

Discretization of the gravitational field

Introduction: The fine structure of the Cosmos

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The last century has left us four unresolved facts about a possible "fine structure" of the Cosmos that, according to some theories already existing at that time, it seems that could shed light on some of the great current frustrations of physics, such as, for example, the unification of Quantum Mechanics with the General Theory of Relativity.

These facts have the characteristics of true mysteries because, although their existence has been verified independently by several researchers, they have not found a satisfactory explanation.

They reveal a "fine structure" like that of the H atom, although it is more "discrete" because even though it has some of the characteristics of the quantization of the atom, it does not have all of them and is also less restrictive, more "soft". Due to these characteristics and the lack of an explanation, these facts have fallen into oblivion.

One of the attempts at explanation wrongly used wave mechanics, due to the similarity with the quantization of the atomic orbitals, but the phenomenon is evidently independent of the mass, which makes the use of Quantum Field Theory more plausible.

Although if the basic models of quantum gravity proposed at that time do not fit properly, a review of them shows that if a different constant is used to compute the field energy, everything square up. The computations of the Solar System orbits discretization seem to confirm it.

This article is just a presentation of these mysteries and how they lead to discover a probable fine structure of the Cosmos, giving only a global view of the integration of theories that help explain it.

Mystery #1: The cosmological constants

The first mystery was discovered by the physicist and astronomer Subramanyan Chandrasekhar who found a non-dimensional relationship between fundamental variables (the Planck constant h , the speed of light c , the universal gravitation constant G and the mass of the proton m_p) that raised to a some power they allowed to obtain the order of magnitude of a maximum mass of the stars (using the exponent 3/2), the galaxies (using the exponent 7/4) and the Universe (using exponent 2):

$$M_v \approx \left(\frac{hc}{G}\right)^v \cdot \frac{1}{m_p^{2v-1}}$$

At first he was not encouraged to publish his discovery because he could only explain it to the stars with his theory of stellar evolution (which earned him the Nobel Prize in 1983), but an article by the mathematician and physicist Paul Dirac in the NATURE magazine, convinced him to publish his discovery. He did it in May 1937 in the same journal, using the same title as Dirac: "Cosmological constants".

His discovery was resumed and expanded a decade later by Albert Wilson.

Mystery #2: The hierarchical structure of the Cosmos

The second mystery was found in the middle of the last century by Albert Wilson, an American astronomer who worked at the Douglas Advanced Research Laboratory.

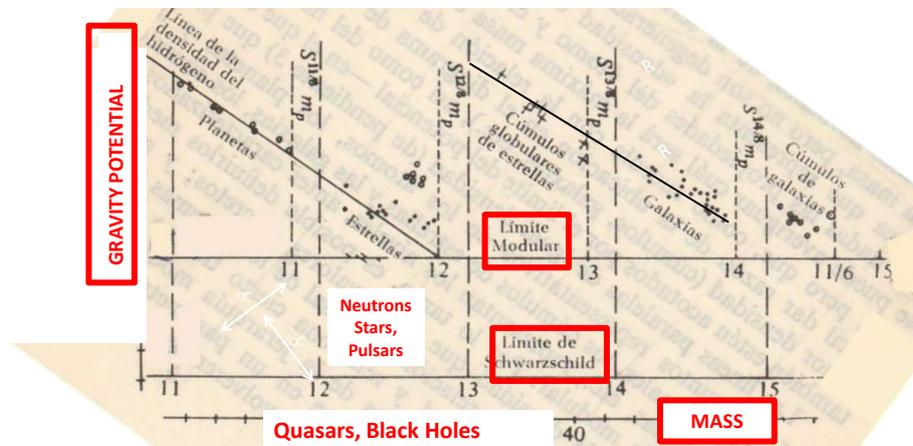
Wilson positioned the main celestial objects known in his time (the Solar System planets, stars, star clusters, galaxies and galaxy clusters), in a graph where he indicated the masses in increasing order on the horizontal axis and in the vertical axis the absolute value of the gravity potential, in decreasing order.

Then he marked the limits of the masses of each category with vertical segmented lines always leaving a range of masses between one category and another.

The result was a clearly defined modular structure of the Cosmos with a hierarchical order from right to left, as it can be seen in the figure below.

In this chart, celestial objects with similar densities are distributed following inclined lines. Since planets and stars in the main sequence have similar densities, the linear distribution of both crosses its categories with a single line. The same happens with star clusters and galaxies. The super-giant stars and the white dwarfs, as well as the galaxy clusters have their own densities, so they are placed in other positions of the graph, but always following sloping lines mostly quite parallel to the others.

Proposal of a hierarchical structure of the Cosmos



Source: Wilson et al.-1967

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What was surprising is that the lower limit of the gravitational potential of all of them was not given by the Schwarzschild Radio but by an upper limit that Wilson called "Modular Limit".

Wilson did neither position neutron stars, nor black holes because, although at that time they had already been theoretically announced, none of them had yet been observed. However, today the existence of these objects has already been confirmed, so they can also be located on the graph below the line of the Modular Limit.

This simple and precise distribution on the chart, of the celestial objects known at the time, convinced Wilson of the "universal" importance of modular structures to the point that in 1968 he organized a two-day conference, sponsored by Douglas, inviting specialists from different types of modular structures (conceptual, inorganic, organic and artifacts) to analyze them in detail and thus compare them.

However, it does not seem to have caught the attention of the scientific community.

The cosmic mass structure of Wilson

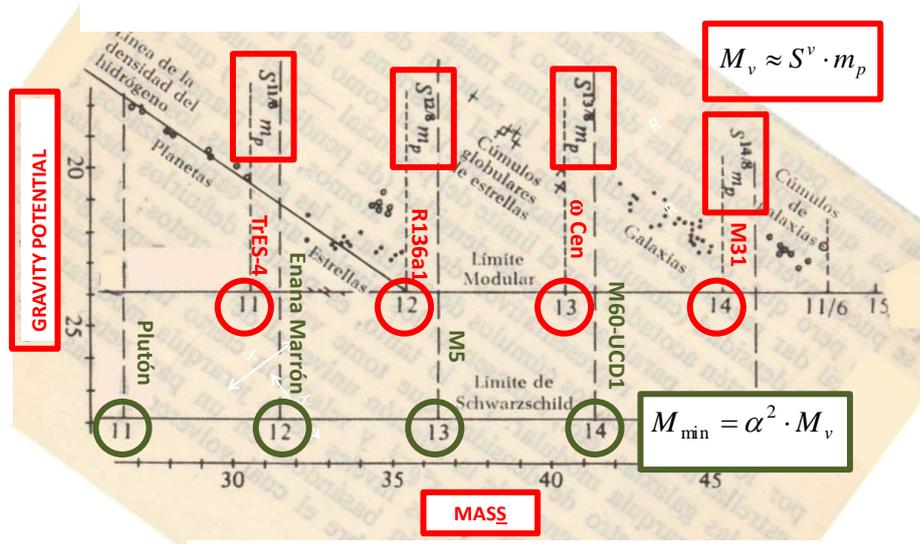
Returning to the formula of Chandrasekhar for the limits of the maximum masses and working on its constants to simplify it, Wilson realized that it was possible to extend it, with very good approximation, to the other stars of his graph by simply multiplying powers of the a-dimensional constant S by the proton mass.

$$M_v \approx \left(\frac{hc}{G}\right)^v \cdot \frac{1}{m_p^{2v-1}} \approx S^v \cdot m_p$$

S results from the relationship between the electric and gravitational attraction forces exerted between a proton and an electron at any distance. The result is a very large number since the electric force is of the order of 10^{38} times greater than the gravitational force.

The exponents ν used by Wilson for the different categories of celestial objects were fractions with denominator 8 and a continuous succession of numerators between 11 and 14 (for planets $11/8$...), keeping those of Chandrasekhar for stars and galaxies. In the galaxy clusters he found different fractions, but always as powers of S. He pointed out the formulas (red boxes) of these limits on the vertical lines of short segments of his graph indicating the numerators of the fractions below the modular limit (red circles).

The limits of the cosmic masses



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The names on the vertical lines with short segments are the celestial objects of maximum mass currently known: Exoplanet TrES-4 (constellation of Hercules), Star R136a1 (constellation of Dorado), Globular cluster Omega Centauri (constellation of Omega Centauro), Galaxy M31 (constellation of Andromeda). Some exceed the established limits, but they are still being verified since the definition of these limits made by Chandrasekhar referred only to celestial objects constituted by a certain type of matter, while those of this example have not yet passed this control.

Wilson also realized that he could obtain the minimum mass of these celestial objects by multiplying the maximum mass by the square of the fine structure constant α and he pointed them out by indicating the numerators of the fractions below the Schwarzschild limit (green circles).

The names on the vertical lines with long segments are the celestial objects of minimum mass currently known: Planet Pluto, Brown dwarfs stars, Globular cluster M5 (Serpens), Galaxy M60-UCD1 (Virgo). Here also some are below the established limits but the same condition indicated in the case of maximum masses applies.

Even today these limits are still valid and their origin remains a mystery (with the exception of the maximum limit of 12/8 in the stars). However, no one talks about it, even though the presence of the fine structure constant α in the structure of the Cosmos would seem to relate it to the H atom.

The gravity potentials of Wilson

As it is well known, the gravitational potential of a celestial object relates its mass to its radius, determining the square of the orbital velocity of anybody that orbits freely at a short distance from its surface.

Wilson realized that the Modular Limit, that is, the minimum gravitational potential of all the celestial objects considered, was proportional to the mass of a proton multiplied by the constant S and divided by the radius of the lower orbital of the H atom indicated as a_0 :

$$-V_{min} = \frac{G \cdot S \cdot m_p}{a_0} = v_{H1}^2 = (\alpha \cdot c)^2$$

This implied that anybody that orbits around a star with that gravitational potential would have a velocity v_{H1} equal to that of the electron in the lower orbital of the H atom, that is, a value equal to the fine structure constant α multiplied by the velocity of the light in a vacuum c and everything squared.

And this is precisely what relates the constant α to the fine structure of the H atom since this structure is not more than the manifestation of the energetic levels of the electron in its possible orbitals from which it falls to lower levels releasing energy and producing colored lines or those that jump absorbing energy and producing black lines in the spectrum.

And the magnitude of the main energy levels is obtained by multiplying the relativistic energy at rest of the electron by the constant α/n squared:

$$E_n = m_e \cdot c^2 \cdot \left[\frac{1}{2} \left(\frac{\alpha}{n} \right)^2 \right]$$

With these structures (the one of the masses and the gravitational potentials) Wilson discovered that the atomic dimensions are present in the structure of the Cosmos, initiating without knowing it, a series of coincidences between the Cosmos and the atom of H.

Although this further reinforces the inexplicable connection of the structure of the Cosmos with that of the atom, this mystery also fell into oblivion.

Mystery #3: Controversy on the differential red shift in pairs of galaxies

In the 1980s, a third mystery appeared: a controversy over the regularity of differential red shift in pairs of apparently morphologically associated galaxies that could be forming a system where galaxies revolve around a common center of gravity.

This orbital spin of galaxies could take thousands or millions of years, so it is not possible to observe it, that is why you must study the morphological associations to determine if there is a connection and form what is called a self-gravitating system, that is, a system where galaxies revolve around a common center of gravity.

What is this controversy about?

In the middle of the last century, two American astronomers, William Tifft of the Steward Observatory in Arizona and Halton Arp of the Palomar Observatory in California, analyzed the spectra of pairs of apparently physically connected galaxies that could be forming a binary system orbiting a common center of gravity, to study the speed differences between them.

They expected a random distribution of them. Moreover, they expected that some of the galaxies of the pair had a movement of approach towards us, that is to say that the individual light had a shift towards the blue because its movement was towards us and its speed exceeded the speed of expansion of the universe in the place where they were, which would give a resultant in the direction of the observer and therefore a shift of the spectral lines toward the blue.

But none of that happened. All galaxies, regardless of their distance, had a red shift and the differences in their speeds an unexpected regularity. Indeed, the difference in the velocities of the pairs of apparently associated galaxies had a maximum value or a submultiple of it with a curious resemblance to the energy levels of the fine structure of the H atom where the electron velocity in the lower orbital is divided by n , as we have seen before.

But the maximum speed was different in each case. Indeed, Arp found a speed of approximately 144 km/s and Tifft half while the speed v_{H1} implicit in α is 2,183 km/s, much higher than both.

However, the comparison with the atom ended in the mathematical formula, since in it the velocities of the electrons are related to their turns around the nucleus, while in galaxies the velocities are related to their movement with respect to the observer and not between them.

This fact and the lack of a shift towards the blue in the spectra, led Halton Arp to think that, in this case, the red shift should not be attributed to the movement in relation to us but to a slower rhythm of time, due to the gravitational potential that one galaxy exerted on the other, an effect predicted by the General Relativity Theory. Regrettably, he never found a model that would provide a satisfactory explanation.

Although many astronomers independently found the same differences in many other pairs of galaxies this mystery was also forgotten, being relegated only to a mention in some catalogs of galaxies.

Mystery #4: Clues to discretization in the Cosmos

The fourth and last mystery appeared in the 1990s, at the end of the last century: Angelo Agnese and Roberto Festa, two physicists from the University of Genoa in Italy, inspired by the quantization of speeds in the pairs of galaxies realized that the same formula, with a reference speed close to that found by Arp, could be applied to different parameters of the cosmic objects, including the orbits of the Solar System and the exoplanets known at that time.

This time it was possible to compare with the atom because, as in it, the velocities were not relative movement with respect to the observer but those of the orbital movement around a center of mass.

But there were still two important differences with the atom:

1. the speed in the lower orbit of ALL GRAVITATORY SYSTEMS, was always equal to 137 km / s, independently of the central mass, while in the atom it depends on the electric charge of the nucleus and also
2. the electrons occupy successive orbits, with $n = 1,2,3, \dots$, while the planets do so with discontinuous values that do not even start at 1. Mercury, Venus, Earth and Mars have continuous values from 3 but the other planets do not!

Clues to discretization in the Cosmos

Planet	<i>n</i>	<i>v_{obs}</i> (km/s)	<i>v_{calc}</i> (km/s)
Mercury	3	47.87	47.90
Venus	4	35.02	35.92
Earth	5	29.78	28.74
Mars	6	24.13	23.95
Jupiter	11	13.06	13.06
Saturn	15	9.64	9.58
Uranus	21	6.80	6.84
Neptune	26	5.43	5.53
Pluto	30	4.74	4.79

Figura 1 – Comparación de velocidades calculadas y observadas
Fuente: Agnese & Festa, ArXiv: Cornell University Library

Source: Agnese A., Festa R.-1997-8

Star	Type	<i>M_{star}</i>	<i>P</i> ^(obs)	<i>a_{star}</i>	<i>M_{j-1}</i>	<i>M_j</i>	<i>M_{j+1}</i>	<i>n</i>	<i>a_j</i> ^(calc)
51 Peg	G2IVa	1.05	4.229	0.050	-	1.30	0.16	1	0.056
ups Androm.	F7V	1.20	4.611	0.057	-	1.42	0.18	1	0.061
55 Cancer	G8V	0.90	14.648	0.110	4.50	0.56	0.17	2	0.097
rho CrB	G0V-G2V	1.05	39.645	0.230	12.18	1.52	0.45	2	0.261
16 Cyg B	G2.5V	1.05	804.000	1.720	1.98	1.14	0.72	6	1.766
47 Uma	G0V	1.05	1088.445	2.110	1.55	0.98	0.65	7	2.050
tau Bootis	F6IV	1.30	3.313	0.046	-	1.02	0.13	1	0.044
70 Virgo	G4V	0.90	116.600	0.430	4.48	1.33	0.56	3	0.512
HD 114762	F9V	1.20	84.050	0.300	3.23	0.96	0.40	3	0.369
HD 110833	K3V	0.73	270.040	0.800	1.30	0.66	0.38	5	0.712
BD-04 782	K5V	0.67	240.920	0.700	1.16	0.59	0.34	5	0.635
HD 112758	K0V	0.79	103.220	0.350	1.17	0.50	0.25	4	0.340
HD 98230	F8.5V	1.30	3.980	0.060	-	1.22	0.15	1	0.052
HD 18445	K2V	0.73	554.670	0.900	1.36	0.79	0.50	6	1.219
HD 29587	G2V	0.98	1157.843	2.500	1.65	1.04	0.69	7	2.180
HD 140913	G0V	1.05	147.940	0.540	1.68	0.71	0.36	4	0.488
HD 283750	K2	0.73	1.790	0.040	-	0.55	0.07	1	0.024
HD 217580	K4V	0.70	454.660	1.000	1.12	0.65	0.41	6	0.999
Alpha Tau	K5III	1.20	654.000	1.350	1.61	0.93	0.59	6	1.437
Prox. Cent.		0.10	42	0.20	0.10	0.06	5	0.111	
Barnard's		0.12	132.0	0.19	0.12	-	7	0.249	

Figura 4 – Comparación de los períodos de traslación calculados y observados en los nuevos planetas extrasolares
Fuente: Agnese & Festa, ArXiv: Cornell University

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This made the discoverers not recognize this effect as a "quantization of the orbits", as in the case of the atom, but as something more "soft", something they called "discretization".

The universality of the relationship between the speed of reference and that of light in a vacuum led them to propose a new universal constant that they called α_g because of its similarity to the constant α of the fine structure.

The discoverers explained this "cosmic discretization" with a theory that is a variant of the model used to explain the quantization in the atom.

Since this variant has no solid bases and has not been proven in other situations, other alternative explanations based on the General Theory of Relativity or the fractals appeared, but none was able to deduce the value of the constant α_g , not even the theory used by the discoverers. In all of them the reference value is obtained by multiplying the orbital speed of the planet Mercury by 3, without giving any reason.

Inexplicably this mystery was also forgotten, although this time the similarity with the H atom was even greater.

The "fine structure of the Cosmos"

The first two mysteries have continuity between Chandrasekhar and Wilson because the latter extended the formula of the maximum masses to the entire Cosmos and then, the third and fourth mysteries had continuity between Arp and Agnese-Festa because the latter realized that the differential velocity of the pairs of galaxies of Arp was approximately three times that of Mercury. But there was a discontinuity between

the second and the third mysteries since neither Arp nor Tifft related their discovery to the hierarchical structure of the Cosmos.

One could then ask if all these mysteries have something in common and the answer is yes, since the constant α_g is present in all of them.

In addition, the mass of the electron also follows Wilson's rule if an exponent equal to 0 is put to the constant S and since according to Chandrasekhar the mass of the Cosmos has an exponent equal to 2, the electron and the Cosmos constitute the extremes of this structure of masses.

Since the value found by Agnese and Festa for α_g is equal to the square of the fine structure constant α multiplied by 3, both constants are related.

The constant α_g also separates the modular limit from the Schwarzschild limit:

$$\text{Limite Modular} = v_{H1}^2 \approx \alpha^2 \cdot c^2 = \frac{G \cdot S \cdot m_p}{a_0}$$

It can be said then that the integration of all these structures could constitute a "fine structure of the Cosmos" and that, as intuited by Agnese and Festa, α_g , just like α in the atom, is who can allow to compute it.

The explanation of this structure should be a challenge for physics and astronomy, as it was at the time to explain the fine structure of the H atom since there are no apparent reasons for it. What is doing that the structure of the atom reproduce itself in the Cosmos?

The little follow-up that these mysteries have had up until now prevented us from connecting them with the existing theories that seem to explain them and that are what would allow us to answer the previous question and to unify Quantum Mechanics with the General Theory of Relativity, one of the greatest challenges of our time.

But this situation has changed, at least at the level of the integration of the theories, their connection to explain the mysteries and their verification in the reality of the Cosmos. This is what will be seen next.

Theoretical foundations of explanation

This explanation could have been made 40 years ago, if the correct constant had been used to determine the gravitational field energy and the correct alternative of Quantum Mechanics had been selected. Because the theories needed to understand the phenomena had already been formulated by Matvey P. Bronstein in 1931 (see Gorelik Gennady -1992), Leon Rosenfeld in the 60s and Hans-Jürgen Treder in the '70s.

In fact, of the two main aspects of Quantum Mechanics, attempts to explain the discretization of the Cosmos were made using wave mechanics, given the similarity of their orbits with those of the H atom.

This choice is wrong because the length of the wave associated with the electron depends not only on its speed but also on its mass, whereas the discretization found in the Cosmos depends, as we have seen, only and exclusively on the speed.

The correct choice is the Quantum Fields Theory that is independent of the mass of the bodies in orbit and introduces the need for the orbital levels to be greater than 1 justifying that the minimum levels found has higher values. That is, finding levels of the order of 100 or 1000, is not strange, on the contrary ensures greater precision in the calculation of energy.

Once these two errors have been corrected, existing theories can be integrated and explain with incredible accuracy the phenomenon of discretization in self-gravitating systems.

The theory that serves as the basis for the explanation of the "Fine Structure of the Cosmos" is within this second quantization and is the Theory of the errors of measurement of the components of the fields, developed in 1933 for electromagnetism, by Niels Bohr and León Rosenfeld.

From here it was that Leon Rosenfeld took out a proposal for a hypothetical weak gravitational radiation that is fundamental to explain the "discretization" of the gravitational field and to complete the quantum gravity model making it equivalent to that of the electromagnetic fields.

Regrettably, he used the wrong constant in his analyzes, so he got results that were not satisfactory. This made him think that the result was not definitive leading him to suggest that the quantization process should be found in an empirical way that is, based on natural facts.

At the time he formulated this proposal, these facts did not yet exist, but as we have seen, the situation had changed and the mysteries can be used to formulate and prove a theory.

Once the handicap of the constant has been corrected, the weak gravitational radiation takes shape through the discretization data of the Solar System. Although if it seems that there is no evidence of this radiation, it is actually likely that we have it in front of our eyes every time we observe objects in motion. The explanation of this "presence" is in the theory of relativity.

In effect, Einstein made it clear that the "apparent" mass increase due to speed is not an increase in mass but energy. This additional energy is enclosed in the volume of the

moving body, giving rise, by the Ampere-Maxwell law applied in the gravitational domain, to an oscillating gravitational field generating a weak gravitational radiation that “drive” the mass motion. This field becomes apparent when analyzing the density of the body's momentum or when it introduces an uncertainty in the intensity of the gravitational field. The latter seems to be the origin of cosmic discretization.

The proposed weak gravitational radiation model is not the only one to consider the possibility that a moving mass is guided by a wave. The same does the De Broglie-Bohm Theory, postulated by physicist David Bohm in 1952, about hidden variables of quantum physics.

In the second half of the last century, the theory of measurement errors was also used by Hans-Jürgen Treder, member of the Academy of Sciences of the German Democratic Republic, in his attempt to unify Quantum Mechanics with the General Theory of Relativity through uncertainty in the measurement of the gravitational field. This theory is fundamental to explain the discretization of the orbits of the self-gravitating systems of the Cosmos which, in turn, is the proof that Quantum Mechanics and Relativity are united.

Treder realized the uselessness of insisting on the integration of the two theories due to the strong mathematical incompatibility between them, reason why he came to the conclusion that he had to look for the unification on other way.

The "inaccuracy" of the Energy-Moment cuadvivector of the General Theory of Relativity, because it was not a true tensor, offered him the opportunity he was looking for since it left room for uncertainty in determining the intensity of the gravitational field, uncertainty that Treder thought that it should be given by the Heisenberg Principle.

In the development of his formalism, Treder used the answer that Niels Bohr gave to Albert Einstein in one of the most famous discussions that physics had in the last century during the 6th Solvay Congress in Belgium, where Einstein challenged quantum theory by proposing a mental experiment where a measurement was made with any error. Bohr replied by showing that the error was introduced by his General Theory of Relativity. This answer served Treder to complete his theory.

The result led him to discover two new variants of the Uncertainty Principle that introduce an error in the relativistic metric of space-time denominated with the variable $g_{i,k}$. As you can see in the following figure, this error is related to α_g and its formulation also shows that this is not really constant since it depends on the density of the orbiting body, indicated with the Greek letter ρ , under the attraction of the gravitational field . The letter K indicates a constant value.

But this does not diminish the validity of the identification of α_g as a universal constant since also the relation between the speed of the lower orbital and the one of the light is different in atoms with nuclei with greater charges than the one of the H.

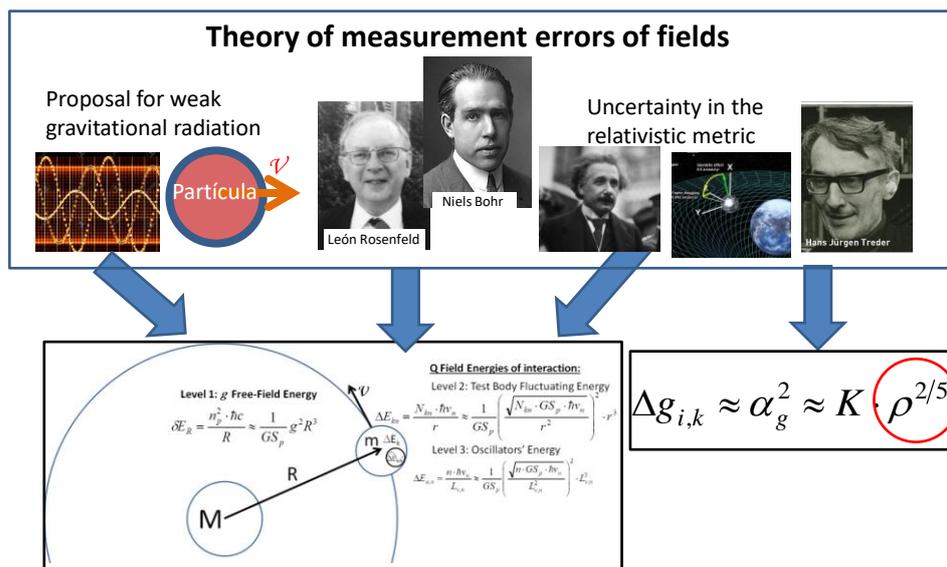
The "universal" value of α_g obtained by Agnese and Festa is due to the fact that the cosmic densities are very similar and that α_g varies based on a very small potency of them. For example, the planet Saturn, with a density of 620 kg/m^3 gives $\alpha_g=0,00043$ while the Earth, with a density of 5.497 kg/m^3 gives $\alpha_g =0,00066$.

The theory that explains the fine structure of the Cosmos could also show that in galaxies with gravitational potential close to the Modular Limit, the value of α_g is similar to α^2 ($\alpha_g \approx \alpha^2$) which it would produce a difference in the gravitational red shift that would give the appearance of a speed equal to those observed. It would be for this reason that the Arp and Tifft formula differ in maximum speed and do not apply to all pairs of associated galaxies. This is the explanation that Halton Arp was looking for.

From the integration of these theories, it appears that the gravitational field produced by the interaction between the central field and the orbiting body is composed of quantum oscillators of dimensions compatible with Rosenfeld's weak radiation. These oscillators are the ones that induce the discretization of the orbits according to the Treder new uncertainty principles.

The following figure is a graphic summary of the process described above and the actors involved.

Theoretical foundations of explanation



Pic. A-Hierarchical structure of the oscillators – Author: J.A. Pardi

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Conclusion

According to the integration of the aforementioned theories, the fine structure of the Cosmos would be the visible manifestation that the space time around a mass is not continuous but discrete, although if in isolated bodies this discontinuity is practically undetectable. In the self-gravitating systems the discretization can increase until manifesting in the dimensions of their orbits and its magnitude will depend on the density of the celestial bodies involved.

Given the discontinuity of quantum numbers and the absence of "quantum leaps" similar to those of electrons, the manifestation of this discretization is not very evident, even in multiple self-gravitating systems such as those existing in the Solar System, requiring analysis of a few orbital parameters to be detected. This characteristic distinguishes the "discretization" of the "quantization", hindering its acceptance, as it happened with the latter when it was discovered in the H atom.

However, its acceptance and subsequent study could reveal new and surprising characteristics, as happened with "quantization". An example could be the explanation of the "double slit" experiment using gravitational radiation hidden in moving bodies.

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