# A discipline of knowledge and the graphical law 

Anindya Kumar Biswas, Department of Physics;<br>North-Eastern Hill University, Mawkynroh-Umshing, Shillong-793022.<br>email:anindya@nehu.ac.in

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#### Abstract

We study English dictionaries of five subjects. We draw in the log scale, number of entries starting with an alphabet vs rank of the alphabet, both normalised. We find that the graphs are closer to the curves of reduced magnetisation vs reduced temperature for various approximations of Ising model.


## I Introduction and Results

Magnetisation and linguistics are two widely separated distinct areas of academic investigations. It was interesting to find a curve of magnetisation existing beneath a written natural language, 1]. The author has studied the word (and verb,adverb,adjective) contents along the letters in a language. The letters were arranged in ascending order of their ranks from the rank one. The letter with the highest number of words starting with, was taken as of rank one. For a natural language, a dictionary from it to English, was a natural choice for that type of study. The author has found that behind each language which was subjected to study, there is a curve of magnetisation. From that the author has conjectured that behind any written natural language there are curves of magnetisation, for words, verbs, adverbs and adjectives respectively. The graphical law was found also to exist in the contemporary chinese usages.

In this work, we continue our search for the graphical law into various disciplines of knowledge. We describe how a graphical law is hidden within a subject in this article. We take the dictionary of five subjects, [2]-[6]. These are science, philosophy, sociology, law and construction. Then we count the independent entries, one by one from the beginning to the end, starting with different alphabets. For science, highest number of entries occur for the alphabet $C$, lowest for $Y$. For philosophy, highest number of entries occur for the alphabet $C$, lowest for $X$. For sociology, highest number of entries occur for the alphabet $S$, lowest for $Y$. For law, highest number of entries occur for the alphabet $C$, lowest for $X$. For construction, highest number of entries occur for the alphabet $S$,


Figure 1: Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{\max }}$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points stands for the subject represented by the titles. For law the reference curve is Bethe line for four neighbours. For sociology the comparator is Bragg-Williams line in absence of magnetic field. For the rest the fit curve is Bragg-Williams in presence of little magnetic field.
lowest for $X$. Then, we assort the alphabets according to their rankings. For science, the alphabet $C$ has the lowest rank one and the rank of the alphabet $Y$ is twentyfive. We take natural logarithm of both number of entries, denoted by $f$ and the respective rank, denoted by $k . k$ is a positive integer starting from one. Since each subject has an alphabet, number of occurances initiating with it being very close to one or, one, we attach a limiting rank, $k_{\text {lim }}$, and a limiting number of occurances for each subject. The limiting rank is just maximum rank (maximum rank plus one) if it is one (close to one) and the limiting number of entries, $f_{\text {lim }}$, is one. For science $k_{\text {lim }}$ is twentysix, $f_{\text {lim }}$ is one and $f_{\text {max }}$ is one thousand one hundred fifty seven. As a result, when $k$ varies from one to $k_{\text {lim }}$, $f$ varies from $f_{\max }$ to one. In other words, $\frac{\ln f}{\ln f_{\max }}$ varies from one to zero with $\frac{\ln k}{\ln k_{l i m} m}$ changing from zero to one. Then we plot $\frac{\ln f}{\operatorname{lnf} f_{\text {max }}}$ against $\frac{\ln k}{\ln k_{l i m}}$. This is fig. We have plotted the points using gnuplot, availing of using facility. Datas for each subject is appended at the end of the paper.
On each plot we superimpose a reference curve. The reference curves are obtained from the magetisation vs temperature curves available from the theory of magnetisation, in particular, from the theory of Ising model. We restrict, moreover, to the Bragg-Williams and Bethe-Peierls approximations of the model. A brief Introduction of the Ising model is given in the appendix.
However, we observe that the points belonging to subjects are not matched with respective comparators fully in fig . We notice that the Bragg-Williams line in
presence of little magnetic field is nearest fit to the resulting points for science, philosophy and construction. For sociology the comparator is Bragg-Williams line in absence of magnetic field. Bethe line for four neighbours is the relevant curve for law.
Naturally, we try doing higher order normalisations with $\ln f_{\text {nextmax }}, \ln f_{\text {nextnextmax }}$, $\ln f_{\text {nextnextnextmax }}$ and try to see whether fits improve and if so whether two consecutive fits agree. We do it in the following way. We ignore the letters with the highest number of occurances and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-maximum $\ln f_{\text {nextmax }}$, and starting from $k=2$. The resulting points come closer to the reference magnetisation curves. We observe, in fig 2, that points for the subject sociology now align with Bragg-Williams line in presence of little magnetic field whereas, for construction, new refrence line is $\gamma=4.1$ Bethe curve.
We then ignore the letters with the highest and next highest number of occurances and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-next-to-maximum $\ln f_{\text {nextnextmax }}$, and starting from $k=3$. We see, in the fig 3 that points come closer for the subject sociology to Bethe line for $\gamma=4$. For construction, refrence line remains $\gamma=4.1$ Bethe curve, with points coming closer.
Looking for better fits, we then ignore the letters with the highest, next highest number and next to next highest number of occurances and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-next-to-maximum $\ln f_{\text {nextnextnextmax }}$, and starting from $k=4$. For sociology Bethe line for $\gamma=4$ continues to be the reference line, albeit closer, in the fig 4 For construction, points have veered liitle bit towards Bethe line for $\gamma=4.2$ though the degree of fitness has decreased. For the subject construction, Bethe line for $\gamma=4.1$ in fig 3 has got the best visual fit.
For science, successive normalisations have brought the points closer and closer to Bethe line for $\gamma=4.05$ in fig $2 \rightarrow$ fig 4 Similarly, For philosophy, successive normalisations have gotten the points closer and closer to Bethe line for $\gamma=4$ in fig $2 \rightarrow$ fig 4. Whereas, for law, successive normalisations have aligned the points closer and closer to Bethe line for $\gamma=4$ in fig $1 \rightarrow$ fig, 4 .


Figure 2: Vertical axis is $\frac{\operatorname{lnf}}{\ln n_{n e x t m a x}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The + points is for the subject in the titles. The Bethe lines are the reference curves for law and philosophy with $\gamma=4$, for science with $\gamma=4.05$ and for construction with $\gamma=4.1$. For sociology, the fit curve is Bragg-Williams in presence of little magnetic field.


Figure 3: The + points represent the subjects as represented by the titles. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{\text {nextnextmax }}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The Bethe lines are the reference curves for law, philosophy and sociology with $\gamma=4$, for science with $\gamma=4.05$ and for construction with $\gamma=4.1$.


Figure 4: The + points represent the subjects as represented by the titles. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{n e x t n e x t n e x t m a x ~}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The Bethe lines are the reference curves for law, philosophy and sociology with $\gamma=4$, for science with $\gamma=4.05$ and for construction with $\gamma=4.2$.

## II Conclusion

From the figures (fig. (1fig(4), we observe that there are curves of magnetisation, behind each subject, enabling us to put the five subjects, depending on the best visual fits, in the following classification

| science | philosophy | sociology | law | construction |
| :--- | :--- | :--- | :--- | :--- |
| Bethe(4.05) | Bethe(4) | Bethe(4) | Bethe(4) | Bethe(4.1) |

In other words, we conclude that the graphical law exists beneath the subjects, [2]-6]. The associated correspondances are,

$$
\begin{aligned}
\frac{\ln f}{\text { Normaliser }} & \longleftrightarrow \frac{M}{M_{\max }}, \\
\frac{\ln k}{\ln k_{l i m}} & \longleftrightarrow \frac{T}{T_{c}}
\end{aligned}
$$

Moreover, k corresponds to temperature in an exponential scale, [7]. As temperature decreases, i.e. $l n k$ decreases, f increases. The alphabets which are recording higher entries compared to those which have lesser entries are at lower temperature. As a subject expands, the alphabets which get enriched more and
more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [8], in another way.

## III Acknowledgement

We would like to thank NEHU library for allowing us to use dictionary of Law and Administration, [6].

## IV appendix

## IV. 1 datas

## IV.1.1 Science

| k | lnk | $\operatorname{lnk} / \ln k_{l i m}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\max }$ | $\operatorname{lnf} / \ln f_{\text {nextmax }}$ | $\operatorname{lnf} / \ln f_{\text {nextnextmax }}$ | $\operatorname{lnf} / \ln f_{\text {nextnextnextmax }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 1157 | 7.05 | 1 | Blank | Blank | Blank |
| 2 | 0.69 | 0.212 | 955 | 6.86 | 0.973 | 1 | Blank | Blank |
| 3 | 1.10 | 0.337 | 943 | 6.85 | 0.972 | 0.999 | 1 | Blank |
| 4 | 1.39 | 0.426 | 877 | 6.78 | 0.962 | 0.988 | 0.990 | 1 |
| 5 | 1.61 | 0.494 | 655 | 6.48 | 0.919 | 0.945 | 0.946 | 0.956 |
| 6 | 1.79 | 0.549 | 573 | 6.35 | 0.901 | 0.926 | 0.927 | 0.937 |
| 7 | 1.95 | 0.598 | 551 | 6.31 | 0.895 | 0.920 | 0.921 | 0.931 |
| 8 | 2.08 | 0.638 | 497 | 6.21 | 0.881 | 0.905 | 0.907 | 0.916 |
| 9 | 2.20 | 0.675 | 478 | 6.17 | 0.875 | 0.899 | 0.901 | 0.910 |
| 10 | 2.30 | 0.706 | 464 | 6.14 | 0.871 | 0.895 | 0.896 | 0.906 |
| 11 | 2.40 | 0.736 | 400 | 5.99 | 0.850 | 0.873 | 0.874 | 0.883 |
| 12 | 2.48 | 0.761 | 394 | 5.98 | 0.848 | 0.872 | 0.873 | 0.882 |
| 13 | 2.56 | 0.785 | 391 | 5.97 | 0.847 | 0.870 | 0.872 | 0.881 |
| 14 | 2.64 | 0.810 | 367 | 5.91 | 0.838 | 0.862 | 0.863 | 0.872 |
| 15 | 2.71 | 0.831 | 292 | 5.68 | 0.806 | 0.828 | 0.829 | 0.838 |
| 16 | 2.77 | 0.850 | 244 | 5.50 | 0.780 | 0.802 | 0.803 | 0.811 |
| 17 | 2.83 | 0.868 | 185 | 5.22 | 0.740 | 0.761 | 0.762 | 0.770 |
| 18 | 2.89 | 0.887 | 122 | 4.80 | 0.681 | 0.700 | 0.701 | 0.708 |
| 19 | 2.94 | 0.902 | 108 | 4.68 | 0.664 | 0.682 | 0.683 | 0.690 |
| 20 | 3.00 | 0.920 | 81 | 4.39 | 0.623 | 0.640 | 0.641 | 0.647 |
| 21 | 3.04 | 0.933 | 74 | 4.30 | 0.610 | 0.627 | 0.628 | 0.634 |
| 22 | 3.09 | 0.948 | 47 | 3.85 | 0.546 | 0.561 | 0.562 | 0.568 |
| 23 | 3.14 | 0.963 | 45 | 3.81 | 0.540 | 0.555 | 0.556 | 0.562 |
| 24 | 3.18 | 0.975 | 26 | 3.26 | 0.462 | 0.475 | 0.476 | 0.481 |
| 25 | 3.22 | 0.988 | 19 | 2.94 | 0.417 | 0.429 | 0.429 | 0.434 |
| 26 | 3.26 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

## IV.1.2 Philosophy

| k | lnk | $\operatorname{lnk} / \ln k_{\text {lim }}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\max }$ | $\operatorname{lnf} / \ln f_{\text {next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-next-max }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 344 | 5.84 | 1 | Blank | Blank | Blank |
| 2 | 0.69 | 0.209 | 321 | 5.77 | 0.988 | 1 | Blank | Blank |
| 3 | 1.10 | 0.333 | 301 | 5.71 | 0.978 | 0.990 | 1 | Blank |
| 4 | 1.39 | 0.421 | 293 | 5.68 | 0.973 | 0.984 | 0.995 | 1 |
| 5 | 1.61 | 0.488 | 262 | 5.57 | 0.954 | 0.965 | 0.975 | 0.981 |
| 6 | 1.79 | 0.542 | 211 | 5.35 | 0.916 | 0.927 | 0.937 | 0.942 |
| 7 | 1.95 | 0.591 | 206 | 5.33 | 0.913 | 0.924 | 0.933 | 0.938 |
| 8 | 2.08 | 0.630 | 194 | 5.27 | 0.902 | 0.913 | 0.923 | 0.928 |
| 9 | 2.20 | 0.667 | 146 | 4.98 | 0.853 | 0.863 | 0.872 | 0.877 |
| 10 | 2.30 | 0.697 | 140 | 4.94 | 0.846 | 0.856 | 0.865 | 0.870 |
| 11 | 2.40 | 0.727 | 128 | 4.85 | 0.830 | 0.841 | 0.849 | 0.854 |
| 12 | 2.48 | 0.752 | 127 | 4.84 | 0.829 | 0.839 | 0.848 | 0.852 |
| 13 | 2.56 | 0.776 | 126 | 4.84 | 0.829 | 0.839 | 0.848 | 0.852 |
| 14 | 2.64 | 0.800 | 125 | 4.83 | 0.827 | 0.837 | 0.846 | 0.850 |
| 15 | 2.71 | 0.821 | 105 | 4.65 | 0.796 | 0.806 | 0.814 | 0.819 |
| 16 | 2.77 | 0.839 | 91 | 4.51 | 0.772 | 0.782 | 0.790 | 0.794 |
| 17 | 2.83 | 0.858 | 73 | 4.29 | 0.735 | 0.744 | 0.751 | 0.755 |
| 18 | 2.89 | 0.876 | 60 | 4.09 | 0.700 | 0.709 | 0.716 | 0.720 |
| 19 | 2.94 | 0.891 | 51 | 3.93 | 0.673 | 0.681 | 0.688 | 0.692 |
| 20 | 3.00 | 0.909 | 35 | 3.56 | 0.610 | 0.617 | 0.623 | 0.627 |
| 21 | 3.04 | 0.921 | 34 | 3.53 | 0.604 | 0.612 | 0.618 | 0.621 |
| 22 | 3.09 | 0.936 | 28 | 3.33 | 0.570 | 0.577 | 0.583 | 0.586 |
| 23 | 3.14 | 0.952 | 25 | 3.22 | 0.551 | 0.558 | 0.564 | 0.567 |
| 24 | 3.18 | 0.964 | 16 | 2.77 | 0.474 | 0.480 | 0.485 | 0.488 |
| 25 | 3.22 | 0.976 | 9 | 2.20 | 0.377 | 0.381 | 0.385 | 0.387 |
| 26 | 3.26 | 0.988 | 5 | 1.61 | 0.276 | 0.279 | 0.282 | 0.283 |
| 27 | 3.30 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

IV.1.3 Sociology

| k | lnk | $\operatorname{lnk} / \ln k_{l i m}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\text {max }}$ | $\operatorname{lnf} / \ln f_{\text {nextmax }}$ | $\operatorname{lnf} / \ln f_{\text {nextnextmax }}$ | $\operatorname{lnf} / \ln f_{\text {nextnextnextmax }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 445 | 6.10 | 1 | Blank | Blank | Blank |
| 2 | 0.69 | 0.212 | 322 | 5.77 | 0.946 | 1 | Blank | Blank |
| 3 | 1.10 | 0.337 | 215 | 5.37 | 0.880 | 0.931 | 1 | Blank |
| 4 | 1.39 | 0.426 | 207 | 5.33 | 0.874 | 0.924 | 0.993 | 1 |
| 5 | 1.61 | 0.494 | 167 | 5.12 | 0.839 | 0.887 | 0.953 | 0.961 |
| 6 | 1.79 | 0.549 | 145 | 4.98 | 0.816 | 0.863 | 0.927 | 0.934 |
| 7 | 1.95 | 0.598 | 142 | 4.96 | 0.813 | 0.860 | 0.924 | 0.931 |
| 8 | 2.08 | 0.638 | 141 | 4.95 | 0.811 | 0.858 | 0.922 | 0.929 |
| 9 | 2.20 | 0.675 | 123 | 4.81 | 0.789 | 0.834 | 0.896 | 0.902 |
| 10 | 2.30 | 0.706 | 111 | 4.71 | 0.772 | 0.816 | 0.877 | 0.884 |
| 11 | 2.40 | 0.736 | 97 | 4.57 | 0.749 | 0.792 | 0.851 | 0.857 |
| 12 | 2.48 | 0.761 | 94 | 4.54 | 0.744 | 0.787 | 0.845 | 0.852 |
| 13 | 2.56 | 0.785 | 92 | 4.52 | 0.741 | 0.783 | 0.842 | 0.848 |
| 14 | 2.64 | 0.810 | 90 | 4.50 | 0.738 | 0.780 | 0.838 | 0.844 |
| 15 | 2.71 | 0.831 | 88 | 4.48 | 0.734 | 0.776 | 0.834 | 0.841 |
| 16 | 2.77 | 0.850 | 74 | 4.30 | 0.705 | 0.745 | 0.801 | 0.807 |
| 17 | 2.83 | 0.868 | 68 | 4.22 | 0.692 | 0.731 | 0.786 | 0.792 |
| 18 | 2.89 | 0.887 | 44 | 3.78 | 0.620 | 0.655 | 0.704 | 0.709 |
| 19 | 2.94 | 0.902 | 36 | 3.58 | 0.587 | 0.620 | 0.667 | 0.672 |
| 20 | 3.00 | 0.920 | 33 | 3.50 | 0.574 | 0.607 | 0.652 | 0.657 |
| 21 | 3.04 | 0.933 | 21 | 3.04 | 0.498 | 0.527 | 0.566 | 0.570 |
| 22 | 3.09 | 0.948 | 15 | 2.71 | 0.444 | 0.470 | 0.505 | 0.508 |
| 23 | 3.14 | 0.963 | 14 | 2.64 | 0.433 | 0.458 | 0.492 | 0.495 |
| 24 | 3.18 | 0.975 | 7 | 1.95 | 0.320 | 0.338 | 0.363 | 0.366 |
| 25 | 3.22 | 0.988 | 2 | 0.69 | 0.113 | 0.120 | 0.128 | 0.128 |
| 26 | 3.26 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

## IV.1.4 Construction

| k | $\operatorname{lnk}$ | $\operatorname{lnk} / \ln k_{\text {lim }}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\max }$ | $\operatorname{lnf} / \ln f_{\text {next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-next-max }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 1289 | 7.16 | 1 | Blank | Blank | Blank |
| 2 | 0.69 | 0.209 | 727 | 6.59 | 0.920 | 1 | Blank | Blank |
| 3 | 1.10 | 0.333 | 634 | 6.45 | 0.901 | 0.979 | 1 | Blank |
| 4 | 1.39 | 0.421 | 593 | 6.39 | 0.892 | 0.970 | 0.991 | 1 |
| 5 | 1.61 | 0.488 | 539 | 6.29 | 0.878 | 0.954 | 0.975 | 0.969 |
| 6 | 1.79 | 0.542 | 519 | 6.25 | 0.873 | 0.948 | 0.966 | 0.984 |
| 7 | 1.95 | 0.591 | 508 | 6.23 | 0.870 | 0.945 | 0.978 |  |
| 8 | 2.08 | 0.630 | 501 | 6.22 | 0.869 | 0.944 | 0.975 |  |
| 9 | 2.20 | 0.667 | 495 | 6.20 | 0.866 | 0.941 | 0.973 |  |
| 10 | 2.30 | 0.697 | 421 | 6.04 | 0.844 | 0.917 | 0.970 |  |
| 11 | 2.40 | 0.727 | 385 | 5.95 | 0.831 | 0.903 | 0.936 | 0.945 |
| 12 | 2.48 | 0.752 | 347 | 5.85 | 0.817 | 0.888 | 0.931 |  |
| 13 | 2.56 | 0.776 | 340 | 5.83 | 0.814 | 0.885 | 0.915 |  |
| 14 | 2.64 | 0.800 | 335 | 5.81 | 0.811 | 0.882 | 0.907 | 0.912 |
| 15 | 2.71 | 0.821 | 283 | 5.65 | 0.789 | 0.857 | 0.909 | 0.884 |
| 16 | 2.77 | 0.839 | 265 | 5.58 | 0.779 | 0.847 | 0.873 |  |
| 17 | 2.83 | 0.858 | 214 | 5.37 | 0.750 | 0.815 | 0.876 | 0.840 |
| 18 | 2.89 | 0.876 | 185 | 5.22 | 0.729 | 0.792 | 0.833 | 0.817 |
| 19 | 2.94 | 0.891 | 154 | 5.04 | 0.704 | 0.765 | 0.781 | 0.789 |
| 20 | 3.00 | 0.909 | 140 | 4.94 | 0.690 | 0.750 | 0.766 | 0.773 |
| 21 | 3.04 | 0.921 | 102 | 4.62 | 0.645 | 0.701 | 0.716 | 0.723 |
| 22 | 3.09 | 0.936 | 78 | 4.36 | 0.609 | 0.662 | 0.676 | 0.682 |
| 23 | 3.14 | 0.952 | 67 | 4.20 | 0.587 | 0.637 | 0.657 | 0.516 |
| 24 | 3.18 | 0.964 | 27 | 3.30 | 0.461 | 0.501 | 0.512 | 0.460 |
| 25 | 3.22 | 0.976 | 19 | 2.94 | 0.411 | 0.446 | 0.456 | 0 |
| 26 | 3.26 | 0.988 | 7 | 1.95 | 0.296 | 0.279 | 0 | 0 |
| 27 | 3.30 | 1 | 1 | 0 | 0 | 0 | 0.305 |  |

## IV.1.5 Law

| k | $\operatorname{lnk}$ | $\operatorname{lnk} / \ln k_{\text {lim }}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\max }$ | $\operatorname{lnf} / \ln f_{\text {next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-max }}$ | $\operatorname{lnf} / \ln f_{\text {next-next-next-max }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 2203 | 7.70 | 1 | Blank | Blank | Blank |
| 2 | 0.69 | 0.209 | 2115 | 7.66 | 0.995 | 1 | Blank | Blank |
| 3 | 1.10 | 0.333 | 2015 | 7.61 | 0.988 | 0.993 | 1 | Blank |
| 4 | 1.39 | 0.421 | 1977 | 7.59 | 0.986 | 0.991 | 0.997 | 1 |
| 5 | 1.61 | 0.488 | 1473 | 7.30 | 0.948 | 0.953 | 0.959 | 0.962 |
| 6 | 1.79 | 0.542 | 1435 | 7.27 | 0.944 | 0.949 | 0.945 | 0.958 |
| 7 | 1.95 | 0.591 | 1342 | 7.20 | 0.935 | 0.940 | 0.904 | 0.949 |
| 8 | 2.08 | 0.630 | 970 | 6.88 | 0.894 | 0.898 | 0.896 | 0.806 |
| 9 | 2.20 | 0.667 | 917 | 6.82 | 0.886 | 0.890 | 0.899 |  |
| 10 | 2.30 | 0.697 | 907 | 6.81 | 0.884 | 0.889 | 0.886 | 0.897 |
| 11 | 2.40 | 0.727 | 842 | 6.74 | 0.875 | 0.880 | 0.875 | 0.877 |
| 12 | 2.48 | 0.752 | 781 | 6.66 | 0.865 | 0.869 | 0.861 | 0.849 |
| 13 | 2.56 | 0.776 | 698 | 6.55 | 0.851 | 0.855 | 0.827 | 0.851 |
| 14 | 2.64 | 0.800 | 642 | 6.46 | 0.839 | 0.843 | 0.817 | 0.819 |
| 15 | 2.71 | 0.821 | 538 | 6.29 | 0.817 | 0.821 | 0.806 |  |
| 16 | 2.77 | 0.839 | 501 | 6.22 | 0.808 | 0.812 | 0.800 |  |
| 17 | 2.83 | 0.858 | 449 | 6.12 | 0.795 | 0.799 | 0.798 | 0.793 |
| 18 | 2.89 | 0.876 | 433 | 6.07 | 0.788 | 0.792 | 0.791 | 0.697 |
| 19 | 2.94 | 0.891 | 412 | 6.02 | 0.782 | 0.786 | 0.750 | 0.572 |
| 20 | 3.00 | 0.909 | 302 | 5.71 | 0.742 | 0.745 | 0.695 | 0.570 |
| 21 | 3.04 | 0.921 | 199 | 5.29 | 0.687 | 0.691 | 0.519 | 0.372 |
| 22 | 3.09 | 0.936 | 77 | 4.34 | 0.564 | 0.567 | 0.315 | 0.145 |
| 23 | 3.14 | 0.952 | 52 | 3.95 | 0.513 | 0.516 | 0 | 0.373 |
| 24 | 3.18 | 0.964 | 17 | 2.83 | 0.368 | 0.369 | 0.145 |  |
| 25 | 3.22 | 0.976 | 11 | 2.40 | 0.312 | 0.313 | 0.144 | 0 |
| 26 | 3.26 | 0.988 | 3 | 1.10 | 0.143 | 0 | 0 | 0 |
| 27 | 3.30 | 1 | 1 | 0 |  | 0. | 0 |  |



Figure 5: Reduced magnetisation vs reduced reduced temperature for BraggWilliams(lower two curves, outer one in presence of magnetic field, $c=0.01$ ), Bethe(upper lines, for $\gamma=4,4.05,4.1,4.2$, outward). Vertical axis refers to reduced magnetisation and horizontal axis represents reduced temperature.

## IV. 2 Magnetisation

We give a brief introduction to the magnetisation curves of Ising model. Ising model is a classical model of lattice of interacting spins at a particular ambient temperature. Each spin has a magnetic moment associated with it. The net magnetic moment of the lattice is referred to as the magnetisation. For an Ising model, in Bragg-Williams approximation( 9 , [10]), spontaneous magnetisation per lattice site, in unit of Bohr magneton, or, reduced spontaneous magnetisation, $\frac{M}{M_{\max }}$ is $\bar{L}_{0}$. $\bar{L}_{0}$ ranges from zero to one in magnitude. Variation of magnetisation with reduced temperature, $\frac{T}{T_{c}}$, is given by

$$
\begin{equation*}
\ln \frac{1+\bar{L}_{0}}{1-\bar{L}_{0}}=\frac{2\left(\bar{L}_{0}+c\right)}{\frac{T}{T_{c}}} \tag{1}
\end{equation*}
$$

Plot of $\bar{L}_{0}$ vs $\frac{T}{T_{c}}$ is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. In an approximation scheme which is improvement over the Bragg-Williams, due to Bethe-Peierls, [9], reduced magnetisation varies with reduced temperature, for $\gamma$ neighbours, as

$$
\begin{equation*}
\frac{0.693}{\ln \frac{\text { factor }-1}{\text { factor }^{\frac{\alpha-1}{\gamma}}-\text { factor }^{\frac{1}{\gamma}}}}=\frac{T}{T_{c}} ; \text { factor }=\frac{\frac{M}{M_{\max }}+1}{1-\frac{M}{M_{\max }}} . \tag{2}
\end{equation*}
$$

The curves relevant for our study are shown in the same figure, fig 5 . We use equations(1) and (2) to generate magnetisation vs temperature tables given here. We plot the tables and obtain the curves of magnetisation. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is
urged to give a google search "reduced magnetisation vs reduced temperature curve".

## IV. 3 Reduced magnetisation vs reduced temperature datas

BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). Bethe(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used, partly, to plot fig Empty spaces in the table mean corresponding point pairs were not used for plotting a line.

| BW | BW $(\mathrm{c}=0.01)$ | Bethe $(4)$ | reduced magnetisation |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0.435 | 0.439 | 0.563 | 0.978 |
| 0.439 | 0.443 | 0.568 | 0.977 |
| 0.491 | 0.495 | 0.624 | 0.961 |
| 0.501 | 0.507 | 0.630 | 0.957 |
| 0.514 | 0.519 | 0.648 | 0.952 |
| 0.559 | 0.566 | 0.654 | 0.931 |
| 0.566 | 0.573 | 0.7 | 0.927 |
| 0.584 | 0.590 | 0.7 | 0.917 |
| 0.601 | 0.607 | 0.722 | 0.907 |
| 0.607 | 0.613 | 0.729 | 0.903 |
| 0.653 | 0.661 | 0.770 | 0.869 |
| 0.659 | 0.668 | 0.773 | 0.865 |
| 0.669 | 0.676 | 0.784 | 0.856 |
| 0.679 | 0.688 | 0.792 | 0.847 |
| 0.701 | 0.710 | 0.807 | 0.828 |
| 0.723 | 0.731 | 0.828 | 0.805 |
| 0.732 | 0.743 | 0.832 | 0.796 |
| 0.756 | 0.766 | 0.845 | 0.772 |
| 0.779 | 0.788 | 0.864 | 0.740 |
| 0.838 | 0.853 | 0.911 | 0.651 |
| 0.850 | 0.861 | 0.911 | 0.628 |
| 0.870 | 0.885 | 0.923 | 0.592 |
| 0.883 | 0.895 | 0.928 | 0.564 |
| 0.899 | 0.918 |  | 0.527 |
| 0.904 | 0.926 | 0.941 | 0.513 |
| 0.946 | 0.968 | 0.965 | 0.400 |
| 0.967 | 0.998 | 0.965 | 0.300 |
| 0.987 |  | 1 | 0.200 |
| 0.997 |  | 1 | 0.100 |
| 1 | 1 | 1 | 0 |


| Bethe3.9 | Bethe4.05 | Bethe4.1 | Bethe4.2 | reduced magnetisation |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 1 |
| 0.550 | 0.574 | 0.582 | 0.598 | 0.978 |
| 0.553 | 0.577 | 0.585 | 0.602 | 0.977 |
| 0.605 | 0.632 | 0.640 | 0.662 | 0.961 |
| 0.616 | 0.642 | 0.653 | 0.669 | 0.957 |
| 0.625 | 0.656 | 0.666 | 0.683 | 0.952 |
| 0.667 | 0.699 | 0.712 | 0.734 | 0.931 |
| 0.674 | 0.709 | 0.720 | 0.736 | 0.927 |
| 0.687 | 0.719 | 0.734 | 0.753 | 0.917 |
| 0.703 | 0.736 | 0.748 | 0.771 | 0.907 |
| 0.709 | 0.743 | 0.754 | 0.777 | 0.903 |
| 0.746 | 0.782 | 0.795 | 0.820 | 0.869 |
| 0.750 | 0.788 | 0.800 | 0.824 | 0.865 |
| 0.758 | 0.796 | 0.807 | 0.833 | 0.856 |
| 0.764 | 0.804 | 0.816 | 0.841 | 0.847 |
| 0.783 | 0.822 | 0.833 | 0.860 | 0.828 |
| 0.796 | 0.837 | 0.851 | 0.878 | 0.805 |
| 0.803 | 0.844 | 0.859 | 0.884 | 0.796 |
| 0.819 | 0.861 | 0.872 | 0.901 | 0.772 |
| 0.834 | 0.876 | 0.894 | 0.925 | 0.740 |
| 0.840 | 0.884 | 0.898 | 0.926 | 0.730 |
|  | 0.885 | 0.900 | 0.935 | 0.720 |
| 0.846 | 0.895 |  | 0.938 | 0.710 |
| 0.852 | 0.899 | 0.910 | 0.943 | 0.700 |
| 0.8611 | 0.905 | 0.917 | 0.947 | 0.690 |
| 0.8608 | 0.907 | 0.919 | 0.957 | 0.680 |
| 0.875 | 0.916 | 0.937 | 0.966 | 0.651 |
| 0.882 | 0.927 | 0.950 | 0.975 | 0.628 |
| 0.889 | 0.942 | 0.961 | 0.990 | 0.592 |
| 0.901 | 0.951 | 0.962 | 0.994 | 0.564 |
|  | 0.956 |  |  | 0.527 |
| 0.905 | 0.971 | 0.971 |  | 0.500 |
|  | 0.989 |  |  | 0.400 |
| 0.924 |  |  |  | 0.350 |
|  |  |  |  | 0.300 |
| 0.938 |  |  |  | 0.250 |
| 0.944 |  |  | 0.200 |  |
| 0.960 |  |  | 0.100 |  |
| 0.9997 |  |  |  |  |
| 1 | 1 |  |  |  |
|  |  |  |  |  |
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