SOME CONJECTURES ON INEQUALITIES IN OPERATOR AXIOMS

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Abstract. The Operator axioms have deduced number systems. In this paper, we conjecture some inequalities in Operator axioms. The general inequalities show the value of Operator axioms.

1. Introduction

In [3], we define the Operator axioms to extend the classical real number system. The classical real number system only includes special irrational numbers. However, Operator axioms import the general irrational numbers to construct a complete real number system. In this paper, we apply Operator axioms to generalize various inequalities. The general Bernoulli's Inequality show the value of Operator axioms.

In [4, TABLE 2], we have defined some replacements for the notations of the Operator axioms. In this paper, we will apply these concise replacements. For convenience, we define two symbols '$>$' and '$\geq$'. Then we add two axioms as follows into Operator axioms:

$$\begin{aligned} & \Psi\{ \\ (OA.116) \quad & (\bar{a} > \bar{b}) \Leftrightarrow (\bar{b} < \bar{a}), \\ (OA.117) \quad & (\bar{a} \geq \bar{b}) \Leftrightarrow (\bar{b} \leq \bar{a}) \\ & \}. \end{aligned}$$

In this paper, we suppose that $n \in N$. In this paper, we suppose that $a \in [1, +\infty)$, $b \in [1, +\infty)$, $c \in [1, +\infty)$ are real numbers.

The paper is organized as follows. In Section 2, we conjecture some commutative inequalities in Operator axioms. In Section 3, we conjecture some distributive inequalities in Operator axioms. In Section 4, we define the general distance inequality in Operator Axioms and conjecture a general distance inequality in Operator Axioms. In Section 5, we conjecture a general Bernoulli's inequality in Operator axioms.

2. Commutative Inequalities In Operator Axioms

Conjecture 2.1. *The following inequality holds:*

$$[[a +'_n a] + [b +'_n b]] \geq [[a +'_n b] + [b +'_n a]]$$

Conjecture 2.2. *The following inequality holds:*

$$[[a +'_n a] + +[b +'_n b]] \geq [[a +'_n b] + +[b +'_n a]]$$

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Conjecture 2.3. *The following inequality holds:*

$$[[a -'_n a] + [b -'_n b]] \leq [[a -'_n b] + [b -'_n a]]$$

Conjecture 2.4. *The following inequality holds:*

$$[[a -'_{[n+1]} a] + +[b -'_{[n+1]} b]] \leq [[a -'_{[n+1]} b] + +[b -'_{[n+1]} a]]$$

Conjecture 2.5. *The following inequality holds:*

$$[[a/'_n a] + [b/'_n b]] \leq [[a/'_n b] + [b/'_n a]]$$

3. Distributive Inequalities In Operator Axioms

Conjecture 3.1. *The following inequality holds:*

$$[[a +'_{[n+2]} [b + c]]] \geq [[a +'_{[n+2]} b] +'_{[n+1]} [a +'_{[n+2]} c]]$$

Conjecture 3.2. *The following inequality holds:*

$$[[a +'_{[n+1]} [b + +c]]] \geq [[a +'_{[n+1]} b] +'_{[n+1]} c]$$

Conjecture 3.3. *The following inequality holds:*

$$[[a -'_{[n+2]} [b + c]]] \leq [[a -'_{[n+2]} b] +'_{[n+1]} [a -'_{[n+2]} c]]$$

Conjecture 3.4. *The following inequality holds:*

$$[[a + b] +'_{[n+1]} c] \geq [[a +'_{[n+1]} c] + [b +'_{[n+1]} c]]$$

Conjecture 3.5. *The following inequality holds:*

$$[[a + +b] +'_{[n+2]} c] \geq [[a +'_{[n+2]} c] + +[b +'_{[n+2]} c]]$$

Conjecture 3.6. *The following inequality holds:*

$$[[a + b] -'_{[n+1]} c] \leq [[a -'_{[n+1]} c] + [b -'_{[n+1]} c]]$$

Conjecture 3.7. *The following inequality holds:*

$$[[a + +b] -'_{[n+1]} c] \leq [[a -'_{[n+1]} c] + +[b -'_{[n+1]} c]]$$

Conjecture 3.8. *The following inequality holds:*

$$[[a + b]/'_{[n+1]} c] \geq [[a/'_{[n+1]} c] + [b/'_{[n+1]} c]]$$

4. General Distance Inequality In Operator Axioms

Definition 4.1. $A = (a_1, a_2, \ldots, a_n)$ and $B = (b_1, b_2, \ldots, b_n)$ are two points in R^n. If $i, k \in N$, then the following formula is the general distance between A and B:

$$\big[\big(\sum_{k=1}^{n} [[a_k - b_k] +'_i 2]\big) -'_i 2\big]$$

Conjecture 4.2. *$A = (a_1, a_2, \ldots, a_n)$ and $B = (b_1, b_2, \ldots, b_n)$ are two points in R^n. If $i, j, k \in N$ and $i < j$, then the following inequality holds:*

$$\big[\big(\sum_{k=1}^{n} [[a_k - b_k] +'_i 2]\big) -'_i 2\big] > \big[\big(\sum_{k=1}^{n} [[a_k - b_k] +'_j 2]\big) -'_j 2\big]$$

5. General Bernoulli's Inequality In Operator Axioms

Conjecture 5.1. *If $d \in [2, +\infty)$, then the following inequality holds:*

$$[[1 + a] +'_{[n+1]} d] \geq [1 + [a +'_n d]]$$

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