‘Supralogic’
or
A method for predicting stochastic mapping outcomes by interpolating their probabilties

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Abstract

In a stochastic mapping model, a method is described for interpolating un-sampled mapping probabilities given a succesive set of observed mappings. The sampled probabilities are calculated from the observed mappings. The previously described method of interpolating values in code space is used to interpolate the un-observed mapping probabilities. The outcomes for subsequent mappings can then be predicted by finding the processes with maximal interpolated probability.

1 Introduction

This paper takes the result from the previous paper [1] and uses it to explain how to interpolate the probabilities of stochastic mapping processes and so predict their outcomes. This is of use, for example, in classifying strings of symbols, guessing truth table outputs for missing rows, or filling out missing database cell values when the database is incomplete and one wishes to find the most likely possible value.

Subsequent papers will implement the results laid down here to demonstrate these facilities.

2 Interpolating values on an undirected graph

2.1 Defining terms

We consider a stochastic mapping, f, which maps the set $M \equiv \{\mathbf{m} = (m_1, m_2, \cdots, m_D)\}$ onto the set $M_0 \equiv \{m_0\}$ where the indices $0 \le m_j \le \mu_j \, \forall j : 0 \le j \le D$,

$$f : M \rightsquigarrow M_0 \tag{1}$$

and where given mappings have an associated normalised probability function $P_f : M \times M_0 \to [0,1]$ such that $\sum_{m_0} P_f(\mathbf{m}, m_0) = 1 \, \forall \, \mathbf{m}$. Suppose a set of a total of N_s stochastic mappings has been observed $\{\mathbf{m}(\tau) \to m_0(\tau)\}$ where $1 \le \tau \le N_\mathrm{s}$. A set of sampled probabilities can then be calculated from these observations,

$$\rho(\mathbf{m}, m_0) = \frac{1}{n(\mathbf{m})} \sum_\tau \delta_{\mathbf{m},\mathbf{m}(\tau)} \, \delta_{m_0, m_0(\tau)} \quad , \quad \forall \, \mathbf{m} : n(\mathbf{m}) \neq 0 \tag{2}$$

where $n(\mathbf{m}) \equiv \sum_\tau \delta_{\mathbf{m},\mathbf{m}(\tau)}$. These probabilities, with $\mathbf{m} : n(\mathbf{m}) \neq 0$ will be considered to be observed or sampled probabilities for the mapping f. The cases of $\mathbf{m}$ where $n(\mathbf{m}) = 0$ will be regarded as unobserved or un-sampled mapping probabilities. This paper will now seek to apply the interpolation equation to give estimated expected probabilities for the unobserved mapping probabilities, given the observed probabilities.

2.2 Application of the interpolation equation

For each of the possible associated values $\{m_0\}$ there is an associated probability either observed by sampling or inferred by interpolation. This necessitates the use of a set of the following independent interpolation equations,

$$\{\Delta\,\rho(\mathbf{m}, m_0) = Q(\mathbf{m}, m_0)\} \quad , \quad \forall\, m_0 : 0 \le m_0 \le \mu_0 - 1 \tag{3}$$

2.3 Conservation of probability

Theorem 2.1. *For normalised distributions at the sample points $\rho(\mathbf{m}(\tau), m_0) : \sum_{m_0} \rho(\mathbf{m}(\tau), m_0) = 1\,\forall\,\tau$ then solutions to the set of interpolation equations (3) have normalised distributions at all the un-sampled points too, i.e. $\rho(\mathbf{m}, m_0) : \sum_{m_0} \rho(\mathbf{m}, m_0) = 1\,\forall\mathbf{m}$.*

Proof. The sampled probability distributions are normalised,

$$\sum_{m_0} \rho(\mathbf{m}, m_0) = 1 \quad \forall\,\mathbf{m} : n(\mathbf{m}) \neq 0 \tag{4}$$

and by the linearity of the interpolation equation,

$$\Delta \sum_{m_0} \rho(\mathbf{m}, m_0) = \sum_{m_0} Q(\mathbf{m}, m_0) \quad \forall\,\mathbf{m} \tag{5}$$

Given that at the sampled points $\mathbf{m} : n(\mathbf{m}) \neq 0$ all the sampled values of $\sum_{m_0} \rho(\mathbf{m}, m_0)$ are unity, then the trivial solution to the interpolation equation (for all sample values being equal) is then,

$$\sum_{m_0} \rho(\mathbf{m}, m_0) = 1 \quad \forall\,\mathbf{m} \tag{6}$$

i.e. *the interpolation equation conserves probability and ensures all the un-sampled distributions $\rho(\mathbf{m}, m_0)$ are normalised over m_0, for all $\mathbf{m}$.* □

So the solutions to the interpolation equations preserve teh conservation of probability and so represent valid stochastic mappings.

2.4 General solution to the interpolation equations

The N_s observed mappings $\{\mathbf{m}(\tau) \to m_0(\tau)\}$ gave rise to a set of N observed probabilities at the points $\{\mathbf{m}_n\} \equiv \cup_\tau \{\mathbf{m}(\tau)\}$. Applying the general solutions to each of the equations in (3) we write,

$$\left\{ \rho(\mathbf{m}, m_0) = \lambda_{m_0} + \sum_{n=1}^{N} Q(\mathbf{m}_n, m_0)\, g(\{d_\mu(\mathbf{m}, \mathbf{m}_n)\}) \right\} \tag{7}$$

The pseudo-charges are determined by the set of linear equations,

$$\left\{ \begin{pmatrix} \mathbf{r}(m_0) \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{g} & \mathbf{1} \\ \mathbf{1}^\mathrm{T} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{q}(m_0) \\ \lambda(m_0) \end{pmatrix} \right\} \tag{8}$$

where,

$$\mathbf{r}(m_0) = \begin{pmatrix} \rho(\mathbf{m}_1, m_0) \\ \rho(\mathbf{m}_2, m_0) \\ \vdots \\ \rho(\mathbf{m}_N, m_0) \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{q}(m_0) = \begin{pmatrix} Q(\mathbf{m}_1, m_0) \\ Q(\mathbf{m}_2, m_0) \\ \vdots \\ Q(\mathbf{m}_N, m_0) \end{pmatrix} \tag{9}$$

and the rank N, symmetric matrix $\mathbf{g}$ has components $g_{n,n'} = g(\{d_\mu(\mathbf{m}_n, \mathbf{m}_{n'})\})$.

2.5 Most likely interpolated mapping

Given the solution (7), then the most likely interpolated outcome for the mapping $\mathbf{m} \to m_0$ is,

$$m_0 = \arg\max_{m_0'} \rho(\mathbf{m}, m_0') \tag{10}$$

References

[1] Bourne, A 2019 *Interpolating Values in Code Space* (http://vixra.org/pdf/1904.0184v5.pdf).