L / x_3) + [\ (D_0 + 2t) / x^2_3\ ]$

$= [ P_L.x_3 + (D_0 + 2t) ] / \ x^2_3 = D' =$ integer number

Since $P_L$ is not divisible by $x_3$ , there exists such an integer number $D_0$ (not equals to zero), such that $x^2_3 \mid (P_L.x_3 + (D_0 + 2t) )$. Whenever $(2t + D_0)$ is divisible by $x_3$. Whenever $(2t + D_0)$ is not divisible by $x^2_3$.

Then by (16): $2.P – C – P_n = P_3. x_3 [ D' ]$ ; where $(P_L / x_3)$ and $(D_0 + 2t) / x^2_3$ are not integers (by 13.1). But we chose $D_0$ and t such that $x^2_3 \mid (P_L.x_3 + (D_0 + 2t) )$.

Then $2.P – C – P_n = P_3. x_3 . D'$ ;

where $D' = ( P_L.x_3 + (D_0 + 2t) ) / x^2_3 =$ integer , but not equals to 0.

Then $P_3 \mid (2.P – C – P_n)$………………………..(17)

by (15), (17): $P_3 \mid ( 2.l + M )$………………(18)

Thus by (i): $P_3$ does not divide $( 2.l + M )$…………………….(19). Thus by (18) and (19): We have a contradiction………………..(20)

Therefore the only possibility is: our assumption (1.0) is false. Therefore there are infinitely many Twin Prime Numbers.

**Proof**

Let's prove that there exists consecutive primes $P_N$ and $(P_N + A_1)$ such that $[ P_3 \mid (A_1 – 2) ]$ for some odd integer $P_3$ which is greater than 1 (when there exist consecutive prime numbers $P_N$ and $(P_N + A_1)$ which both are greater than $[P_{n-1} + 2]$ ) through this 'Proof' as below.

By 2nd reference: $P_{N-1} = (P_N + A_1 ) = 2 + \sum_{j=1}^{N-2} hj$ ,where $h_j = P_{j+1} – P_j$ for all $j \in \{1 , 2,.., (N-2) \}$ or $P_{N+1} = (P_N + A_1 ) = 2 + \sum_{j=1}^{N} hj$ when $j \in \{1 , 2,…., N\}$. Here $(P_N + A_1 ) = P_{N+1}$ or $P_{N-1}$ , depends on the sign of $A_1$.

$P_{N-1} = P_N + A_1 = 2 + \sum_{j=1}^{N-2} hj$ (when $A_1 < 0$). If $A_1 > 0$, $P_{N+1} = P_N + A_1 = 2 + \sum_{j=1}^{N} hj$

Consider the case that $A_1 < 0$.

Then $(A_1 – 2) = - P_N + \sum_{j=1}^{N-2} hj$ . Then $(2 – A_1) = P_N – \sum_{j=1}^{N-2} hj$

Then $(2 - A_1) = P_N - k' - \sum_{j=1}^{N-2} hj$ ……………………(21) Because here inside the term $\sum_{j=1}^{N-2} hj$ , I have included (+ k') term.

But by 2nd reference: for all $\epsilon > 0$, there is a natural number 'm' such that for all (N- 2) > m;

$h_{N-2} < P_{N-2} . \epsilon$

Let $\epsilon_s$ is a positive real number $\epsilon_s = [ - B + C_s + k' + P_N + B_2. P_3 ] / P_s > 0$, such that $h_s < P_s * \epsilon_s$ for all s > (N - 3) . Let here the chosen $\epsilon_s$ implies that m = (N - 3) (Here s is going from 1 to (N - 2). Then " for all s > (N - 3)" means s = (N – 2). Where k' is an integer number which not equals to 0 and we choose k' such that k' / (N - 2) is an integer. Here the chosen k' integer number is responsible for $h_s < P_s * \epsilon_s$ for all s > (N - 3) (i.e. s = N- 2) and $\epsilon_s > 0$. That means here the value of k' is responsible to say " $\epsilon_s$ is existing such that $h_s < P_s * \epsilon_s$ , for s = (N - 2) " .

Here $h_j = b_j - [ k'/ (N- 2) ]$ for all j < (N – 2) = s. And where $\Sigma b_j = B$ for j < (N – 2) = s. Then for some $C_s$ , $h_s = P_s * \epsilon_s - C_s$ ; here s ≡ (N – 2). *** the meaning of 'j' is the order number and $h_j$ is the prime gap between $P_{j+1}$ and $P_j$ . Please refer the below content and the 2nd reference. But here we chose $C_{N-2}$ such that $h_{N-2} = P_{N-2} * \epsilon_{N-2} - C_{N-2}$

But $h_{N-2} = P_{N-2} * \epsilon_{N-2} - C_{N-2} = ( - B + k' + P_N + B_2. P_3)$. Where k' is a natural number. Now let's use the 2nd reference to proceed further. By (21):

$(A_1 - 2) = - P_N + \sum_{j=1}^{N-2} hj = - P_N + ( - B + k' + P_N + B_2. P_3) + B - (N - 2).[ k'/ (N- 2) ] = B_2. P_3$ …………………………..(22)

Thus by (22): $(A_1 - 2) = P_3.B_2$ . Thus there exist consecutive prime numbers $P_N$ and $(P_N + A_1)$ both greater than $(P_{n-1} + 2)$ such that $(A_1 - 2) = B_2.P_3$ ; for integer $B_2$ ($\neq 0$).

Similar to above, if $A_1 > 0$, we can proceed with the similar steps to prove that $(A_1 - 2) = B_2.P_3$ ; for integer $B_2$ ($\neq 0$) when $A_1 > 0$.

**Proof 1**

Let's prove the existence of an integer ($P_N$ - 2) (> $P_{n-1}$ + 2 ) such that ($P_N$ - 2) = $P_3.x_3$ such that $P_3$ is divisible by $x_3$. But $x^2_3$ does not divide $P_3$ as below.

By 2nd reference: $P_N = 2 + \sum_{j=1}^{N-1} gj$ , where $g_j = P_{j+1} - P_j$ for all j ϵ {1 , 2, ……., N- 1 }

Then $(P_N - 2) = \sum_{j=1}^{N-1} gj$ ……………………….(23)

But by 2nd reference: for all Є > 0, there is a natural number '$m_0$' such that for all N > $m_0$; $g_N < P_N$ .Є.

Let $Є_s$ is a positive real number $Є_s = [ - A + C_s + x^2_3 . k_1 ] / P_s > 0$ , such that $h_s < P_s* Є_s$ for all s > (N - 2) . Let here the chosen $Є_s$ implies that $m_0$ = (N - 2) (Here s is going from 1 to N-1. Then " for all s > (N - 2)" means s = (N – 1) ). Where $k_1$ is an integer number which is not divisible by $x_3$. And let $k_1 = P_Q. (x_3 - 1) / 2t$. Where $P_Q$ is an integer which is not divisible by $x_3$. Here the chosen $P_Q$ and t integer numbers (≠ 0) are responsible for $g_s < P_s* Є_s$ for all s > (N - 2) (i.e. s = N- 1 ) and $Є_s > 0$. OR in another words, the previously chosen integer M and t are responsible to say that above fact. That means here the value of $k_1$ (or in another words, $P_Q$ and t) is responsible to say " $Є_s$ is existing such that $g_s < P_s* Є_s$ , for s = N - 1 ". Here $g_j = a_j$ for all j < (N – 1) = s. And where $\Sigma a_j$ = A for j < (N – 1) = s. Then for some $C_s$ , $g_s = P_s* Є_s - C_s$ ; here s ≡ (N – 1). *** the meaning of 'j' is the order number and $g_j$ is the prime gap between $P_{j+1}$ and $P_j$ . Please refer the below content and the 2nd reference. But here we chose $C_{N-1}$ such that $g_{N-1}$ = $P_{N-1} * Є_{N-1} – C_{N-1}$.

But $g_{N-1} = (- A + x^2_3. k_1)$. Now let's use the 2nd reference to proceed further.

By (23): $(P_N - 2) = \sum_{j=1}^{N-1} gj$. But $\sum_{j=1}^{N-1} gj = A + ( - A + x^2_3. k_1) = x^2_3. k_1$ ………………….(24)

Thus by (23) and (24): $(P_N - 2) = x^2_3. k_1$ ; where $k_1$ is not divisible by $x_3$.

Then $(P_N - 2) = x_3.(x_3.k_1) = x_3. P_3$ ; where $P_3$ is divisible by $x_3$.

Thus $(P_N - 2) = P_3. x_3$ ; where $P_3$ is divisible by $x_3$. But $P_3$ is not divisible by $x^2_3$ .

Thus there exists an integer set {$P_3. x_3$ : where $P_3$ is divisible by $x_3$ but $P_3$ is not divisible by $x^2_3$}.

**Discussion**

We assumed initially that there are finitely many twin primes. After proceeding with that, I ended up with a contradiction. But to get the contradiction, I used that $P_N$ as a prime number greater than ($P_{n-1}$ + 2). And we chose $P_n$ odd integer ( > 1) and also we chose an integer $A_1$ such that $P_3 \mid (A_1 - 2)$. Also to get the contradiction, I used the facts that ($P_N$ + 2) and ($P_N$ - 2) as non-primes since $P_N - 2 > (P_{n-1} + 2)$ . And also I have used that $x_2$ and $x_3$ as natural numbers (since, ($P_N$ + 2) and ($P_N$ – 2) are not prime numbers). And also I have used the fact (to get the contradiction as in (20) ): The difference between any two consecutive prime numbers (which are greater than ($P_{n-1}$ + 2) ) is greater than 2. Therefore to get the contradiction, I have used the facts got from our assumption (1.0). Then the only possibility is our assumption (1.0) is false.

## Results

Therefore I have used our assumption (1.0) to get the contradiction finally, as showed in (20). Therefore it is possible to conclude that our assumption (1.0) is false. Thus the negation of the assumption (1.0) is true.

**Thus there are infinitely many twin prime numbers.**

## Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote ‘i’ th prime gap $g_i = P_{i+1} – P_i$

Then according to the $2^{nd}$ reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} gj$

Also by $2^{nd}$ reference: for all $\epsilon > 0$, there is a natural number ‘n’ such that for all N -1 > n;

$g_{N-1} < P_{N-1} . \epsilon$

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