Infinity and Pythagorean theorem

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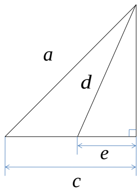
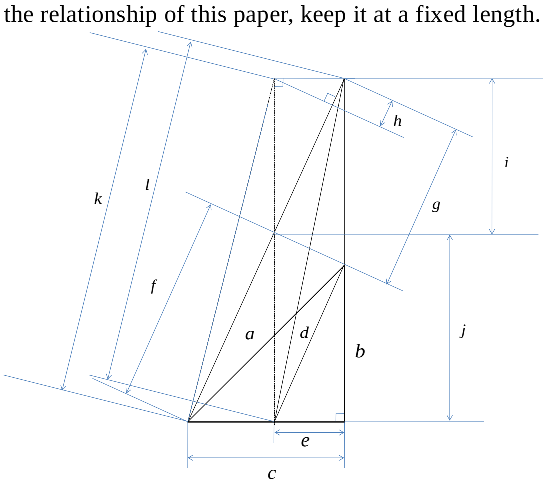
Introducing infinity into the Pythagorean theorem provides the Pythagorean theorem even for triangles that are not right triangles.

First, the Pythagorean theorem holds for the three sides of a right triangle.



$$a^2-d^2=c^2-e^2$$

Second, As shown in the figure, the positions of the intersections of the side a and the side b are moved sufficiently large upward while maintaining the lengths of c and d. Due to the relationship of this paper, keep it at a fixed length.



By using the relation shown from the first figure,

$$k^2-i^2=(f+g)^2-(g-h)^2$$
$$l^2-g^2=(i+j)^2-i^2$$
$$k^2-(i+j)^2=(c-e)^2$$

Then (i+j)=∞⟶h = 0.
The following relationship is obtained from the above three equations.

$$\boxed{(f+g)^2=l^2+(c-e)^2} \qquad ①$$

From the above equation, by introducing ∞ into a right triangle, the Pythagorean theorem holds for triangles that are not strictly right triangles.

【Proof】

Then introduce ∞ including imaginary numbers and compare.
First, ±∞ is constant at any observation point (position).
If a set of real numbers is R,then,

$$R\times(\pm\infty)=\pm\infty$$
$$R+(\pm\infty)=\pm\infty$$
$$(-1)\times(\pm\infty)\neq\mp\infty$$

On the other hand, when x (∈R)is taken on a number line, the absolute value X becomes larger toward ± ∞ as the absolute value X is expanded.
Similarly, as the size decreases, the absolute value X decreases toward 0.Furthermore, x (-1) represents the reversal of the direction of the axis.

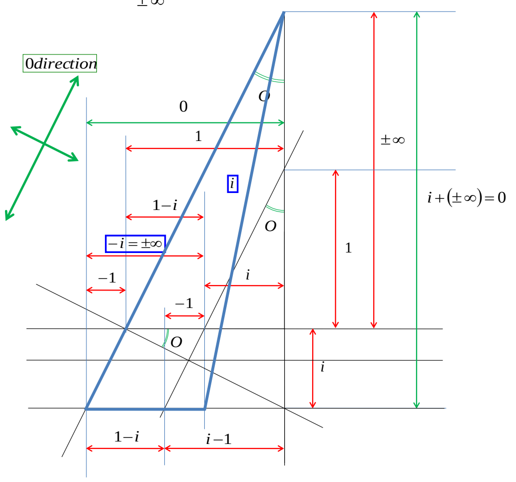
$$\boxed{\begin{aligned}&\frac{1}{\pm\infty}=(-1)\cdot(\pm\infty)=i\\&(\pm\infty)\cdot i-1=0\end{aligned}}$$

$$(-1)\cdot(\pm\infty)=\frac{1}{\pm\infty}$$
$$i^2=(\pm\infty)^2\to i=\pm(\pm\infty)$$
$$\therefore i=-(\pm\infty)=(-1)(\pm\infty)=\frac{1}{\pm\infty},(\because i\neq+(\pm\infty))$$

Next,

$$\boxed{\pi=\frac{2}{\pi}+2\arctan\left(\frac{1}{\tan\left(\frac{1}{x}\right)}\right),\left(\because x\geq\frac{1}{\pi}\right)}$$

$$x=\frac{2}{x}\left(\geq\frac{1}{x}\right)$$
$$\pi=\frac{2}{\left(\frac{2}{\pi}\right)}+2\arctan\left(\frac{1}{\tan\left(\frac{\pi}{2}\right)}\right)=\pi+2\arctan\left(\frac{1}{\pm\infty}\right)$$
$$\arctan\left(\frac{1}{\pm\infty}\right)=\arctan(i)=0$$
$$\therefore\tan 0=\frac{1}{\pm\infty}=(-1)(\pm\infty)=i$$



From the above figure,

$$\boxed{(i)^2+(-i)^2=0,(\because 0=0direction)} \qquad ②$$

$(\because i+j=0,e=i,c=0,c-e=-i,\ f=g=h=0(\because 0direction))$
Therefore, it shows that Equation ① holds in a special triangle in geometry that introduces ∞ and an imaginary number i to the Pythagorean theorem.