A motivic sterile neutrino

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Despite the resounding experimental success of the Standard Model, the mystery of neutrino mass and neutrino oscillations must be approached from a framework for quantum gravity. Using well established results in condensed matter physics and in motivic mathematics, we present a new view of the quantum vacuum based on neutrino braid diagrams in quantum computation. The prediction of an effective 1.29 eV non local sterile state from the Koide matrix for $\overline{\nu}$ masses fits known observational constraints.

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I. INTRODUCTION

As we know, three spin 1/2 Pauli operators define a quaternion basis $i, j, k \in \mathbb{H}$ for the spatial directions of Minkowski space. Thus the Pauli exclusion principle means that quantum information in a fermion generates spatial degrees of freedom. In contrast, bosons accumulate in the ground state of a Bose-Einstein condensate. Any theory with supersymmetry must explain how gravity distinguishes these regimes, and why Bose-Einstein condensates can exhibit antigravity and give photons mass.

Our approach[40–47] assumes that the categorical axioms of quantum logic, which are foundational to condensed matter physics, can greatly simplify the mathematical formulation of the Standard Model. This idea is justified by the success of polytope and operad methods in the computation of scattering amplitudes.

An arrow of time in a large physical system is inherently thermodynamic, emerging from local laws that are symmetric in time. The largest system, the observable universe, is in a state defined by the temperature of the CMB. If a photon reaches us from a region of higher temperature, we interpret its origin in an earlier cosmic time. In 2010, it was observed by us[14,42] that a good candidate for a neutrino rest mass at 0.00117 eV exactly matches the present day CMB temperature, using only the relation

$$mc^2 = \frac{hc}{2\pi\lambda} = \beta kT, \tag{1}$$

where $\beta = 4.965$ is Wien's constant for black body radiation. This coincidence was intriguing, given the expectation that a neutrino mass mechanism beyond the Standard Model should be related to a natural measure of cosmic time.

By the uncertainty principle, any knowledge of a precise mass must correspond to an indefinite time, forcing models of mass generation to employ all possible scales[48]. We considered[46] the information content of dyonic states for Standard Model particles, complementing the usual local states with a maximally non local (mirror) set of states associated to a cosmological horizon. A good analogy is the dyon mirror pair[38] of topological surface states. This mirror is literally a mirror, when particles are represented by chiral ribbon diagrams[3], which is quite appropriate in the axioms of quantum computation.

Thinking about this pairing of local and cosmological horizons, with their natural UV and IR cutoffs, we can now elevate the mass model of quantum inertia[23,24,33–35] to a foundational statement about the quantum vacuum: the localisation of mass in an electroweak vertex pairs local and cosmological information. Neutrino masses are foundational, suggesting a derived Higgs mass $m_H \equiv \sqrt{\mu m_P}$, where μ is a neutrino mass scale and m_P the Planck scale[47]. This would fix the quartic coupling parameter in the Higgs Lagrangian to be $\lambda = \mu m_P / 2^{3/2} G_F$. Quantum inertia recovers a MOND description for galactic rotation curves.

The prediction of a sterile neutrino mass is fixed by precise rest masses for the active states, which we get from the Koide-Brannen scheme[5,31,32]. Quantum mechanics is of course axiomatised by symmetric monoidal categories[1,13], where the symmetry condition is imposed on an initial braided monoidal category. Thus the introduction of a braiding is the right way to break quantum mechanics. A non trivial braiding is required in pertinent ribbon categories, such as the category for Fibonacci anyons. The ribbon twist will denote charge. Pairs of Chern-Simons field theories for gravity[15] are often considered.

Once we accept ribbon categories as a diagnostic for any physical scale, we remember the knotted field lines in the intergalactic media of plasma cosmologies[2,30]. In some sense, gravity is merely a cosmological form of electromagnetism.

Section II covers the quantum information behind Standard Model states in this framework, and in section III we introduce our non local sterile neutrino candidate.

II. THE STANDARD MODEL

A. Lepton and quark braids

The chiral lepton and quark states of the Standard Model were given as ribbon diagrams[3] based on preon models, where each particle consists of three anyons. Let σ_1 and σ_2 be the generators of B_3, the braid group on three strands, with relation $\sigma_1 \sigma_2 \sigma_1 = \sigma_2 \sigma_1 \sigma_2$.

The start of each row in Table I is the B_3 braid for the neutrino. Along each row, three electric anyon charges are added to the underlying braid, as ribbon twists. Standard Model right handed singlets are also B_3 diagrams. Massless neutrinos have a fixed helicity, but both states are possible when neutrinos gain mass. Each row in Table I defines a parity cube with 8 vertices, listing states for three qubits.

Observe that mirror braids for charged leptons and quarks, with opposite charges for a given neutrino diagram, are not included in Table I. We may think of these states as supply-

TABLE I. Standard Model electric braid states

ν_L	e_L^-	$\overline{u}_L(1)$	$\overline{u}_L(2)$	$\overline{u}_L(3)$	$d_L(1)$	$d_L(2)$	$d_L(3)$
$\sigma_1\sigma_2^{-1}$	$---$	$0--$	$-0-$	$--0$	-00	$0-0$	$00-$
$\overline{\nu}_R$	e_R^+	$u_R(1)$	$u_R(2)$	$u_R(3)$	$\overline{d}_R(1)$	$\overline{d}_R(2)$	$\overline{d}_R(3)$
$\sigma_2\sigma_1^{-1}$	$+++$	$0++$	$+0+$	$++0$	$+00$	$0+0$	$00+$
$\overline{\nu}_L$	e_L^+	$u_L(1)$	$u_L(2)$	$u_L(3)$	$\overline{d}_L(1)$	$\overline{d}_L(2)$	$\overline{d}_L(3)$
$\sigma_1^{-1}\sigma_2$	$+++$	$0++$	$+0+$	$++0$	$+00$	$0+0$	$00+$
ν_R	e_R^-	$\overline{u}_R(1)$	$\overline{u}_R(2)$	$\overline{u}_R(3)$	$d_R(1)$	$d_R(2)$	$d_R(3)$
$\sigma_2^{-1}\sigma_1$	$---$	$0--$	$-0-$	$--0$	-00	$0-0$	$00-$

ing magnetic information. The dyon is associated to holographic surfaces, in analogy to a topological insulator[38], but our abstract surfaces define the fundamental quantum vacuum in a $2+1$ dimensional theory. Braid composition of a particle and antiparticle annihilates to a neutral photon identity diagram. Holography from $2+1$ dimensions makes perfect sense in a quantum theory that sets up a direction of propagation prior to a global spacetime, wherein distance along the line of propagation is a sufficient measure of local time.

These particle states correspond to an algebra of ideals for $\mathbb{C}\otimes\mathbb{O}$[21,22,26], where the complex factor introduces the ribbon twist for charge, so that $\mathbb{C}\otimes\mathbb{O}$ accounts for the charge $U(1)$ and $SU(3)$ color groups. An alternative but equivalent braid description uses the quantum group $SL_q(2)$[20]. It is natural to use B_4 for $SU(3)$, since $SU(3)$ carries the B_4 representation for the Fibonacci anyon[19,29], which is universal for quantum computation. Here B_3 fills $SU(2)$, viewed either as a compactified component of spacetime or an adjoint representation for mass.

Braid groups are also represented by Majorana operators[28]. In particular, a cyclic B_3 group is generated by

$$\sigma_1 = \frac{1}{\sqrt{2}}(1+i), \;\; \sigma_2 = \frac{1}{\sqrt{2}}(1+j), \;\; \sigma_{12} = \frac{1}{\sqrt{2}}(1+k), \tag{2}$$

where i, j, k are the quaternion units. The Fibonacci anyon B_3 is a rotation of the $\pi/4$ phase defined by the $1/\sqrt{2}$ to the generators

$$\sigma_1 = e^{7\pi i/10}, \quad \sigma_2 = (\phi i + \sqrt{\phi}k)\sigma_1(\phi i + \sqrt{\phi}k)^{-1}, \tag{3}$$

where $\phi = (\sqrt{5}-1)/2$ is the inverse golden ratio.

Tracing the boundary of two linked loops[4], we form a Hopf link and evaluate the Jones polynomial at the quantum dimension $\phi + 1$, giving an estimate for the fine structure constant[40,45]

$$\sqrt{\alpha}^{-1} = 4\cosh\frac{2\pi}{\phi+3}, \tag{4}$$

with $\alpha^{-1} = 137.096$.

The flavor lepton numbers L_e, L_μ and L_τ may be conserved locally at an electroweak vertex, but not in neutrino oscillations.

B. Fourier supersymmetry

Standard Model bosons and fermions are related by Fourier supersymmetry[43,47]. Each neutrino braid state in the last section is reduced to a 3×3 matrix representation of the underlying permutation in $C_3 \subset S_3$. The identity I_3 is the photon matrix γ. The two neutrinos are

$$\nu = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \overline{\nu} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{5}$$

Electric charge on each anyon strand is represented by one of three symbols: 1 for neutral, ω for $+1/3$, or $\overline{\omega}$ for $-1/3$. Then the charged leptons are

$$e_L^- = \overline{\omega} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^+ = \omega \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{6}$$

which compose to the identity. Similarly,

$$e_L^+ = \omega \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^- = \overline{\omega} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{7}$$

Quarks use different charges on individual strands, as in

$$u_L(1) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix}, \quad u_L(2) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega & 0 & 0 \end{pmatrix}, \quad u_L(3) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix}. \tag{8}$$

The $W^{\pm}$ bosons are represented by

$$W^{-} = \overline{\omega} I_3, \quad W^{+} = \omega I_3. \tag{9}$$

For the Z boson there are six remaining neutral boson matrices, which are

$$Z_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix}, \quad \overline{\omega} Z_1 = \begin{pmatrix} \overline{\omega} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \omega Z_1 = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

and their three conjugates. Altogether, when $\omega = 2\pi/3$ is the cubed root of unity, there are 27 matrices of the form $(\omega)^a (Z_1)^b (\nu)^c$ for $a, b, c \in \{0, 1, 2\}$. This algebra defines a basis for the 27 dimensional exceptional Jordan algebra over $\mathbb{O}^{27}$, showing that the $\nu\gamma$ copy of C_3 is a baby representation of triality.

The twisted Fourier transform $\mathbf{F}$ is defined on e_L^- by

$$\mathbf{F}(e_L^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & \overline{\omega} & 0 \\ 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^{-} \tag{11}$$

and on right handed states by

$$\mathbf{F}(e_R^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \\ 0 & \overline{\omega} & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^{-}. \tag{12}$$

Thus the full lepton states map to electroweak bosons

$$e^{\pm} \mapsto W^{\pm}, \qquad \nu, \overline{\nu} \mapsto \gamma. \tag{13}$$

Particle braid groups are truncated by relations[26] of the form $\sigma_i^8 = I$, satisfied by (2). Taking a 27 dimensional three qutrit state space, labeled by the matrices (a, b, c), we can define the 24 off-diagonal elements of a Jordan matrix as a basis for the 24 dimensional Leech lattice[49], accounting for extra dimensions in bosonic M theory.

C. Mass and mixing matrices

The Pauli matrices i, j, k for qubits may be replaced by their mutually unbiased bases[8,39,50]. Such bases exist in any prime power dimension d, and are generated by a circulant $d \times d$

TABLE II. neutrino masses (eV)

$\nu(A)$	0.0507 0.0089 0.0004
$\nu(B)$	0.0582 0.00117 0.0006

matrix, which for qubits and qutrits are

$$R_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}, \quad R_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & 1 \\ 1 & 1 & \omega \\ \omega & 1 & 1 \end{pmatrix}. \tag{14}$$

Since $R_2^8 = I$ and $R_3^{12} = I$, the common geometric phase is $\pi/12$, well known to number theorists as the phase in the Dedekind eta function.

Assuming that each local particle state defines a mass triplet, the double set of neutrino helicities in Table I allows for two distinct triplets of mass states for the neutrinos. We assign the $+\pi/12$ phase to the correct helicity neutrinos (case A) and the $-\pi/12$ phase to the wrong helicity ones (case B), noting that there is no observed local CPT violation, but the apparent CPT violation agrees precisely with initial results from MINOS[7] in 2010. Both triplets sum to a scale of 0.06 eV.

The 3×3 Fourier transform of the diagonal triplet $(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ of square root lepton masses is defined by the Koide matrix

$$\sqrt{M} = \frac{\sqrt{\mu}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \delta & \overline{\delta} \\ \overline{\delta} & \sqrt{2} & \delta \\ \delta & \overline{\delta} & \sqrt{2} \end{pmatrix} \tag{15}$$

for a dimensionful scale μ and complex phase δ. Koide's relation[31,32] from the 1980s, which correctly predicted the τ mass, fixes the $\sqrt{2}$ parameter. In 2006, Brannen[5] found that $\sqrt{2}$ also accounts for the neutrino masses, as fitted by oscillation data. One is able to select the neutrino phases $\delta + \pi/12$ and $\delta - \pi/12$ relative to the charged lepton δ, which is close to $2/9$. The resulting neutrino masses are listed in Table II.

We interpret the double ν phases as the interaction of two kinds of clock. A Keplerian clock employs the arithmetic mean of t_1 and t_2 in special relativity. A second cosmic clock is associated to a geometric mean $\sqrt{T_1 T_2}$, which is natural when there is an origin for time[48]. Kepler's law differs only by a factor of 2π from the equivalence principle in Sciama's law

$GU = c^2R$, where U is the mass of the observable universe and R its characteristic Hubble radius.

The PMNS and CKM mixing matrices are estimated using circulant operators and the golden ring numbers ϕ^{-1} and $\rho = \sqrt{\phi+3}$. Assuming a lepton quark complementarity between the two sets of angles, the relation between the two tribimaximal angles and the golden angles is given by[47]

$$\delta_a \equiv \frac{\pi}{4} - \tan^{-1}(\phi^{-1}) = 13.28^\circ, \tag{16}$$

$$\delta_b \equiv \frac{\pi}{6} - \tan^{-1}(\rho^{-1}) = 2.3^\circ.$$

Observe that δ_a is close to the Cabibbo angle, and $2.3^\circ = 135^\circ - \delta$ is the neutrino phase, where δ is the charged lepton phase above. This gives a new relation

$$\delta_\nu = \delta + \frac{\pi}{12} = \tan^{-1}(\rho^{-1}), \tag{17}$$

implying that $\delta = 0.2222206$ radians, completing the parameterisation of the charged lepton masses.

Mixing matrices are also modeled by three dimensional representations of modular form symmetries[9,12,18], which include a golden ratio mixing for the group A_5.

III. THE NON LOCAL STERILE

Although null results for sterile neutrinos appear to exclude large parts of the oscillation parameter space, good arguments for eV range mass states remain[10,11,25]. For instance, upward moving showers observed in 2018 by the ANITA experiment[17] must pass through the Earth, and cannot be explained by heavier neutrinos[6].

Our non local vacuum scenario evades the usual Lagrangian formalism, and can provide a sterile candidate without adding any further local particle states to the Standard Model. We may assume either a $3+3$ mixing scenario for the six neutrino states, or a $3+1$ scenario when the CMB state is singled out.

The central $\overline{\nu}$ mass in Table II is 0.00117 eV, precisely the peak present day CMB temperature by (1). Redshifting this temperature back to the CMB creation time at $z = 1100$ we obtain a (wrong helicity) mass at 1.29 eV, in good agreement with global fits[10] to oscillation data. The local neutrinos tend to keep their normal helicity, and when they flip it is thought of as an early universe state, because the local equivalent under supersymmetry is a CMB photon.

It is important to understand that we are *not* varying local masses over the history of the classical universe. We consider changes of scale in the sense of the renormalization group. One possible objection here is the use of only one $\nu(B)$ mass state, rather than three. But CMB temperatures distinguish the past, present and future, and we have simply chosen to observe the present. This distinction between space and time also explains the use of right handed singlets in the Higgs mechanism. Other interstellar CMB temperatures are also worth considering, such as the 20 K that corresponds to a redshifted 0.0089 eV.

These empirical coincidences are impressively consistent within the axiomatic framework. Pandey[37] has considered ν oscillations using a broken equivalence principle, and in breaking this principle, quantum inertia can also explain[46] discrepancies in the Hubble parameter over cosmic scales[16]. An analysis[36] of extra dark energy components for ΛCDM favours a model that selects the critical CMB creation time. It is unfortunate that neutrino mass is often studied using Standard Model Lagrangian techniques, when neutrino mass only makes sense beyond the Standard Model.

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