Prime numbers and Twin prime

Algorithm

For the first time a method that generates all prime numbers.

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Abstract: This article develops an old and well-known expression to obtain prime numbers, composite numbers and twin prime numbers. The conditioning (n) will be the key to make the formula work and the conditioning of the letter (z) will be important for the formula to be efficient.

Keywords: Prime numbers, twin prime numbers, composite numbers.

Introduction

The study of the prime numbers is wonderful and exciting, in the absence of an expression that involves all of them I have investigated and I have discovered a brilliant expression that contains all the prime numbers greater than 3. This expression comes from investigating first how they are distributed the composite numbers, this allowed me to explore its order and understand its mechanism. The expression of the prime numbers is its result.
The expression to obtain the prime numbers is similar to how we use the sieve of Eratosthenes although all that infinite procedure expressed in symbols in a formula.
From here it was very easy to get to the Twin prime numbers.

Methods

The prime numbers greater than three can be expressed under the expressions $(6\,n\,+\,1)$ and $(6\,n-\,1)$, for some values of (n). This paper demonstrates in chapter 1, 2 and 3 how to obtain the correct (n) values to obtain all the prime numbers.

Also, Twin prime numbers greater than 3 are expressed under the same expression, so once the values of (n) are obtained for obtaining the prime numbers, combining both expressions we obtain all the twin prime numbers in chapter 6 and 7.

The algorithm presented in this paper does not calculate prime numbers or twin prime numbers only generates the mechanisms to obtain them.

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Chapter 1

A) Sequence of: $\boldsymbol{\beta} = \mathbf{6n \pm 1}$

The $\boldsymbol{\beta}$ sequence will be fundamental since this will allow us to obtain all the composite values and once excluded these will only be the prime numbers.

All prime numbers greater than 3 are within this sequence, the problem is that there are also composite numbers that are products of prime numbers greater than 3.

The formula enunciated in the following point is intended to separate the composite numbers from the prime numbers and does it marvellously.

$n > 0$

Sequence numbers $\beta_{(a)}$	Sequence numbers $\beta_{(b)}$
$\beta_{(a)} = 6\text{n} + 1 = 7{,}13{,}19{,}25{,}31, \ldots.$	$\beta_{(b)} = 6\text{n} - 1 = 5{,}11{,}17{,}23{,}29{,}35, \ldots..$

$$\boldsymbol{\beta} = \mathbf{6n \pm 1 = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, \ldots..}$$

B) Formula to obtain prime numbers

$$P_{(a,b)} = 6n_{\substack{n \neq 0 \\ n \neq B \pm \left(\frac{B \pm 1}{6}\right) + B * z_{<\left(\frac{B \pm 1}{6}\right)}}} \pm 1$$

This algorithm allows the obtaining of all the prime numbers greater than three. This formula allows to obtain the correct values of (n), for which we can establish how the prime numbers are located and distributed.

$P_{(a,b)} = Prime\ numbers\ > 3$
$B = 6n \pm 1$
$Z \in \mathbb{N}$
$Z < \left(\frac{B \pm 1}{6}\right)$

The formula is divided into 2 variables:
Two for the form $B_{(b)} = 6n + 1$
Two for the form $B_{(a)} = 6n - 1$

Inside, each formula is again in two more variables.

$P_{(a)} = 6n_{\substack{n \neq 0 \\ n \neq B_{(b)} - \left(\frac{B_{(b)}+1}{6}\right) + B_{(b)} * z_{<\left(\frac{B_{(b)}+1}{6}\right)} \\ n \neq B_{(a)} + \left(\frac{B_{(a)}-1}{6}\right) + B_{(a)} * z_{<\left(\frac{B_{(a)}-1}{6}\right)}}} + 1$	$P_{(b)} = 6n_{\substack{n \neq 0 \\ n \neq B_{(b)} + \left(\frac{B_{(b)}+1}{6}\right) + B_{(b)} * z_{<\left(\frac{B_{(b)}+1}{6}\right)} \\ n \neq B_{(a)} - \left(\frac{B_{(a)}-1}{6}\right) + B_{(a)} * z_{<\left(\frac{B_{(a)}-1}{6}\right)}}} - 1$

Chapter 2

Primes Numbers in $\beta_{(a)}$

A) Example and demonstration of the formula

With this formula we obtain the prime numbers greater than 3 that are within the sequence $\beta_{(a)}$

$$P_{(a)} = 6n \underset{\substack{n \neq 0 \\ n \neq B_{(b)} - \left(\frac{B_{(b)}+1}{6}\right) + B_{(b)} * z_{<\left(\frac{B_{(b)}+1}{6}\right)} \\ n \neq B_{(a)} + \left(\frac{B_{(a)}-1}{6}\right) + B_{(a)} * z_{<\left(\frac{B_{(a)}-1}{6}\right)}}}{} + 1$$

$P_{(a)}$ = $Prime\ numbers\ in\ \beta(a)$
$Z \in \mathbb{N}$

Sequence numbers $\beta_{(a)}$

$$\beta_{(a)} = 6n + 1 = 7,13,19,25,31,37,43,49,55 \ldots$$

Sequence numbers $\beta_{(b)}$

$$\beta_{(b)} = 6n - 1 = 5,11,17,23,29,35,41,47,53,59,65 \ \ldots..$$

B) Demonstration of how to look for prime numbers <190 in $\beta_{(a)}$

$x > 7$

$$n = \frac{x-1}{6}$$

$$n = \frac{190-1}{6} = 31{,}5$$

Minimum values according to Beta

$\beta_{(b)}$	$Composite = \beta_{(b)} * 5$
5	25
11	55
17	85
23	115
29	145
35	175
41	205
47	235
53	265
59	295
65	325
71	355
77	385

I look in the table for the next value of 31,5 in the column $\beta_{(b)}$.

The value is 35

This result helps us to determine how far we should expand the formula.

$$n \neq \beta_{(a,b)}$$

$$P_{(a)} = 6n \begin{array}{c} n\neq 0 \\ n\neq 5-\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq 7+\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq 11-\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq 13+\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq 17-\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq 19+\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)} \\ n\neq 23-\left(\frac{23+1}{6}\right)+23*z_{<\left(\frac{23+1}{6}\right)} \\ n\neq 25+\left(\frac{25-1}{6}\right)+25*z_{<\left(\frac{25-1}{6}\right)} \\ n\neq 29-\left(\frac{29+1}{6}\right)+29*z_{<\left(\frac{29+1}{6}\right)} \\ n\neq 31+\left(\frac{31-1}{6}\right)+25*z_{<\left(\frac{31-1}{6}\right)} \\ n\neq 35-\left(\frac{35+1}{6}\right)+35*z_{<\left(\frac{35+1}{6}\right)} \end{array} + 1$$

We expanded the formula until $\beta_{(b)}$=35

(z) values are conditioned so that the formula is more efficient and repeats the results as little as possible.

Therefore solving the operations we obtain:

$$P_{(a)} = 6n \begin{array}{c} n\neq 0 \\ n\neq 4+5*z_{<1} \\ n\neq 8+7*z_{<1} \\ n\neq 9+11*z_{<2} \\ n\neq 15+13*z_{<2} \\ n\neq 14+17*z_{<3} \\ n\neq 22+19*z_{<3} \\ n\neq 19+23*z_{<4} \\ n\neq 29+25*z_{<4} \\ n\neq 24+29*z_{<5} \\ n\neq 36+31*z_{<5} \\ n\neq 29+35*z_{<6} \end{array} + 1 = 6n \begin{array}{c} n\neq 0 \\ n\neq 4 \\ n\neq 8 \\ n\neq 9,20 \\ n\neq 15,28 \\ n\neq 14,31,45 \\ n\neq 22,41,60 \\ n\neq 19,42,65,88 \\ n\neq 29,54,79,104 \\ n\neq 24,53,82,111,140 \\ n\neq 36,67,98,129,160 \\ n\neq 29,64,99,134,169,204 \end{array} + 1$$

We take the values ℕ: $n \geq 0 \;\wedge\; n < 31{,}5$

$$\boldsymbol{n} \neq [\mathbf{0}, \mathbf{4}, \mathbf{8}, \mathbf{9}, \mathbf{14}, \mathbf{15}, \mathbf{19}, \mathbf{20}, \mathbf{22}, \mathbf{24}, \mathbf{28}, \mathbf{29}, \mathbf{31}]$$

By replacing these values in 6n + 1, composite numbers <190 are formed, for that same reason these values are not valid to get the prime numbers.

Using the missing natural numbers in the previous set we obtain:

$\boldsymbol{n}_{(p)} = values\ of\ (n)\ that\ generate\ prime\ numbers$

$\boldsymbol{n}_{(\boldsymbol{p})} = [1, 2, 3, 5, 6, 7, 10, 11, 12, 13, 16, 17, 18\ , 21, 23, 25, 26, 27{,}30]$

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We get the following prime numbers:

$$\mathbf{P}_{(\mathbf{a})<190} = \mathbf{6}\ \boldsymbol{n}_{(\boldsymbol{p})} + \mathbf{1}$$

Primes numbers<190

$$\mathbf{P}_{(\mathbf{a})<190} = [7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139, 151, 157, 163, 181]$$

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C) Triangles of values for (n), composite numbers within the sequence $\boldsymbol{\beta a = 6n + 1}$

The triangle is armed with the following horizontal form formula:

$$n \neq 0$$

$$(\,Triangle\ 1), n \neq \beta_{(b)} - \left(\frac{\beta_{(b)} + 1}{6}\right) + \beta_{(b)} * z_{<\left(\frac{\beta_{(b)}+1}{6}\right)}$$

$$(\,Triangle\ 2), n \neq \beta_{(a)} + \left(\frac{\beta_{(a)} - 1}{6}\right) + \beta_{(a)} * z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}$$

But we can easily expand it, we only add to the last number the value corresponding to the diagonal sequence β

Example: 9+5=14; 20+11=31

Triangle 1

							+5							
								+11			Sequence numbers $\beta_{(b)}$			
					4				+17					
				9		20				+23				
			14		31		48				+29			
		19		42		65		88				+35		
	24		53		82		111		140				+41	
29		64		99		134		169		204				+47

The green numbers are the initial sequence of the triangle and are formed under the following formula:

$$4\ x\ pentagonal\ numbers.$$

$$Pentagonal\ numbers = 1{,}5{,}12{,}22{,}35, \ldots$$

<u>It can also be calculated as:</u>

$$n > 0$$

$$a\,(n) = 2n(3n - 1) = 4{,}20{,}48{,}88, \ldots$$

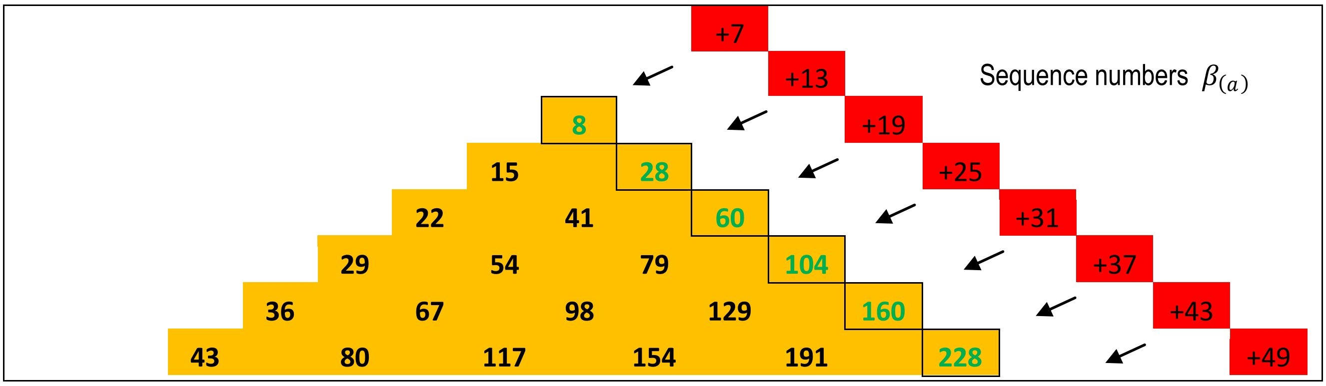
These numbers replaced in $(6n - 1)$ form the sequence $\beta_{(b)}{}^{2}$= 25,121,289,529,...

<u>We can also form the sequence 4,20,48,88..., taking the numbers:</u> $\beta_{(b)}$

$$\beta_{(b)} * \left(\frac{\beta_{(b)} + 1}{6}\right) - \left(\frac{\beta_{(b)} + 1}{6}\right) =$$

$$\beta_{(5)} = 5 * 1 - 1 = 4$$
$$\beta_{(11)} = 11 * 2 - 2 = 20$$
$$\beta_{(17)} = 17 * 3 - 3 = 48$$
$$\beta_{(23)} = 23 * 4 - 4 = 88$$

Triangle 2



The green numbers are the initial sequence of the triangle and are formed under the following formula:

$$4 \, * (second\ pentagonal\ numbers)$$

$$Second\ pentagonal\ numbers = \ 2,7,15,26,40, \ldots.$$

<u>It can also be calculated as:</u>

$$n > 0$$

$$a\ (n) = \ 2n\ (3n\ +\ 1) = 8,28,60,104,160, \ldots.$$

These numbers replaced in $(6n\ +\ 1)$ form the sequence $\beta_{(a)}{}^{2}$= 49,169,361,625,….

<u>We can also form the sequence 8,28,60,104, …. taking the $\beta_{(a)}$ numbers:</u>

Example:

$$\beta_{(a)} * \left(\frac{\beta_{(a)} - 1}{6}\right) + \left(\frac{\beta_{(a)} - 1}{6}\right) =$$

$$\beta_{(7)} = 7 * 1 + 1 = 8$$
$$\beta_{(13)} = 13 * 2 + 2 = 28$$
$$\beta_{(19)} = 19 * 3 + 3 = 60$$
$$\beta_{(25)} = 25 * 4 + 4 = 104$$

Chapter 3

Primes Numbers in $\beta_{(b)}$

A) Demonstration of how to look for prime numbers <190 in $\beta_{(b)}$

$x > 7$

$$n = \frac{x-1}{6}$$

$$n = \frac{190-1}{6} = 31{,}5$$

Minimum values according to Beta

$\beta_{(a)}$	$Composite = \beta_{(a)} * 7$
7	49
13	91
19	133
25	175
31	217
37	259
43	301
49	343
55	385
61	427
67	469
73	511
79	553

I look in the table for the next value of 31,5 in the column $\beta_{(a)}$.

The value is 37

This result helps us to determine how far we should expand the formula

$$n \neq \beta_{(a,b)}$$

With this formula we obtain the prime numbers greater than 3 that are within the sequence $\beta_{(b)}$

$$P_{(b)} = 6n_{\substack{n \neq 0 \\ n \neq \beta_{(b)} + \left(\frac{\beta_{(b)}+1}{6}\right) + \beta_{(b)} * z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n \neq \beta_{(a)} - \left(\frac{\beta_{(a)}-1}{6}\right) + \beta_{(a)} * z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}}} - 1$$

$P_{(b)}$ = $Prime\ numbers\ in\ \beta_{(b)}$
$Z \in \mathbb{N}$

Sequence numbers $\beta_{(a)}$

$$\beta_{(a)} = 6n + 1 = 7,13,19,25,31,37,43,49,55 \ldots$$

Sequence numbers $\beta_{(b)}$

$$\beta_{(b)} = 6n - 1 = 5,11,17,23,29,35,41,47,53,59,65 \ \ldots..$$

$$P_{(b)} = 6\mathrm{n}_{\begin{array}{c} n\neq0 \\ n\neq5+\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq7-\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq11+\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq13-\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq17+\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq19-\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)} \\ n\neq23+\left(\frac{23+1}{6}\right)+23*z_{<\left(\frac{23+1}{6}\right)} \\ n\neq25-\left(\frac{25-1}{6}\right)+25*z_{<\left(\frac{25-1}{6}\right)} \\ n\neq29+\left(\frac{29+1}{6}\right)+29*z_{<\left(\frac{29+1}{6}\right)} \\ n\neq31-\left(\frac{31-1}{6}\right)+31*z_{<\left(\frac{31-1}{6}\right)} \\ n\neq35+\left(\frac{35+1}{6}\right)+35*z_{<\left(\frac{35+1}{6}\right)} \\ n\neq37-\left(\frac{37-1}{6}\right)+37*z_{<\left(\frac{37-1}{6}\right)} \end{array}} - 1$$

we expanded the formula until $\beta_{(a)37}$

(z) values are conditioned so that the formula is more efficient and repeats the results as little as possible.

Therefore solving the operations we obtain:

$$P_{(b)} = 6n_{\begin{array}{c} n\neq0 \\ n\neq6+5*z_{<1} \\ n\neq6+7*z_{<1} \\ n\neq13+11*z_{<2} \\ n\neq11+13*z_{<2} \\ n\neq20+17*z_{<3} \\ n\neq16+19*z_{<3} \\ n\neq27+23*z_{<4} \\ n\neq21+25*z_{<4} \\ n\neq34+29*z_{<5} \\ n\neq26+31*z_{<5} \\ n\neq41+35*z_{<6} \\ n\neq31+37*z_{<6} \end{array}} - 1 = 6n_{\begin{array}{c} n\neq0 \\ n\neq6 \\ n\neq6 \\ n\neq13,24 \\ n\neq11,24 \\ n\neq20,37,54 \\ n\neq16,35,54 \\ n\neq27,50,73,96 \\ n\neq21,46,71,96 \\ n\neq34,63,92,121,150 \\ n\neq26,57,88,119,150 \\ n\neq41,76,111,146,181,216 \\ n\neq31,68,105,142,179,216 \end{array}} - 1$$

We take the values $\mathbb{N}$: $n \geq 0 \ \wedge \ n < 31{,}5$

$$n \neq [0,6,11,13,16,20,21,24,26,27,31]$$

By replacing these values in 6n - 1, composite numbers <190 are formed, for that same reason these values are not valid to get the prime numbers.

<u>Using the missing natural numbers in the previous set we obtain:</u>

$\boldsymbol{n}_{(p)} = values\ of\ (n)\ that\ generate\ prime\ numbers$

$\boldsymbol{n}_{(\boldsymbol{p})} = [1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 14, 15\ , 17, 18, 19, 22, 23, 25, 28, 29, 30]$

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<u>We get the following prime numbers:</u>

$$\mathbf{P}_{(\mathbf{b})<190} = \mathbf{6}\ \boldsymbol{n}_{(\boldsymbol{p})} - \mathbf{1}$$

$$P_{(b)<190} = [5, 11,17,23,29,41,47,53,59,71,83,89,101,107,113,131,137,149,167,173,179]$$

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B) Triangles of values for (n), composite numbers within the sequence $\boldsymbol{\beta b} = \mathbf{6n-1}$

The triangle is armed with the following formulas horizontally:

$$n \neq 0$$

$$(\,Triangle\ 3), n \neq B_{(b)} + \left(\frac{B_{(b)}+1}{6}\right) + B_{(b)} * z_{<\left(\frac{B_{(b)}+1}{6}\right)}$$

$$(\,Triangle\ 4), n \neq B_{(a)} - \left(\frac{B_{(a)}-1}{6}\right) + B_{(a)} * z_{<\left(\frac{B_{(a)}-1}{6}\right)}$$

But we can easily expand it, we only add to the last number the value that corresponds to the diagonal of the sequence β

Triangle 3

							+7							
								13				sequence $B_{(a)}$		
					6				+19					
				13		24				+25				
			20		37		54				+31			
		27		50		73		96				+37		
	34		63		92		121		150				+43	
41		76		111		146		181		216				+49

The yellow numbers in boths triangles are the initial sequence and are formed under the following formula:

$$n > 0$$

$$a\ (n) \;=\; 6n^2 = 6{,}24{,}54{,}96{,}150{,}216$$

These numbers replaced in $6n - 1$ form the sequence: 35,143,323,....

(5*7=35), (11*13=143), (17*19=323), (23*25=575),.....

Triangle 4

							+5							
								+11				sequence βb		
					6				+17					
				11		24				+23				
			16		35		54				+29			
		21		46		71		96				+35		
	26		57		88		119		150				+41	
31		68		105		142		179		216				+47

Chapter 4

How to find out if a number is prime using triangles.

The even numbers are excluded from the set of prime numbers, we also exclude those numbers with two or more digits that have the number 5 in their unit.

Example: number 529

A) 529 must be within the sequence $6n \pm 1$ to be a possible prime number.

B)To know if it is within this sequence we can do the following operation:

Nº/9= 529/9= 58,777777

C) Classification of your decimal numbers

If the result has decimals 3,6 or 9 it is out of sequence $6n \pm 1$ for which it is composite.
If the result has decimals 1,4 or 7 it is inside of sequence $6n + 1$
If the result has decimals 2,5 or 8 it is inside of sequence $6n - 1$

58,777777, is inside the sequence $6n + 1$

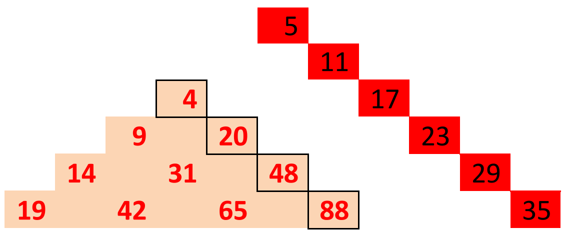
For sequence $6n + 1$, we use triangle 1 and 2
For sequence $6n - 1$, we use triangle 3 and 4

D) We look for a result that is within the natural numbers, if calculating the result is not found inside both triangles the number will be prime.

$$n = \frac{\text{Number} \pm 1}{6}$$

For Triangle 1:

$n = \frac{529-1}{6}$= **88** (is inside the **Triangle 1**, in the fourth row, which is a multiple of 23) **529 is a composite number.**



For Triangle 2:

$n = \frac{529+1}{6}$= 88,33..

Chapter 5

Graphics tables

These graphs (Table 1 and table 2) show the behavior of (n) with respect to the prime numbers and compound numbers.
The brown colors are related to the β numbers, where they fall generate composite numbers for (n)
The columns are sorted as appropriate (the addition or subtraction) of the twin natural numbers with β.
The columns are ordered every 7 boxes on one side and the other every 5 boxes.

Table1

7	6	5	4	3	2	1		1	2	3	4	5	6	7
43	37	31	25	19	13	7	N	5	11	17	23	29	35	41
							1							
							2							
							3							
							4							
							5	1						
							6	2						
							7	3						
							8	4						
						1	9	5						
						2	10	1	1					
						3	11	2	2					
						4	12	3	3					
						5	13	4	4					
						6	14	5	5					
						7	15	1	6	1				
					1	1	16	2	7	2				
					2	2	17	3	8	3				
					3	3	18	4	9	4				
					4	4	19	5	10	5				
					5	5	20	1	11	6	1			
					6	6	21	2	1	7	2			
					7	7	22	3	2	8	3			
				1	8	1	23	4	3	9	4			
				2	9	2	24	5	4	10	5			
				3	10	3	25	1	5	11	6	1		
				4	11	4	26	2	6	12	7	2		
				5	12	5	27	3	7	13	8	3		
				6	13	6	28	4	8	14	9	4		
				7	1	7	29	5	9	15	10	5		
			1	8	2	1	30	1	10	16	11	6	1	
			2	9	3	2	31	2	11	17	12	7	2	
			3	10	4	3	32	3	1	1	13	8	3	
			4	11	5	4	33	4	2	2	14	9	4	
			5	12	6	5	34	5	3	3	15	10	5	
			6	13	7	6	35	1	4	4	16	11	6	1
			7	14	8	7	36	2	5	5	17	12	7	2
		1	8	15	9	1	37	3	6	6	18	13	8	3
		2	9	16	10	2	38	4	7	7	19	14	9	4
		3	10	17	11	3	39	5	8	8	20	15	10	5
		4	11	18	12	4	40	1	9	9	21	16	11	6
		5	12	19	13	5	41	2	10	10	22	17	12	7
		6	13	1	1	6	42	3	11	11	23	18	13	8
		7	14	2	2	7	43	4	1	12	1	19	14	9

Table 2

7	6	5	4	3	2	1		1	2	3	4	5	6	7
43	37	31	25	19	13	7	N	5	11	17	23	29	35	41
							1							
							2							
							3							
							4							
							5							
							6							
						1	7	1						
						2	8	2						
						3	9	3						
						4	10	4						
						5	11	5						
					1	6	12	1						
					2	7	13	2						
					3	1	14	3	1					
					4	2	15	4	2					
					5	3	16	5	3					
				1	6	4	17	1	4					
				2	7	5	18	2	5					
				3	8	6	19	3	6					
				4	9	7	20	4	7					
				5	10	1	21	5	8	1				
			1	6	11	2	22	1	9	2				
			2	7	12	3	23	2	10	3				
			3	8	13	4	24	3	11	4				
			4	9	1	5	25	4	1	5				
			5	10	2	6	26	5	2	6				
		1	6	11	3	7	27	1	3	7				
		2	7	12	4	1	28	2	4	8	1			
		3	8	13	5	2	29	3	5	9	2			
		4	9	14	6	3	30	4	6	10	3			
		5	10	15	7	4	31	5	7	11	4			
	1	6	11	16	8	5	32	1	8	12	5			
	2	7	12	17	9	6	33	2	9	13	6			
	3	8	13	18	10	7	34	3	10	14	7			
	4	9	14	19	11	1	35	4	11	15	8	1		
	5	10	15	1	12	2	36	5	1	16	9	2		
1	6	11	16	2	13	3	37	1	2	17	10	3		
2	7	12	17	3	1	4	38	2	3	1	11	4		
3	8	13	18	4	2	5	39	3	4	2	12	5		
4	9	14	19	5	3	6	40	4	5	3	13	6		
5	10	15	20	6	4	7	41	5	6	4	14	7		
6	11	16	21	7	5	1	42	1	7	5	15	8	1	
7	12	17	22	8	6	2	43	2	8	6	16	9	2	

Table 3

In yellow we have the prime numbers in red the composite numbers.

$\beta_{(a)} = 6\mathrm{n} + 1$ $\beta_{(b)} = 6\mathrm{n} - 1$

Prime Numbers					
β (a)				β (b)	
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144
145	146	147	148	149	150

Chapter 6

A) Formula to obtain twin prime numbers.

A twin prime is a prime number that is either 2 less or 2 more than another prime number, for example, either member of the twin prime pair (41, 43). In other words, a twin prime is a prime that has a prime gap of two. Sometimes the term *twin prime* is used for a pair of twin primes; an alternative name for this is prime twin or prime pair.

This formula allows the obtaining of all the Twin prime numbers >3. This formula is armed with the combination of the two main variables of prime numbers. Both formulas come together to condition (n) and obtain the correct values for (n).

This formula does not pretend to demonstrate the infinity of twin prime numbers, it only shows and proves how to obtain them.

$$Tp = 6n_{\substack{n\neq 0 \\ n\neq\beta_{(b)}\mp\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(a)}\mp\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{\left(\frac{\beta_{(a)}-1}{6}\right)}}} \pm 1$$

$\beta = 6\text{n} \pm 1 = 5,7,11,13,17,19,23,25,29,31,35, \dots \dots \dots$

$Z \in \mathbb{N}$

$Tp = twin\ prime\ numbers > 3$

This formula has 2 main formats with 4 variables each. In both formulas we obtain the same result. For which we can choose one of them.
In the variables we must repeat each Beta value twice. Once subtracting again adding.

$Tp_{(a)} = 6n_{\substack{n\neq 0 \\ n\neq\beta_{(b)}-\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(b)}+\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(a)}-\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)} \\ n\neq\beta_{(a)}+\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}}} + 1$	$Tp_{(b)} = 6n_{\substack{n\neq 0 \\ n\neq\beta_{(b)}-\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(b)}+\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(a)}-\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)} \\ n\neq\beta_{(a)}+\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}}} - 1$

This formula allows composite numbers and non-twin prime numbers to be excluded.

<u>Chapter 7</u>

<u>Application of the formula.</u>

$Twin\ prime\ number\ > 3.$

A) Demonstration of how to look for Twin prime numbers <100 in $\beta_{(a,b)}$

$x > 7$

$$n = \frac{x-1}{6}$$

$$n = \frac{100-1}{6} = 16{,}5$$

Minimum values according to Beta		
$\beta_{(a)}$	Composite	I look in the table for the next value of 16,5 in the column $\beta_{(a)}$. The value is 19 This result helps us to determine how far we should expand the formula $n \neq \beta_{(a,b)}$
7	49	
13	91	
19	133	
25	175	
31	217	
37	259	
43	301	
49	343	
55	385	
61	427	
67	469	
73	511	
79	553	

$$Tp_{(a)} = 6n_{\substack{n\neq 0 \\ n\neq\beta_{(b)}-\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(b)}+\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq\beta_{(a)}-\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)} \\ n\neq\beta_{(a)}+\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}}} + 1$$

Replacement with the first value of$\beta_{(b)}$ twice, then replacement with the first value value of $\beta_{(a)}$ also 2 times, then repeat the same procedure with the following values $\beta_{(b)}$and $\beta_{(a)}$ as many times as necessary. With the following example it will be much more clearer.

B) Twin Prime numbers in $in\ \boldsymbol{\beta}_{(\mathbf{b})}$.

$$Tp_{(a)} = 6n \underset{\substack{n\neq 5-\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq 5+\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq 7-\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq 7+\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq 11-\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq 11+\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq 13-\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq 13+\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq 17-\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq 17+\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq 19-\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)} \\ n\neq 19+\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)}}}{n\neq 0} + 1$$

We expanded the formula until 19.

$$Tp_{(a)} = 6n \underset{\substack{n\neq 4+5*Z_{<1} \\ n\neq 6+5*Z_{<1} \\ n\neq 6+7*Z_{<1} \\ n\neq 8+7*Z_{<1} \\ n\neq 9+11*Z_{<2} \\ n\neq 13+11*Z_{<2} \\ n\neq 11+13*Z_{<2} \\ n\neq 15+13*Z_{<2} \\ n\neq 14+17*Z_{<3} \\ n\neq 20+17*Z_{<3} \\ n\neq 16+19*Z_{<3} \\ n\neq 22+19*Z_{<3}}}{n\neq 0} + 1 = 6n \underset{\substack{n\neq 4 \\ n\neq 6 \\ n\neq 6 \\ n\neq 8 \\ n\neq 9,20 \\ n\neq 13,24 \\ n\neq 11,24 \\ n\neq 15,28 \\ n\neq 14,31,48 \\ n\neq 20,37,54 \\ n\neq 16,35,54 \\ n\neq 22,41,60}}{n\neq 0} + 1$$

We take the values ℕ: $n \geq 0 \ \wedge \ n < 16{,}5$

$$n \neq [0,4,6,8,9,11,13,14,15,16]$$

Using the missing natural numbers in the previous set we obtain:

$n_t = values\ of\ (n)\ that\ generate\ twin\ prime\ numbers$

$n_t = [\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}, \mathbf{7}, \mathbf{10}, \mathbf{12}]$

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By replacing we get the following Twin prime numbers

$$Tp_{(a)} = 6n_t + 1$$

$Tp_{(a)} = [7,13,19,31,43,61,73]$

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C) Twin Prime numbers in $in\ \boldsymbol{\beta}_{(\mathbf{b})}$.

Everything is ordered in the same way as in the previous exercise with the difference that in the central formula the number 1 is subtracting.

$$Tp_{(b)} = 6n_{\substack{n\neq 0 \\ n\neq \beta_{(b)}-\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq \beta_{(b)}+\left(\frac{\beta_{(b)}+1}{6}\right)+\beta_{(b)}*Z_{<\left(\frac{\beta_{(b)}+1}{6}\right)} \\ n\neq \beta_{(a)}-\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)} \\ n\neq \beta_{(a)}+\left(\frac{\beta_{(a)}-1}{6}\right)+\beta_{(a)}*Z_{<\left(\frac{\beta_{(a)}-1}{6}\right)}}} - 1$$

$$Tp_{\,(b)} = 6n_{\substack{n\neq 0 \\ n\neq 5-\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq 5+\left(\frac{5+1}{6}\right)+5*z_{<\left(\frac{5+1}{6}\right)} \\ n\neq 7-\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq 7+\left(\frac{7-1}{6}\right)+7*z_{<\left(\frac{7-1}{6}\right)} \\ n\neq 11-\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq 11+\left(\frac{11+1}{6}\right)+11*z_{<\left(\frac{11+1}{6}\right)} \\ n\neq 13-\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq 13+\left(\frac{13-1}{6}\right)+13*z_{<\left(\frac{13-1}{6}\right)} \\ n\neq 17-\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq 17+\left(\frac{17+1}{6}\right)+17*z_{<\left(\frac{17+1}{6}\right)} \\ n\neq 19-\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)} \\ n\neq 19+\left(\frac{19-1}{6}\right)+19*z_{<\left(\frac{19-1}{6}\right)}}} - 1$$

we expanded the formula until 19

$$Tp_{(b)} = 6n_{\substack{n\neq 0\\ n\neq 4+5*Z_{<1}\\ n\neq 6+5*Z_{<1}\\ n\neq 6+7*Z_{<1}\\ n\neq 8+7*Z_{<1}\\ n\neq 9+11*Z_{<2}\\ n\neq 13+11*Z_{<2}\\ n\neq 11+13*Z_{<2}\\ n\neq 15+13*Z_{<2}\\ n\neq 14+17*Z_{<3}\\ n\neq 20+17*Z_{<3}\\ n\neq 16+19*Z_{<3}\\ n\neq 22+19*Z_{<3}}} - 1 = 6n_{\substack{n\neq 0\\ n\neq 4\\ n\neq 6\\ n\neq 6\\ n\neq 8\\ n\neq 9,20\\ n\neq 13,24\\ n\neq 11,24\\ n\neq 15,28\\ n\neq 14,31,48\\ n\neq 20,37,54\\ n\neq 16,35,54\\ n\neq 22,41,60}} - 1$$

We take the values ℕ: $n \geq 0$ ^ $n < 16{,}5$

$n \neq [0,4,6,8,9,11,13,14,15,16]$

Using the missing natural numbers in the previous set we obtain:

$n_t = [\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}, \mathbf{7}, \mathbf{10}, \mathbf{12}]$

We get the same values as in $Tp_{(a)}$

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By replacing we get the following prime numbers

$$Tp_{(b)} = 6\, n_t - 1$$

$Tp_{(b)} = [5,11,17,29,41,59,71]$

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D) Twin prime numbers:

In this chapter we will order the results of the previous exercise forming the pairs of twin prime numbers.

Twin prime numbers = $Tp > 3$

$$Tp = (Tp_{(b)}; Tp_{(a)})$$

Tp <100

$Tp_{(a)}$= Twin prime numbers in $\beta_{(a)}$ $Tp_{(a)} = [7,13,19,31,43,61,73]$

$Tp_{(b)}$ = Twin prime numbers $in\ \beta_{(b)}$ $Tp_{(b)} = [5,11,17,29,41,59,71]$

$$Tp = (5,7),(11,13),(17,19),(29,31),(41,43),(59,61),(71,73)$$

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Graphics table

The twin prime numbers in light blue, the prime numbers in yellow and the composite numbers in red.

Table 4

Twin Prime Numbers >3					
β (a)				β (b)	
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144
145	146	147	148	149	150

Conclusion

These surprising formulas, beyond calculating the prime numbers and the twin prime numbers with excellent accuracy, show that these numbers are governed by a very significant pattern, in which the signs of the formula alternate according to the Beta sequence. The two formulas are totally polarized one with respect to the other. This shows that both columns of prime and composite numbers $(\mathrm{B_a}\ \mathrm{y}\ \mathrm{B_b})$ are incredibly connected. These are complementary opposites.

These wonderful formulas generate what has been sought throughout history. For the first time we can find an expression that generates absolutely all prime numbers greater than three, composite numbers and also all prime prime numbers greater than three, and Twin prime numbers. These formulas are simple and easy although extensive, and infinite. Understanding the behavior of (n) is equivalent to understanding how prime numbers and composite numbers are distributed.

This is the expression that is combined with the mechanism of the sieve of Eratosthenes and the sieve of Sundaram.

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