Diving deep into the crypts of physics

A SELF-CREATING MODEL OF PHYSICAL REALITY

Hans van Leunen

Colophon

Written by J.A.J. van Leunen MSc

The main subject of this book is a purely mathematical model of physical reality.

This book is written as an e-book. It contains hyperlinks that become active in the electronic version, which is archived at http://vixra.org/author/j_a_j_van_leunen.

The most recent version can be accessed at http://www.e-physics.eu.

At this site, the same file is available as .docx file.

Last update of this (published) version: Wednesday, November 6, 2019

For personal use, you can bring this file to a local print shop, so that they can turn it into an affordable, easily readable A4-sized ring band book. The file starts with a front cover and ends with a back cover.

You can order this document as a print on demand book at https://www.boekenbestellen.nl/boek/the-mathematics-of-physical-reality/33143?lang=en

A Self-creating Model of Physical Reality

by Hans van Leunen

6-11-2019

Summary

The main subject of this book is a purely mathematical model of physical reality. The book acts as a survey of the Hilbert Book Model Project. The project concerns a well-founded, purely mathematical model of physical reality. The project relies on the conviction that physical reality owns its own kind of mathematics and that this mathematics guides and restricts the extension of the foundation to more complicated levels of the structure and the behavior of physical reality. This results in a model that more and more resembles the physical reality that humans can observe. The book treats several subjects that are directly related to the main subject. The book introduces new physics and new mathematics.

The selected approach results in a self-creating model that offers a creator's view and a far more restricted observer's view. Observers get their information via the dynamic field that physicists call their universe. Observers only get historical information. The creator has access to the complete model. Most physical theories only provide the observer's view.

Contents

1 The initiator of the project

The Hilbert Book Model Project is an ongoing project. Hans van Leunen is the initiator of this project. The initiator was born in the Netherlands in 1941. He will not live forever. This project will contain his scientific inheritance.

The project is introduced in a Wikiversity project [1]. In the opinion of the initiator, a Wikiversity project is a perfect way of introducing new science. It especially serves the needs of independent or retired scientific authors.

The initiator maintains a ResearchGate project that considers the Hilbert Book Model Project. The ResearchGate site supports a flexible way of discussing scientific subjects [2] [3].

The initiator has generated some documents that contain highlights as excerpts of the project, and he stored these papers on his personal e-print archive http://vixra.org/author/j_a_j_van_leunen [4].

The private website http://www.e-physics.eu contains most documents both in pdf as well as in docx format [5]. None of these documents claims copyright. Everybody is free to use the content of these papers.

1.1 Trustworthiness

Introducing new science always introduces controversial and unorthodox text. The Hilbert Book Model Project is an ongoing enterprise. Its content is dynamic and is revised regularly.

The content of this project is not peer-reviewed. It is the task of the author to ensure the correctness of what he writes. In the vision of the author, the reader is responsible for checking the validity of what he/she reads. The peer-review process cannot cope with the dynamics of revisions and extensions that become possible via publishing in freely accessible e-print archives. In comparison to openly accessible publication on the internet, the peer-review process is a rather slow process. In addition, it inhibits the usage of revision services, such as offered by vixra.org and by arxiv.org/

Reviewers are always biased, and they are never omniscient. The peer-review process is expensive and often poses barriers to the renewal of science.

One way to check the validity of the text is to bring parts of the text to open scientific discussion sites such as ResearchGate. [2]

The initiator challenges everybody to disprove the statements made in this report. He promises a fine bottle of XO cognac to anyone that finds a significant flaw in the presented theories.

This challenge stands already for several years [6]. Up to so far, nobody claimed the bottle.

1.2 The author

Hans is born in Helmond in 1941 and visited the Eindhoven HTS in chemistry from 1957-1960.

After his military service in 1960-1963, Hans started at the Technical Highschool Eindhoven (THE) which is now called the Technical University Eindhoven (TUE) for a study in applied physics.

Hans finished this study in 1970 and then joined Philips Elcoma EOD in the development of image intensifier tubes. Later this became a department of Philips Medical Systems division.

In 1987 Hans switched to an internal software house. In 1995 Hans joined the Semiconductor division of Philips. In this period Hans designed a system for modular software generation.

In 2001 Hans retired.

From 1983 until 2006 Hans owned a software company "Technische en Wetenschappelijke Programmatuur" (TWP).

A private website treats my current activities [5].

I store my papers at a freely accessible e-print archive [4].

To investigate the foundations and the lower levels of physical reality, Hans started in 2009 a personal research project that in 2011 got its current name "The Hilbert Book Model Project."

The Hilbert Book Model is a purely mathematical unorthodox and controversial model of the foundations and the lower levels of the structure of physical reality.

Hans's motto: If you think, then think twice.

Hans's conviction: We live in a universe that recurrently renews its content at a high regeneration rate.

1.3 Early encounters

I am born with a deep curiosity about my living environment. When I became aware of this, I was astonished why this environment appeared to be so complicated, and at the same time, it behaved in such a coherent way. In my childhood, I had no clue. Later some unique experiences offered me some indications. After my retirement, I started in 2009 a personal research project to discover and formulate some of the clues. The "Hilbert Book Model" is the name of my personal research project.

My interest in the structure and phenomena of physical reality started in the third year of my physics study when the configuration of quantum mechanics confronted me for the first time with its special approach. The fact that its methodology differed fundamentally from the way that physicists did classical mechanics astonished me. So, I asked my very wise lecturer, professor Broer, on what origin this difference is based. His answer was that the superposition principle caused this difference. I was not very happy with this answer because the superposition principle was indeed part of the methodology of quantum mechanics, but in those days, I did not comprehend how that could present the main cause of the difference between the two methodologies. I decided to dive into literature, and after some search, I encountered the booklet of Peter Mittelsteadt, "Philosophische Probleme der modernen Physik" (1963). This booklet contained a chapter about quantum logic and that appeared to me to contain a more appropriate answer. Later, this appeared a far too quick conclusion. In 1936 Garrett Birkhoff and John von Neumann published a paper that described their discovery of what they called "quantum logic." [7] Quantum logic is since then in mathematical terminology known as an orthomodular lattice [8]. Another name is Hilbert lattice. The relational structure of this lattice is, to a large extent, quite like the relational structure of classical logic. That is why the duo gave their discovery the name "quantum logic." This name was an unlucky choice because no good reason exists to consider the orthomodular lattice as a system of logical propositions. In the same paper, the duo indicated that the set of closed subspaces of a separable Hilbert space has exactly the relational structure of an orthomodular lattice. John von Neumann long doubted between Hilbert spaces and projective geometries. In the end, he selected Hilbert spaces as the best platform for developing quantum physical theories. That appears to be one of the main reasons why quantum physicists prefer Hilbert spaces as a realm in which they do their modeling of quantum physical systems. Another habit of quantum physicists also intrigued me. My lecturer thought me that all observable quantum physical quantities are eigenvalues of Hermitian operators. Hermitian operators feature real eigenvalues. When I looked around, I saw a world that had a structure that configures from a three-dimensional spatial domain and a one-dimensional and thus, scalar time domain. In the quantum physics of that time, no operator represents the time domain, and no operator was used to deliver the spatial domain in a compact fashion. After some trials, I discovered a four-dimensional number system that could provide an appropriate normal operator with an eigenspace that represented the full four-dimensional representation of my living environment. At that moment, I had not yet heard from quaternions, but an assistant professor Boudewijn Verhaar quickly told me about the discovery of Rowan Hamilton that happened more than a century earlier. Quaternions appear to be the number system of choice for offering the structure of physical reality its powerful capabilities.

The introductory paper of Birkhoff and von Neumann already mentioned quaternions. Much later Maria Pia Solèr offered a hard prove that Hilbert spaces can only cope with members of an associative division ring. Quaternions form the most versatile associative division ring. To my astonishment, I quickly discovered that physicists preferred a spacetime structure that features a Minkowski signature instead of the Euclidean signature of the quaternions. The devised Hilbert Book Model shows that in physical reality, the Euclidean structure, as well as the spacetime structure, appear in parallel. Observers only see the spacetime structure. Physics is a science that focusses on observable information. My university, the TUE, targeted applied physics, and there was not much time nor support for diving deep into the fundamentals of quantum physics. After my study, I started a career in the high-tech industry, where I joined the development of image intensifier devices. There followed my confrontation with optics and with the actual behavior of elementary particles. See: http://www.ephysics.eu/#_What_image_intensifiers reveal.

In the second part of my career, I devoted my time to establish a better way of generating software. I saw how the industry was very successful in the modular construction of hardware. The software was still developed as a monolithic system. My experiences in this trial are reported in the chapter "Story of a War Against Software Complexity"; and the chapter "Managing the Software Generation Process." It taught me the power of modular design and modular construction [4].

Only after my retirement, I got enough time to dive deep into the foundations of physical reality. In 2009 after the recovery of a severe disease, I started my personal research project that in 2011 got its current name “The Hilbert Book Model.” For the rest of his life, the author takes the freedom to upgrade the related papers at a steady rate.

2 Intention

Theoretical physics still contains unresolved subjects. These deficiencies of the theory are caused by the way that physics was developed and by the attitude of the physicists that designed the current theory. Scientists take great care to secure the trustworthiness of their work, which ends in the publication of the results. They take measures to prevent that their publications get intermingled with badly prepared publications or even worse, with descriptions of fantasies. For that reason, they invented the scientific method [7]. In applied physics, the scientific method founds on observations. Applied physics flourishes because the descriptions of observations help to explore these findings, especially when formulas extend the usability of the observations beyond direct observation. In theoretical physics, this is not always possible because not all aspects of physical reality are observable. The only way of resolving this blockade is to start from a proper foundation that can be extended via trustworthy methods that rely on deduction. This approach can only be successful if the deduction process is guided and restricted such that the extensions of the foundation still describe physical reality. Thus, if a mathematical deduction is applied, then mathematics must guide and restrict this process such that a mathematically consistent extension of the model is again a valid model of physical reality. After a series of development steps, this approach must lead to a structure and behavior of the model that more and more conforms to the reality that we can observe.

This guidance and restriction are not self-evident. On the other hand, we know that when we investigate deeper, the structure becomes simpler and easier comprehensible. So, finally, we come to a fundamental structure that can be considered as a suitable foundation. The way back to more complicated levels of the structure cannot be selected freely. Mathematics must pose restrictions onto the extension of the fundamental structure. This happens to be true for a foundation that was discovered about eighty years ago by two scholars. They called their discovery quantum logic [8]. The scholar duo selected the name of this relational structure because its relational structure resembled closely the relational structure of the already known classical logic. Garrett Birkhoff was an expert in relational structures. These are sets that precisely define what relations are tolerated between the elements of the set. Mathematicians call these relational structures lattices, and they classified quantum logic as an orthomodular lattice [9]. John von Neumann was a broadly oriented scientist that together with others, was searching for a platform that was suitable for the modeling of quantum mechanical systems. He long doubted between two modeling platforms. One was a projective geometry, and the other was a Hilbert space [10] [11] [12].Finally, he selected Hilbert spaces. In their introductory paper, the duo showed that quantum logic emerges into a separable Hilbert space. The set of closed subspaces inside a separable Hilbert space has exactly the relational structure of an orthomodular lattice. The union of these subspaces equals the Hilbert space. A separable Hilbert space applies an underlying vector space [13], and between every pair of vectors, it defines an inner product [14]. This inner product can only apply numbers that are taken from an associative division ring [15] [16]. In a division ring, every non-zero member owns a unique inverse. Only three suitable division rings exist. These are the real numbers, the complex numbers, and the quaternions. Depending on their dimension, these number systems exist in several versions that differ in the way that Cartesian and polar coordinate systems sequence their members [17] [18].

In the Hilbert space, operators exist that can map the Hilbert space onto itself. In this way, the operator can map some vectors along themselves. The inner product of a normalized vector with such a map produces an eigenvalue. This turns the vector into an eigenvector. Together the eigenvalues of an operator form its eigenspace. This story indicates that mathematics guides and restricts the extension of the selected foundation into more complicated levels of the structure. It shows that the scholar duo started a promising development project.

However, this initial development was not pursued much further. Axiomatic models of physical reality are not popular. Most physicists mistrust this approach. Probably these physicists consider it naïve to suspect that an axiomatic foundation can be discovered that like the way that a seed evolves in a certain type of plant, will evolve into the model of the physical reality that we can observe.

Most quantum physicists decided to take another route that much more followed the line of the physical version of the scientific method. As could be suspected, this route gets hampered by the fact that not every facet of physical reality can be verified by suitable experiments.

Mainstream quantum physics took the route [20] of quantum field theory [21], which diversified into quantum electrodynamics [22] and quantum chromodynamics [23]. It bases on the principle of least action [24], the Lagrangian equation [25], and the path integral [26] However, none of these theories apply a proper foundation.

In contrast, the Hilbert Book Model Project intends to provide a purely and self-consistent mathematical model of physical reality [1] [20]. It uses the orthomodular lattice as its axiomatic foundation and applies some general characteristics of reality as guiding lines. An important ingredient is the modular design of most of the discrete objects that exist in the universe. Another difference is that the Hilbert Book Model relies on the control of coherence and binding by stochastic processes that own a characteristic function instead of the weak and strong forces and the force carriers that QFT, QED, and QCD apply [21] [22] [23].

Crucial to the Hilbert Book Model is that reality applies quaternionic Hilbert spaces as structured read-only archives of the dynamic geometric data of the discrete objects that exist in the model. The model stores these data before they can be accessed by observers. This fact makes it possible to interpret the model as the creator of the universe. The classification of modules as observers introduces two different views; the creator's view and the observer's view. Time reversal is only possible in the creator's view. It cannot be perceived by observers because observers must travel with the scanning time window.

The most amazing discovery of the Hilbert Book Model is that the initial assumption that the Hilbert Book Model creates itself indeed appears to be true.

3 The Hilbert Book Base Model

The Hilbert Book Model Project deviates considerably from the mainstream physics approaches. It tries to stay inside a purely mathematical model that can be deduced from the selected foundation. First, it designs a base model that is configured from a huge set of quaternionic separable Hilbert spaces that all share the same underlying vector space. We call this the standard base model. It shows an interesting resemblance to the Standard Model that current physics applies.

3.1 The standard base model

The standard base model is a completely countable model that archives the countable part of the Hilbert Book Model. This model emerges directly from the orthomodular lattice. It adds an infinite dimensional vector space and a set of versions of the quaternionic number system. The versions differ in the coordinate systems that they apply to sequence the members of the number system. These versions are applied to specify the inner products of pairs of vectors, such that the subspaces of the vector space can be closed. Sets of these subspaces have the lattice structure of the orthomodular lattice. In this way leads every applied version of the number system to a corresponding separable Hilbert space. A dedicated normal operator that will be called reference operator manages the applied version of the number system as its eigenspace. This eigenspace acts as the private parameter space of the separable Hilbert space. The vector space supports a huge number of these separable Hilbert spaces. One of them acts as a background platform and it supports the background parameter space. All other separable Hilbert spaces act as floating platforms. They float with the geometric center of their parameter space over the background parameter space. Each of these Hilbert spaces causes a contribution to a field in the background Hilbert space. That contribution is represented by a source or a sink at the image of the geometric center of the floating platform. The source or sink is characterized by a symmetry related charge. The value of the charge relates to the difference in symmetry of the floating platform with respect to the symmetry of the background platform. The symmetry is determined by the way the selected coordinate system sequences the elements of the version of the number system along the axes of the coordinate system. On each floating platform a dedicated footprint operator manages an ongoing hopping path. The hop landing locations of the hopping path are archived as a combination of a timestamp and a three-dimensional spatial location that is taken from the parameter space of the platform. A private stochastic process that owns a characteristic function generates the hop landing locations of the hopping path and ensures that the hopping path recurrently regenerates a coherent hop location swarm that is described by a stable location density distribution. The characteristic function of the process equals the Fourier transform of the location density distribution. In this way the floating platform represents a point-like hopping particle that moves with the floating platform. The location density distribution equals the square of the modulus of what physicists would call the wavefunction of the particle. The eigenspace of the dedicated footprint operator archives the complete life story of the particle. After sequencing the real parts of these eigenvalues, the archive tells the life story of the point-like object as an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm.

Mainstream quantum physics attributes the name elementary particles to the described point-like particles. They behave as elementary modules, but mainstream physics does not exploit that interpretation. In contrast, the Hilbert Book Model Project exploits the modular design of the model.

The collection of allowable versions of the quaternionic number system provides more versions than the short list of electrical charges in the standard model suggests. This is repaired by requiring that the axes of the Cartesian coordinate systems in the floating platforms run in parallel with the Cartesian axes in the background platform. The ranking along the axes may still differ in sign. The only indication that the Hilbert Book model provides for this restriction is the fact that the system can thus relate the differences between the symmetries to the electric charges. This is explained in more detail in section 14.4.

3.1.1 Questions about the standard base model

The floating platforms move with the image of their geometric center of their private parameter space over the background parameter space. No straightforward reason can be found for this movement of the floating platforms relative to the background platform. From observations we know that in free space conglomerates of objects move at uniform speed. Theoretically it is not clear why the floating platforms move uniformly relative to the background platform when they do not interact with other objects.

No reason is found why a private stochastic process that owns a static characteristic function generates the content of the eigenspace of a footprint operator.

Together these facts constitute the reason of existence of an artificial field that tries to prevent its first order change. This field compensates the change of the uniform movement of the conglomerate of field excitations with the gradient of the function that characterizes the deformation that these objects cause in the field that embeds them. This artificial field is treated in sections 8.8 and 8.12.

3.2 The full base model

The full base model adds a unique companion non-separable Hilbert space to the background platform. The eigenspaces of operators of this companion Hilbert space need not to be countable and can represent continuums.

The non-separable Hilbert space embeds its separable companion. Together these companion Hilbert spaces form the background platform of the base model.

The sequencing of the real parts of the eigenvalues of the footprint operators defines a subspace of the underlying vector space that scans as a function of progression over the whole base model. This scanning window divides the model into a historical part, a window that represents the current static status quo, and a future part. In this way, the dynamic model resembles the paging of a book in which each page tells a universe-wide story of what currently happens in this continuum. This explains the name of the Hilbert Book Model. Together with the requirement that all applied separable Hilbert spaces share the same vector space the fact that a window scans the Hilbert Book Base Model as a function of a progression parameter results in the fact that these quaternionic separable Hilbert spaces share the same real number based separable Hilbert space. After sequencing the eigenvalues, the eigenspace of the reference operator of this Hilbert space acts as a model wide proper time clock.

In contrast to the Hilbert Book Model, most other physical theories apply only a single Hilbert space that applies complex numbers for defining its inner product, or they apply a Fock space [27], which is a tensor product of complex number based Hilbert spaces. A tensor product of quaternionic Hilbert spaces [28] results in a real number based Hilbert space. In the Hilbert Book Base Model, the quaternionic separable Hilbert spaces share the same real-number based Hilbert space.

The coherence of the hop landing location swarm that configures the footprint of an elementary module is ensured by the fact that the mechanism that generates the hop landing locations is a stochastic process that owns a characteristic function. This characteristic function is the Fourier transform of the location density distribution of the hop landing location swarm. The mechanism reflects the effect of the ongoing embedding of the separable Hilbert space of the elementary module into the background non-separable Hilbert space. A continuum eigenspace of a dedicated operator registers the embedding of the hop landings of all elementary modules into this continuum. The continuum corresponds to the dynamic field that physicists call the universe. This field acts as the living space of all discrete objects that exist in the universe.

3.3 Open questions

The suggested Hilbert Book Base Model raises some questions. The fact that the set of rational numbers is countable is used to suggest that a proper time clock exists and that this clock ticks with a fixed and model wide minimal period. The Hilbert Book Model does not offer an explanation or a suggestion for this minimal period. The known value of the frequency of the photon that is generated at the annihilation of an elementary particle offers some indication. For the electron that means a frequency of about 10^{20} Hertz. However, this elementary particle category exists in three known generations: electron, muon, and tau.

Further, it is suggested that the private stochastic process generates a new hop landing location at each clock tick. It is possible that the stochastic process acts slower than the proper time clock, and its rate differs for each generation.

Also, the mass of different type categories of elementary particles differs. Currently, the Hilbert Book Model has no detailed explanation for that difference.

The generations in which fermions exist may also be explained by the binding mechanism that holds composite modules together.

3.4 Selfies of an infinite-dimensional vector space

It is possible to see the discrete part of the base model as a collection of self-images of the underlying vector space. Each operator corresponds with such a selfie. The selfie can cover the full vector space or can restrict to a subspace. Vectors that are mapped along themselves are eigenvectors. Each combination of an eigenvector and an operator results in a storage bin that is attached to the eigenvector. Operators use a version of a selected number system. The storage bin is filled with an element of that version. The separable Hilbert spaces structure the coverage of the vector space with operators that represent selfies.

3.5 Categories

The selfies can be categorized in groups. The selfies in a Hilbert space form a group. The selfies that are reference operators form a group. The selfies that are footprint operators form a group. The selfies that are footprint operators and belong to the same symmetry type form a subgroup of the footprint group. Fourier transforms of the members of a group form a group. An example of a group that resides in the Fourier space are the selfies that archive displacement generators. These groups also reside in the background platform.

4 Modeling dynamic fields and discrete sets

The eigenspace of a dedicated footprint operator in a quaternionic separable Hilbert space can represent the dynamic geometric data of the point-like object that resides on this Hilbert space. The eigenspace of operators in a quaternionic non-separable Hilbert space can, in addition, represent the description of a dynamic continuum. We already met the eigenspace of the reference operator, which represents the private parameter space of the Hilbert space. In the separable Hilbert space this eigenspace is countable and contains only the rational values of the version of the quaternionic number system that the separable Hilbert space can apply as eigenvalues. In the non-separable Hilbert space, the eigenspace of the reference operator also contains all the limits of the convergent series of rational values. Consequently, this eigenspace is no longer countable. In each of the applied Hilbert spaces, it is possible to use the reference operator to define a category of newly defined operators by taking for each eigenvector of the reference operator a new eigenvalue that equals the target value of a selected quaternionic function for the parameter value that equals the corresponding eigenvalue of the reference operator. In the quaternionic separable Hilbert space, the new eigenspace represents the sampled field that is described by the selected quaternionic function. In the quaternionic non-separable Hilbert space the new eigenspace represents the full continuum that is described by the selected quaternionic function. Continuum eigenspaces can represent the mathematical equivalent of a dynamic physical field. The private parameter space of a quaternionic Hilbert space represents a flat field. The dynamics of a field can be described by quaternionic differential equations.

Quaternionic second-order partial differential equations describe the interaction between point-like actuators and a dynamic field. Physical fields differ from mathematical fields by the fact that the value of the physical field is represented in physical units. All basic fields obey the same quaternionic differential and integral equations. The basic fields differ in their start and boundary conditions.

4.1 Quaternionic differential calculus

The first order partial differential equations divide the change of a field in five different parts that each represent a new field. We will represent the field change operator by a quaternionic nabla operator. This operator behaves as a quaternionic multiplier.

A quaternion can store a timestamp in its real part and a three-dimensional spatial location in its imaginary part. The quaternionic nabla ∇ acts as a quaternionic multiplying operator. Quaternionic multiplication obeys the equation

$$\begin{aligned} c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{4.1.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{4.1.2}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia. [30] [31]

$$\begin{aligned} \phi = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)\left(\psi_r + \vec{\psi}\right) \\ = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \tag{4.1.3}$$

The differential $\nabla \psi$ describes the change of field ψ. The five separate terms in the first-order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{4.1.4}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{4.1.5}$$

$\vec{\nabla} f$ is the gradient of f.

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{4.1.6}$$

$$\begin{aligned} \zeta = \nabla^* \phi &= \left(\frac{\partial}{\partial \tau} - \vec{\nabla} \right) \left(\phi_r + \vec{\phi} \right) \\ &= \nabla_r \phi_r + \langle \vec{\nabla}, \vec{\phi} \rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \end{aligned} \tag{4.1.7}$$

4.2 Field excitations

Field excitations are solutions of second-order partial differential equations.

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned} \zeta = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi &= \left(\nabla_r + \vec{\nabla} \right) \left(\nabla_r - \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \left(\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi \end{aligned} \tag{4.2.1}$$

Integration over the time domain results in the Poisson equation

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \tag{4.2.2}$$

Under isotropic conditions, a very special solution of the Poisson equation is the Green's function $\frac{1}{4\pi |\vec{q} - \vec{q}'|}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\left|\vec{q}-\vec{q}'\right|} = -\frac{\left(\vec{q}-\vec{q}'\right)}{\left|\vec{q}-\vec{q}'\right|^3} \tag{4.2.3}$$

$$\begin{aligned} \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left|\vec{q}-\vec{q}'\right|} &\equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q}-\vec{q}'\right|} \right\rangle \\ &= -\left\langle \vec{\nabla}, \frac{\left(\vec{q}-\vec{q}'\right)}{\left|\vec{q}-\vec{q}'\right|^3} \right\rangle = 4\pi\delta\left(\vec{q}-\vec{q}'\right) \end{aligned} \tag{4.2.4}$$

Under isotropic conditions, the dynamic spherical pulse response of the field is a solution of a special form of the equation (4.2.1)

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q}-\vec{q}'\right)\theta(\tau \pm \tau') \tag{4.2.5}$$

Here $\theta(\tau)$ is a step function and $\delta(\vec{q})$ is a Dirac pulse response [33] [34].

After the instant τ', this solution is described by

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c(\tau-\tau')\vec{n}\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.2.6}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the Green’s function. This means that the expanding pulse injects the volume of the Green’s function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the Green’s function and sucks it out of the field. The $\pm$ sign in equation (4.2.5) selects between injection and subtraction.

Apart from the spherical pulse response equation (4.2.5) supports a one-dimensional pulse response that acts as a one-dimensional shock front. This solution is described by

$$\psi = f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\vec{n}\right) \tag{4.2.7}$$

Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (4.2.1) and (4.2.2) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle$ are valid second-order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation [35].

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi \tag{4.2.8}$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{4.2.9}$$

$$\psi = f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\right) \tag{4.2.10}$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions [31 [35].

By splitting the field into the time-dependent part $T(\tau)$ and a location-dependent part, $A(\vec{q})$, the homogeneous version of the wave equation can be transformed into the Helmholtz equation [36].

$$\frac{\partial^2\psi}{\partial\tau^2} = \langle\vec{\nabla},\vec{\nabla}\rangle\psi = -\omega^2\psi \tag{4.2.11}$$

$$\psi(\vec{q},\tau) = A(\vec{q})T(\tau) \tag{4.2.12}$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial \tau^2} = \frac{1}{A}\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \tag{4.2.13}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A \tag{4.2.14}$$

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep the moving objects within a bounded region [37].

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm} j_l(kr)\right) + b_{lm} Y_l^m(\theta,\varphi)\right\} \tag{4.2.15}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics [38] [39]. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions of the equation (4.2.11)

$$\psi(\vec{q},\tau) = \exp\left\{\vec{n}\left(\left\langle \vec{k}, \vec{q} - \vec{q}_0 \right\rangle - \omega\tau + \varphi\right)\right\} \tag{4.2.16}$$

$$\psi(\vec{q},\tau) = \frac{\exp\left\{\vec{n}\left(\left\langle \vec{k}, \vec{q} - \vec{q}_0 \right\rangle - \omega\tau + \varphi\right)\right\}}{\left|\vec{q} - \vec{q}_0\right|} \tag{4.2.17}$$

A more general solution is a superposition of these basic types.

Two quite similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equation

(4.2.5) and equation (4.2.8). The first equation has spherical shock front solutions with a spin vector that behaves like the spin of elementary fermionic particles. The second equation has spherical shock front solutions that behave more like elementary bosons.

The inhomogeneous pulse activated equations are

$$\left(\nabla_r\nabla_r \pm \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi = 4\pi\delta\left(\vec{q}-\vec{q'}\right)\theta\left(\tau \pm \tau'\right) \qquad (4.2.18)$$

The paper treats quaternionic differential equations more extensively in chapter 14.

5 Photons

Photons are objects that still offer significant confusion among physicists. The mainstream interpretation is still that photons are electromagnetic waves [40]. This interpretation conflicts with the known behavior of photons. Photons that are emitted by a nearby star can be detected by a human eye. Since the space between the star and the earth does not contain waveguides, waves cannot do this trick. Electromagnetic fields require the nearby presence of electric charges. Both conditions forbid that photons are implemented by electromagnetic waves.

5.1 Photon structure

Photons are one-dimensional objects that are strings of equidistant energy packages, such that the string obeys the Einstein-Planck relation

$$E = h\nu \qquad (5.1.1)$$

The energy packages are implemented by one-dimensional shock fronts that possess a polarization vector.

Where the light speed c indicates the speed at which shock fronts travel, will Planck’s constant indicate the period during which one-dimensional shock fronts will be emitted. We know the frequency of the photon that is emitted at the annihilation of an electron. Thus, we know the rate at which the energy packages that constitute this photon are produced. However, no data are available on the duration D of the photon emission or on the spatial length $L = D/c$ of photons.

$$E = h\nu = N_p E_p \tag{5.1.2}$$

$$E_p = \frac{h\nu}{N_p} \tag{5.1.3}$$

$$D = \frac{N_p}{\nu} \tag{5.1.4}$$

$$\nu = \frac{N_p}{D} \tag{5.1.5}$$

$$E = \frac{hN_p}{D} = N_p E_p \tag{5.1.6}$$

$$E_p = \frac{h}{D} \tag{5.1.7}$$

$$h = \frac{E_p}{D} = \frac{cE_p}{L} \tag{5.1.8}$$

Thus, Planck's constant equals the energy E_p of the standard energy packages divided by the emission duration of the photons.

5.2 One-dimensional pulse responses

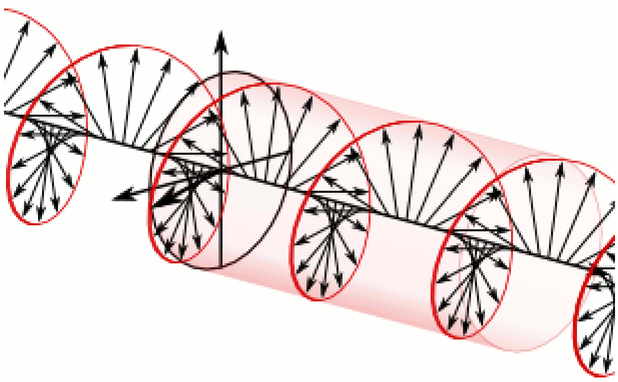
One-dimensional pulse responses that act as one-dimensional shock fronts and possess a polarization vector are solutions of the equation (4.2.5) and are described by the equation (4.2.7).

$$\psi = f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right) \tag{5.1.9}$$

During travel, the front $f(\vec{q})$ keeps its shape and its amplitude. So also, during long-range trips, the shock front does not lose its integrity. The one-dimensional pulse response represents an energy package that travels with speed c through its carrier field. The energy of the package has a standard value.

In the animation of this left handed circular polarized photon, the black arrows represent the moving shock fronts [41]. The red line connects the vectors that indicate the amplitudes of the separate shock fronts. Here the picture of a guided wave is borrowed to show the similarity with such EM waves. However,

photons are not EM waves!



5.3 Photon integrity

Except for its speed, the photon emitter determines the properties of the photon. These properties are its frequency, its energy, and its polarization. The energy packages preserve their own integrity. They travel at a constant speed and follow a worldline. Photon emission possesses a fixed duration. It is not an instant process. During emission, the emitter must not move and can only rotate around the direction of travel. Failing these requirements will compromise the integrity of the photon and make it impossible for a distant, tiny absorber to capture the full photon. In that case, the energy packages will spray and fly to multiple locations. Consequently, they will act like dark energy objects.

The absorption of a photon by an atom requires an incredible aiming precision of the emitter. In fact, this absorption can only be comprehended when it is interpreted as the time-reversal of the corresponding emission process. If the absorbing atom cannot cope with the full energy of the photon, then it might absorb only part of the energy packages of the photon. The rest will stay on its route to the next absorber. Absorbing individual energy packages will result in an increase in the kinetic energy of the absorber. Absorbing the full photon or a part of it will result in an increase in the potential energy of the absorber. Usually, this results in a higher oscillation mode of one or more of the components of the absorber.

5.4 Light

Light is a dynamic spatial distribution of photons. Often the location density distribution of photons owns a Fourier transform. In that case, the light may show wave behavior. Photons are one-dimensional particles that feature private frequency and energy. Single photons do not show wave behavior. Photons and light waves will feature different frequencies.

5.5 Optics

Optics is the science of imaging distributions of particles that can be characterized by a location density distribution and a corresponding Fourier transform of that location density distribution. Even though photons have a fixed non-zero spatial length, optics will treat these particles as point-like objects. Another name for the location density distribution is point spread function (PSF). Another name for the Fourier transform of the PSF is the optical transfer function (OTF) [42]. Apart from a location density distribution, the swarm of the particles is also characterized by an angular distribution and by an energy distribution. In the case of photons, the energy distribution is also a chromatic distribution.

A linearly operating imaging device can be characterized by its point spread function or alternatively by its OTF. This point spread function is an image of a point-like object. The PSF represents the blur that is introduced by the imaging device. For a homogeneous distribution of particle properties, the OTF of a chain of linearly operating imaging devices equals the product of the OTF's of the separate devices.

The imaging properties of an imaging device may vary as a function of the location and the orientation in the imaging surface.

Without the presence of the traveling particles, the imaging devices keep their OTF. Small apertures and patterns of apertures feature an

OTF. That OTF handles single particles similarly as this feature handles distributions of particles.

6 Modular design and construction

The discrete objects that exist in the universe show a modular design. In modular configurations, elementary particles behave as elementary modules. Together they constitute all modules that exist in the universe. Some modules constitute modular systems.

Also, photons show a composite structure.

6.1 Elementary modules

6.1.1 Symmetry-related charge

Elementary modules are very complicated objects that reside on a private platform, which possesses some of the characteristic properties of the elementary module. These properties establish the type of elementary module.

Elementary modules reside on a private Hilbert space, which uses a selected version of the quaternionic number system to specify its inner products. Consequently, the operators in this Hilbert space apply members of this version to specify its eigenvalues. The eigenspace of this operator reflects the properties of this version. Thus, the eigenspace of the reference operator reflects the symmetry of the Hilbert space. Its geometric center floats over the background parameter space. The symmetry is defined relative to the symmetry of the background platform. Mathematics can compare these differences when the axes of the Cartesian coordinate systems in these parameter spaces are parallel to each other. The model applies the Stokes theorem and the Gauss theorem to determine the effect of the symmetry differences [43] [44]. See section 14.3. The only freedoms that are left are the locations of the geometric centers of the parameter spaces and the way that the elements of the versions of the number systems are sequenced along the axes. These restrictions reduce the list of symmetry differences to a shortlist. It means that the elementary modules exist in a small number of different symmetry-related categories. The symmetry difference is represented by a symmetry-related charge that resides at the geometric center of the private parameter space. The opposed restrictions that determine the allowable versions of the quaternionic number system restrict the list of values of symmetry-related charges to $-3,-2,-1,0,+1,+2,+3$. The isotropic symmetry differences are represented by $-3,0,+3$

The symmetry-related charges correspond to symmetry-related fields. At the location of the charge, a source or a sink generates a corresponding potential.

The anisotropic differences spread over the three coordinate axes and are indicated by corresponding RGB color charges. If we extend this distinguishing to the real axis of the parameter spaces, then the anti-color charges add to the three RGB color charges. Further, the product rule of the quaternions introduces diversity in the handiness of the version of the number system. The polar coordinate system also allows the polar angle and the azimuth to run up or down. The range of the polar angle is π radians. The range of the azimuth is 2π radians. This freedom of choice adds to the freedom that is left by the Cartesian coordinate system.

The first conclusion is that elementary modules exist in a shortlist of categories that differ in their symmetry-related properties, in their angular range properties, and in their arithmetic properties.

6.2 Modular configuration

The elementary modules can combine into composed modules. Some modules combine into modular systems. However, not all modules can compose with arbitrary other modules. For example, symmetry-related charges that have the same sign will repel each other, while symmetry-related charges with a different sign will attract. Composition applies internal oscillation of the components of the module. This is explained in the next section. Only elementary modules with the proper angular symmetry can take part in the modular composition process. These elementary modules are called fermions. The other elementary modules are called bosons. Inside a composed module, fermions cannot share the same oscillation mode and cannot share the same angular properties, such as spin. The binding via internal oscillation must be supported by the attraction that is caused by deformation of the embedding field. The symmetry-related charges also influence the efficiency of the bond. The anisotropic elementary modules cannot themselves deform the embedding field. They must first combine into colorless hadrons before their combination can deform the embedding field. Physicists call this phenomenon color confinement.

The hop landings of isotropic elementary modules can produce spherical pulse responses that deform the embedding field. Similarly, the hop landings of hadrons can produce such spherical pulse responses.

6.2.1 Open question

The Hilbert Book Model does not explain why fermions feature an exclusion principle, while bosons do not possess such property. This phenomenon determines the structure of atoms and is known as the Pauli exclusion principle.

6.3 Stochastic control

For each elementary module, a private stochastic process generates the hop landing locations in the ongoing hopping path that recurrently regenerates the coherent hop landing location swarm that constitutes the footprint of the elementary module. Only for isotropic elementary modules, the hop landings can deform the embedding field. The footprints of anisotropic elementary modules must first combine into colorless hadrons before these footprints can deform the embedding field. This phenomenon is known as color confinement.

The type of stochastic process that generates the footprint of elementary modules owns a characteristic function that equals the

Fourier transform of the location density distribution of the coherent hop landing location swarm. It is possible to interpret the stochastic process as a spatial Poisson point process in $\mathbb{R}^3$ [45]. The intensity function of this process is implemented by a spatial point spread function that equals the location density distribution of the generated hop landing location swarm. The eigenspace of the footprint operator archives the target values of a quaternionic function, whose spatial part describes the point spread function. A cyclic random distribution describes the real parts of these target values. After sequencing these real parts, the eigenspace describes the ongoing hopping path of the elementary module.

The location density distribution can be interpreted as a detection probability density distribution. If it has a Fourier transform, then a kind of uncertainty principle exists between the standard deviation of the detection probability density distribution and the standard deviation of the modulus of this Fourier transform [46]. If the standard deviation of the modulus of this Fourier transform increases, then the standard deviation of the detection probability density distribution decreases (and vice versa).

The second type of stochastic process controls composed modules. This process also owns a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the module. The superposition coefficients act as displacement generators. In this way, these coefficients control the internal positions of the components. Inside atoms, these components perform their own oscillation mode. All modules attach an extra displacement generator to their characteristic function. This displacement generator determines the location of the full module.

This analysis tells that the characteristic functions, which reside in Fourier space, define the constitution of the module. In Fourier space, the spatial locality in configuration space has no meaning. It means that the components of a module can be far apart. The phenomenon is known as entanglement [47]. Only the attracting influences of potentials can keep components closely together.

6.3.1 Superposition

The way that superposition is implemented in the Hilbert Book Model explains the most important difference between classical physics and quantum physics. Superposition of field excitations occurs in Fourier space and is controlled by the characteristic functions of stochastic processes. Color confinement inhibits the generation and subsequent superposition of the field excitation for quarks. They must first combine into colorless hadrons before they can generate the required pulse responses. Also, this combination is controlled by oscillations that are managed by the characteristic functions of the corresponding stochastic processes.

Since the definition of a composed module is defined in Fourier space, the location of the components of the modules in configuration space is not important for this definition. This definition does not depend on this location. Entanglement is the phenomenon that allows components of a module to locate far apart. This fact becomes observable when these components possess exclusive properties.

6.3.2 Open questions

The Hilbert Book Model offers no detailed explanation of why the ongoing embedding of elementary modules is represented by a private stochastic process that owns a characteristic function. Similarly, the Hilbert Book Model offers no explanation for the fact that binding of modules inside composed modules is controlled by a stochastic process that owns a characteristic function that is a dynamic superposition of the characteristic functions of its components. In effect, this means that the HBM does not explain why the superposition of modules is defined in Fourier space.

6.4 Benefits of modular design and construction

The modular design hides relations that are only relevant inside the module from the outside of the module. In this way, the modular design reduces the relational complexity of the construction of composed modules. This is further improved by the possibility to gather relations in standard interfaces. This standardization promotes the reusability of modules. The fact that composed modules can be generated from lower-level modules has an enormously beneficial effect on the reduction of the relational complexity of the modular composition process.

By applying modular design, the creator has prepared the universe for modular construction, which is a very efficient way of generating new objects. However, modular configuration of objects involves the availability of modules that can be joined to become higher-level modules or modular systems. This means that enough resources must be available at the proper place and the proper time. The generation of a module out of composing modules makes sense when the new module has a profitable functionality. An advantage can be that the new module or modular system has a better chance of survival in a competitive environment. In that case, stochastic modular design can easily win from monolithic design. Evolution can evolve with a pure stochastic modular design. However, as soon as intelligent species are generated as modular systems, then these individuals can take part in the control of evolution by intelligent modular design. Intelligent modular design and construction occur much faster than stochastic modular design and construction. However, intelligent modular design and construction only occur where intelligent species exist. These locations are not widespread in the universe.

6.4.1 Modular hierarchy

The modular hierarchy starts with elementary modules. Elementary modules exist in several types that differ in their basic properties.

These basic properties are their symmetry-related charge, their spin, and their mass.

6.4.2 Compound modules

Compound modules are composed modules for which the geometric centers of the platforms of the components coincide. The charges of the platforms of the elementary modules establish the binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions [48].

In free compound modules, the symmetry-related charges do not take part in the oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution of the Helmholtz equation [36]. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or the absorption of a photon. The center of emission coincides with the geometrical center of the compound module. During the emission or absorption, the oscillation mode and the hopping path halt, such that the emitted photon does not lose its integrity. Since all photons share the same emission duration, that duration must coincide with the regeneration cycle of the hop landing location swarm. Absorption cannot be interpreted so easily. In fact, it can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

6.4.2.1 Open questions

The Hilbert Book Model does not reveal the fine details of the photon emission, and consequently, it does not reveal the fine details of photon absorption.

6.4.3 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center [49]. However, electron oscillations are shared among the compound modules. Together with the symmetry-related charges, this binds the compound modules into the molecule.

6.4.4 Consciousness and intelligence

In the Hilbert Book Model, all modules are considered to act as observers. That does not mean that these modules react to the perceived information in a conscious or intelligent way. In the hierarchy of modular systems, compared to intelligence, consciousness already enters at lower levels of complexity [50] [51]. However, consciousness cannot be attributed to non-living modular systems. Primitive life forms have primitive degrees of consciousness.

Intelligent species show self-reflection and can create strategies that guard their type-community or their social-community. Conscious species can also develop such guarding measures, but that is usually a result of trial and error instead of a developed strategy. The strategy is then inherited via genes.

For intelligent species, the modular design strategy of the creator can be an inspiration.

- Modular design is superior to monolithic design.
- Modular construction works economically with resources.

- It is advantageous to have access to a large number and a large diversity of suitable modules.
- Create module-type communities.
- Type communities survive far longer than the corresponding individual modules.
- Members must guard their module type community.
- Type communities may inherit and cultivate the culture of their members.
- Modular systems must care about the type communities on which they depend.
- Modular systems must care about their living environment.
- Darwin’s statement that the fittest individual will survive must be replaced by the statement that the module-type community survives that cares best for its members, its resources, and its environment.

In the modern human activity, the hardware is often designed and constructed in a modular way. In contrast, the software is typically designed and constructed in a non-modular way. In comparison, the software is far less robust than hardware.

7 Dark objects and progression zigzag

The generation of shock fronts is described by the equation (4.2.18). The effects of the shock fronts that are caused by pulses are so tiny that no measuring instrument will ever be able to detect the presence of the single shock fronts. Thus, these field excitations can rightfully be called dark objects or more in detail dark energy and dark matter [52] [53]. These objects become noticeable in huge coherent ensembles that may contain about 10^{10} elements. The one-dimensional shock fronts combine in photons, and the spherical shock fronts combine in the footprints of elementary particles. They can exchange roles in pair production and pair annihilation events. For observers, these events pose interpretation problems. However, the model can interpret these events as time-reversal that converts a particle into its antiparticle or vice versa. This interpretation relies on the mass-energy equivalence and on the fact that during the conversion each one-dimensional shock front is exchanged against a spherical shock front. In this interpretation, elementary particles can zigzag through the time domain. This vision suggests that elementary particles never die, but at the utmost change the direction of their life story and turn into its antiparticle. The conversion does not happen instantaneously. It takes the full regeneration cycle of the hop landing location swarm of the elementary particle. The universe-wide proper time clock ticks with a frequency of about 10^{20} ticks per second and the regeneration then takes about 10^{10} proper time clock ticks.

In huge numbers, spurious dark objects may still cause noticeable influences. The halo of dark matter around galaxies is known to produce gravitational lensing effects.

Even though the Hilbert Book Model does not consider the shock fronts as the lowest level of modules, the shock fronts together constitute all discrete objects that exist in the universe.

The Hilbert Book model considers elementary modules as the lowest level modules. They are complicated constructs that consist of a quaternionic separable Hilbert space, a selected version of the quaternionic number system and a private stochastic process that generates their life story.

8 Gravity

Mainstream physics considers the origin of the deformation of our living space as an unsolved problem [54]. It presents the Higgs mechanism as the explanation of why some elementary particles get their mass [55] [56]. The Hilbert Book Model relates mass to deformation of the field that represents our universe. This deformation causes the mutual attraction of massive objects [57].

8.1 Difference between the Higgs field and the universe field

The Higgs field corresponds with a Higgs boson. The dynamic field that represents our universe does not own a field generating particle like the Higgs boson that is supposed to generate the Higgs field. The universe field exists always and everywhere. In fact, a private stochastic process generates each elementary particle. The stochastic process produces quaternions that break the symmetry of the background parameter space. Consequently, the embedded quaternion breaks the symmetry of the functions that apply this parameter pace. Thus, the quaternion breaks the symmetry of the field that represents the universe. However, only isotropic symmetry breaks can produce the spherical pulse responses that temporarily deform the universe field. These spherical pulse responses act as spherical shock fronts. The pulse injects volume into the field, and the shock front distributes this volume over the whole field. The volume expands the field persistently, but the initial deformation fades away. The front wipes the deformation away from the location of the pulse.

8.2 Center of mass

In a system of massive objects $p_i, i = 1,2,3,...n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from

$$\sum_{i=1}^{n} m_i \left(\vec{r}_i - \vec{R} \right) = \vec{0} \tag{8.2.1}$$

Thus

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \tag{8.2.2}$$

Where

$$M = \sum_{i=1}^{n} m_i \tag{8.2.3}$$

In the following, we will consider an ensemble of massive objects that own a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that locates at $\vec{R}$. $\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. In physical reality, this construct has no point-like equivalent that owns a fixed mass. The problem with the treatise in this paragraph is that in physical reality, point-like objects that possess a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses constitute all massive objects that exist in universe.

8.3 Newton

Newton's laws are nearly correct in nearly flat field conditions. The main formula for Newton's laws is

$$\vec{F} = m\vec{a} \tag{8.3.1}$$

Another law of Newton treats the mutual attraction between massive objects.

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = M_1\vec{a} = \frac{G M_1 M_2 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{8.3.2}$$

Newton deduced this universal law of gravitation from results of experiments, but this gravitational attraction can also be derived theoretically from the gravitational potential that is produced by spherical pulse responses.

Massive objects deform the field that embeds these objects. At large distances, a simplified form of the gravitational potential describes properly what occurs.

The following relies heavily on the chapters on quaternionic differential and integral calculus.

8.4 Gauss law

The Gauss law for gravitation is

$$\oiint_{\partial V} \langle \vec{g}, dA \rangle = \iiint_V \langle \vec{\nabla}, \vec{g} \rangle dV = -4\pi G \iiint_V \rho dV = -4\pi G M \tag{8.4.1}$$

Here $\vec{g}$ is the gravitational field. G is the gravitational constant. M is the encapsulated mass. ρ is the mass density distribution. The differential form of Gauss law is

$$\langle \vec{\nabla}, \vec{g} \rangle = \langle \vec{\nabla}, \vec{\nabla} \rangle \phi = -4\pi G \rho \tag{8.4.2}$$

$$\vec{g} = -\vec{\nabla} \phi \tag{8.4.3}$$

ϕ is the gravitational field. Far from the center of mass this gravitation potential equals

$$\phi(r) = \frac{MG}{r} \tag{8.4.4}$$

8.5 A deforming field excitation

A spherical pulse response is a solution of a homogeneous second-order partial differential equation that was triggered by an isotropic pulse. The corresponding field equation and the corresponding solution are repeated here.

$$\left(\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle\right) \psi = 4\pi \delta\left(\vec{q} - \vec{q'}\right) \theta\left(\tau \pm \tau'\right) \tag{8.5.1}$$

Here the $\pm$ sign represents time inversion.

$$\psi = \frac{f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right) \vec{n}\right)}{\left|\vec{q} - \vec{q'}\right|} \tag{8.5.2}$$

The spherical pulse response integrates over time into the Green's function of the field. The Green's function is a solution of the Poisson equation.

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{8.5.3}$$

The Green's function occupies some volume.

$$g(\vec{q}) = \frac{1}{4\pi \left| \vec{q} - \vec{q'} \right|} \tag{8.5.4}$$

This means that locally the pulse pumps some volume into the field, or it subtracts volume out of the field. The selection between injection and subtraction depends on the sign in the step function in the equation (8.5.1). The dynamics of the spherical pulse response shows that the injected volume quickly spreads over the field. In the case of volume subtraction, the front first collects the volume and finally subtracts it at the trigger location. Gravitation considers the case in which the pulse response injects volume into the field.

Thus, locally and temporarily, the pulse deforms the field, and the injected volume persistently expands the field.

This paper postulates that the spherical pulse response is the only field excitation that temporarily deforms the field, while the injected volume persistently expands the field.

The effect of the spherical pulse response is so tiny and so temporarily that no instrument can ever measure the effect of a single spherical pulse response in isolation. However, when recurrently regenerated in huge numbers in dense and coherent swarms, the pulse responses can cause a significant and persistent deformation that instruments can detect. This is achieved by the stochastic processes that generate the footprint of elementary modules.

The spherical pulse responses are straightforward candidates for what physicists call dark matter objects. A halo of these objects can cause gravitational lensing.

8.6 Gravitational potential

A massive object at a large distance acts as a point-like mass. Far from the center of mass, the gravitational potential of a group of massive particles with combined mass M is [59]

$$\phi(r) \approx \frac{GM}{r} \tag{8.6.1}$$

At this distance the gravitation potential shows the shape of the Green's function of the field; however, the amplitude differs. The formula does not indicate that the gravitational potential can cause acceleration for a uniformly moving massive object. However, the gravitational potential is the gravitational potential energy per unit mass. The relation to Newton's law is shown by the following.

The potential ϕ of a unit mass m at a distance r from a point-mass of mass M can be defined as the work W that needs to be done by an external agent to bring the unit mass in from infinity to that point [59].

$$\phi(\vec{r}) \approx \frac{W}{m} = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \vec{F}, d\vec{r} \right\rangle = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \frac{GmM\,\vec{r}}{|\vec{r}|^3}, d\vec{r} \right\rangle = \frac{GM}{|\vec{r}|} \tag{8.6.2}$$

8.7 Pulse location density distribution

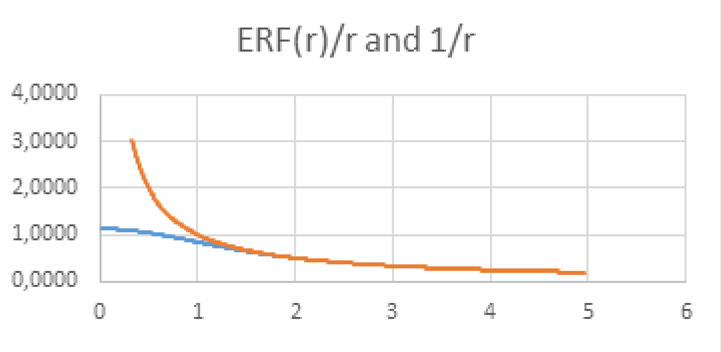
It is false to treat a pulse location density distribution as a set of point-like masses as is done in formulas (8.2.1) and (8.2.2). Instead, the gravitational potential follows from the convolution of the location density distribution and the Green's function. This calculation is still not correct, because the exact result depends on the fact that the deformation that is due to a pulse response quickly fades away and the result also depends on the density of the distribution. If these effects can be ignored, then the resulting gravitational potential of a Gaussian density distribution would be given by [60]

$$g(r) \approx GM \frac{ERF(r)}{r} \tag{8.7.1}$$

Where $ERF(r)$ is the well-known error function. Here the gravitational potential is a perfectly smooth function that at some distance from the center equals the approximated gravitational potential that was described above in equation (8.6.1). As indicated above, the convolution only offers an approximation because this computation does not account for the influence of the density of the swarm and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in three generations [61] [62] [63].



Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that is contained in the red curve, which shows the Green’s function.

In physical reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the gravitational center of the distribution the deformation of the field is characterized by the here shown simplified form of the gravitation potential

$$\phi(r) \approx \frac{GM}{r} \qquad (8.7.2)$$

Warning: This simplified form shares its shape with the Green's function of the deformed field. This does not mean that the Green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

8.8 Inertia

The relation between inertia and mass is complicated. We apply a field that resists its changing. The condition that for each type of massive object, the gravitational potential is a static function and the condition that in free space, the massive object moves uniformly, establish that inertia rules the dynamics of the situation. These conditions define an artificial quaternionic field that does not change. The real part of the artificial field is represented by the gravitational potential, and the uniform speed of the massive object represents the imaginary (vector) part of the field.

The change of the quaternionic field can be divided into five separate changes that partly can compensate each other.

The first-order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ξ changes, indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (8.8.1)$$

Thus

$$\zeta_r = \nabla_r\xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle = 0 \qquad (8.8.2)$$

$$\vec{\zeta} = \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (8.8.3)$$

These formulas can be interpreted independently. For example, according to equation (8.8.2) the variation in time of ξ_r must equal the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla}\times\vec{\xi}$ of the vector field $\vec{\xi}$ is expected to be zero. The resulting terms of equation (8.8.3) are

$$\nabla_r\vec{\xi}+\vec{\nabla}\xi_r=0 \tag{8.8.4}$$

In the following text plays $\vec{\xi}$ the role of the vector field and ξ_r plays the role of the scalar gravitational potential of the considered object. For elementary modules, this special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process, and the uniform movement in free space of the floating platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change zero. We call ξ the ***conservation field***.

At a large distance r, we approximate this potential by using formula

$$\phi(r)\approx\frac{GM}{r} \tag{8.8.5}$$

The new artificial field $\xi=\left\{\frac{GM}{r},\vec{v}\right\}$ considers a uniformly moving mass as a normal situation. It is a combination of the scalar potential $\frac{GM}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{\frac{GM}{r},\vec{v}\right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{GM}{r}$ of the new field. According to the equation (8.8.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{GM}{r}\right) = \frac{GM\,\vec{r}}{|\vec{r}|^3} \tag{8.8.6}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two uniformly moving masses M_1 and M_2 exist in each other's neighborhood, then any disturbance of the situation will cause the gravitational force

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = M_1\vec{a} = \frac{GM_1M_2\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{8.8.7}$$

The disturbance by the ongoing expansion of the embedding field suffices to put the gravitational force into action. The description also holds when the field ξ describes a conglomerate of platforms and M represents the mass of the conglomerate.

The artificial field ξ represents the habits of the underlying model that ensures the constancy of the gravitational potential and the uniform floating of the considered massive objects in free space.

Inertia ensures that the third-order differential (the third-order change) of the deformed field is minimized. It does that by varying the speed of the platforms on which the massive objects reside.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the gravitational potential behaves as the Green's function of the field. There the formula $\xi_r = \frac{m}{r}$ applies.

Further, it bases in the intention of modules to keep the gravitational potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change ζ of the conservation field ξ equals zero.

Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second-order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the gravitational potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

The applied artificial field also explains the gravitational attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and pits that can be described with the Green’s function.

8.9 Elementary particles

For elementary particles, a private stochastic process generates the hop landing locations of the ongoing hopping path that recurrently forms the same hop landing location density distribution. The characteristic function of the stochastic process ensures that the same location density distribution is generated. This does not mean that the same hop landing location swarm is generated! The squared modulus of the wavefunction of the elementary particle equals the generated location density distribution. This explanation means that all elementary particles and all conglomerates of elementary particles are recurrently regenerated.

8.10 Mass

Mass is a property of objects, which has its own significance. Since at large distance, the gravitational potential always has the shape $\phi(r) \approx \frac{GM}{r}$, it does not matter what the massive object is. The formula can be used to determine the mass, even if only is known that the object in question deforms the embedding field. In that case, the formula can still be applied. This is used in the chapter about mixed fields.

In physical reality, no static point-like mass object exists.

8.11 Hop landing generation

The generation of the hopping path is an ongoing process. The generated hop landing location swarm contains a huge number of elements. Each elementary module type is controlled by a corresponding type of stochastic process. For the stochastic process, only the Fourier transform of the location density distribution of the swarm is important. Consequently, for a selected type of elementary module, it does not matter at what instant of the regeneration of the hop landing location swarm the location density distribution is determined. Thus, even when different types are bonded into composed modules, there is no need to synchronize the regeneration cycles of different types. This freedom also means that the number of elements in a hop landing location swarm may differ between elementary module types. This means that the strength of the deformation of the embedding field can differ between elementary module types. The strength of deformation relates to the mass of the elementary modules according to formula (8.6.1).

The requirement for regeneration represents a great mystery. All mass that by elementary modules generate appears to dilute away and must be recurrently regenerated. This fact conflicts with the conservation laws of mainstream physics. The deformation work done by the stochastic processes vanishes completely. What results is the ongoing expansion of the field. Thus, these processes must keep generating the particle to which they belong. The stochastic process accurately regenerates the hop landing location swarm, such that its rest mass stays the same.

Only the ongoing embedding of the content that is archived in the floating platform into the embedding field can explain the activity of the stochastic process. This supposes that at the instant of creation, the creator already archived the dynamic geometric data of his creatures into the eigenspaces of the footprint operators. These data consist of a scalar timestamp and a three-dimensional spatial location. The quaternionic eigenvalues act as storage bins.

After the instant of creation, the creator left his creation alone. The set of floating separable Hilbert spaces, together with the background Hilbert space, act as a read-only repository. After sequencing the timestamps, the stochastic processes read the storage bins and trigger the embedding of the location into the embedding field in the predetermined sequence.

8.11.1 Open question

If the instant of archival proceeds the passage of the window that scans the Hilbert Book Base Model as a function of progression, then the behavior of the model does not change. This indicates a freedom of the described model.

8.12 Symmetry-related charges

Symmetry-related charges only appear at the geometric center of the private parameter space of the separable Hilbert space that acts as the floating platform for an elementary particle. These charges represent sources or sinks for the corresponding symmetry-related field. Since these phenomena disturb the corresponding symmetry-related field in a static way that can be described by the Green’s function of the field, the same trick that was used to explain inertia can be used here to explain the attraction or the repel of two symmetry-related charges Q_1 and Q_2.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{Q}{r}\right) = \frac{Q\,\vec{r}}{|\vec{r}|^3} \tag{8.12.1}$$

$$\vec{F}(\vec{r}_1 - \vec{r}_2) = Q_1\vec{a} = \frac{Q_1 Q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{8.12.2}$$

8.13 Color confinement

Some elementary particle types do not possess an isotropic symmetry. Mainstream physics indicates this fact with a corresponding color charge. Spherical pulse responses require an isotropic pulse. Thus, colored elementary particles cannot generate a gravitational potential. They must first cling together into colorless conglomerates before they can manifest as massive objects. Mesons and baryons are the colorless conglomerates that become noticeable as particles that attract other massive particles.

9 The concept of time

9.1 Proper time

The range of proper time corresponds to the range of archived timestamps. Observers can only receive information from events that were stored with for them historic timestamps.

The notion of time in the Hilbert book model only means something in relation to the archived timestamps. This means that things could still take place before the first proper time instant. This includes, among other things, the preparation and archival of the dynamic geometric data of the elementary particles.

9.2 Clock rates

Proper time ticks with a minimum step. However, that does not mean that this minimum step is the same in the whole universe. It may depend on the local expansion rate of the universe, and that local expansion rate varies with the nearby occurrence of deformation. So, traversing a closed path through a deformed region can result in a difference in time count at the return point between the traveler and the object that stayed at that location because the traveler experienced a different expansion rate of the part of the universe that the traveler traversed. During his trip, the clock of the traveler ran at a different rate than the clock of the staying object. These effects have been measured with accurate clocks.

The metaphor that the Hilbert Book Model steps through the universe with universe-wide progressions steps remains valid, but the page thicknesses in this metaphor can vary from place to place in a fluid way.

9.3 A self-creating model

By restricting the notion of proper time in such a way, it is possible to classify the Hilbert Book Model as a self-creating model. It is now possible to weld a preparatory phase, in which the creation and storage of the dynamic geometrical data of the elementary modules are arranged. Only after this stage can observers obtain information. They get this information via the field that embeds them.

9.4 In the beginning

Before the stochastic processes started their action, the content of the universe was empty. It was represented by a flat field that in its spatial part, was equal to the parameter space. In the beginning, a huge number of these stochastic processes started their triggering of the dynamic field that represents the universe. From that moment on the universe started expanding. This did not happen at a single point. Instead, it happened at a huge number of locations that were distributed all over the spatial part of the parameter space of the quaternionic function that describes the dynamic field.

Close to the begin of time, all distances were equal to the distances in the flat parameter space. Soon, these islands were uplifted with volume that was emitted at nearby locations. This flooding created growing distances between used locations. After some time, all parameter space locations were reached by the generated shock waves. From that moment on the universe started acting as an everywhere expanded continuum that contained deformations which in advance were very small. Where these deformations grew, the distances grew faster than in the environment. A more uniform expansion appears the rule and local deformations form the exception. Deformations make the information path longer and give the idea that time ticks slower in the deformed and expanded regions. This corresponds with the gravitational redshift of photons.

Composed modules only started to be generated after the presence of enough elementary modules. The generation of photons that reflected the signatures of atoms only started after the presence of these compound modules. However, the spurious one-dimensional shock fronts could be generated from the beginning.

This picture differs considerably from the popular scene of the big bang that started at a single location.

The expansion is the fastest in areas where spherical pulse responses are generated. For that reason, it is not surprising that the measured Hubble constant differs from place to place.

10 Relational structures

Lattice theory is a branch of mathematics [66].

10.1 Lattice

A lattice is a set of elements $a, b, c, \ldots$ that is closed for the connections $\cap$ and $\cup$. These connections obey:

- The set is ***partially ordered***.
    - This means that with each pair of elements a, b belongs to an element c, such that $a \subset c$ and $b \subset c$.
- The set is a $\cap$ ***half lattice.***
    - This means that with each pair of elements a, b an element c exists, such that $c = a \cap b$.
- The set is a $\cup$ half lattice.
    - This means that with each pair of elements a, b an element c exists, such that $c = a \cup b$.
- The set is a lattice.
    - This means that the set is both a $\cap$ half lattice and a $\cup$ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{10.1.1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{10.1.2}$$

$$a \cap (a \cup b) = a \tag{10.1.3}$$

$$a \cup b = b \cup a \tag{10.1.4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{10.1.5}$$

$$a \cup (a \cap b) = a \tag{10.1.6}$$

The lattice has a ***partial order inclusion*** $\subset$:

$$a \subset b \Leftrightarrow a \cap b = a \tag{10.1.7}$$

10.2 Lattice types

A ***complementary lattice*** contains two elements n and e, and with each element *a;* it contains a complementary element a' such that [67]:

$$a \cap a' = n \tag{10.2.1}$$

$$a \cap n = n \tag{10.2.2}$$

$$a \cap e = a \tag{10.2.3}$$

$$a \cup a' = e \tag{10.2.4}$$

$$a \cup e = e \tag{10.2.5}$$

$$a \cup n = a \tag{10.2.6}$$

An ***orthocomplemented lattice*** contains two elements n and e ,and with each element *a;* it contains an element a'' such that [68]:

$$a \cup a'' = e \tag{10.2.7}$$

$$a \cap a'' = n \tag{10.2.8}$$

$$\left(a''\right)'' = a \tag{10.2.9}$$

$$a \subset b \Leftrightarrow b'' \subset a'' \tag{10.2.10}$$

e is the ***unity element***; n is the ***null element*** of the lattice

A ***distributive lattice*** supports the distributive laws [69]:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \tag{10.2.11}$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \tag{10.2.12}$$

A ***modular lattice*** supports [70]:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \tag{10.2.13}$$

Every distributive lattice is modular.

An ***orthomodular lattice*** supports instead [71]:

There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c) \tag{10.2.14}$$

where d obeys:

$$(a \cup b) \cap d = d \tag{10.2.15}$$

$$a \cap d = n \tag{10.2.16}$$

$$b \cap d = n \tag{10.2.17}$$

$$(a \subset g) and (b \subset g) \Leftrightarrow d \subset g \tag{10.2.18}$$

In an ***atomic lattice*** holds [72]

$$\exists \{p \ni L\} \forall \{x \ni L\} \{x \subset p \Rightarrow x = n\} \tag{10.2.19}$$

$$\forall \{a \ni L\} \forall \{x \ni L\} \left\{ \begin{pmatrix} a \subseteq x \subseteq (a \cap p) \\ \qquad \Rightarrow \left[(x = a) or (x = a \cap p) \right] \end{pmatrix} \right\} \tag{10.2.20}$$

p is an atom

10.3 Well known lattices

Boolean logic, also called classical logic, has the structure of an orthocomplemented distributive and atomic lattice [73] [74].

Quantum logic has the structure of an orthocomplemented weakly modular and atomic lattice [75].

It is also called an ***orthomodular lattice*** [71].

11 Quaternions

Quaternions were discovered by Rowan Hamilton in 1843 [77] [76]. Later, in the twentieth century, quaternions fell in oblivion.

Hilbert spaces can only cope with number systems whose members form a divisions ring [14]. Quaternionic number systems represent the most versatile associative division ring. Quaternionic number systems exist in many versions that differ in the way that coordinate systems can sequence them. Quaternions can store a combination of a scalar timestamp and a three-dimensional spatial location. Thus, they are ideally suited as storage bins for dynamic geometric data.

In this paper, we represent quaternion q by a real one-dimensional part q_r and a three-dimensional imaginary part $\vec{q}$. The summation is commutative and associative

The following quaternionic multiplication rule describes most of the arithmetic properties of the quaternions.

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{11.1.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system.

A quaternionic conjugation exists

$$q^* = (q_r + \vec{q})^* = q_r - \vec{q} \tag{11.1.2}$$

$$\left(ab\right)^* = b^* a^* \tag{11.1.3}$$

The norm $|q|$ equals

$$|q| = \sqrt{q_r^2 + \langle \vec{q}, \vec{q} \rangle} \tag{11.1.4}$$

$$q^{-1} = \frac{1}{q} = \frac{q}{|q|^2} \tag{11.1.5}$$

$$q = |q| \exp\left(q_{\varphi} \frac{\vec{q}}{|\vec{q}|} \right) \tag{11.1.6}$$

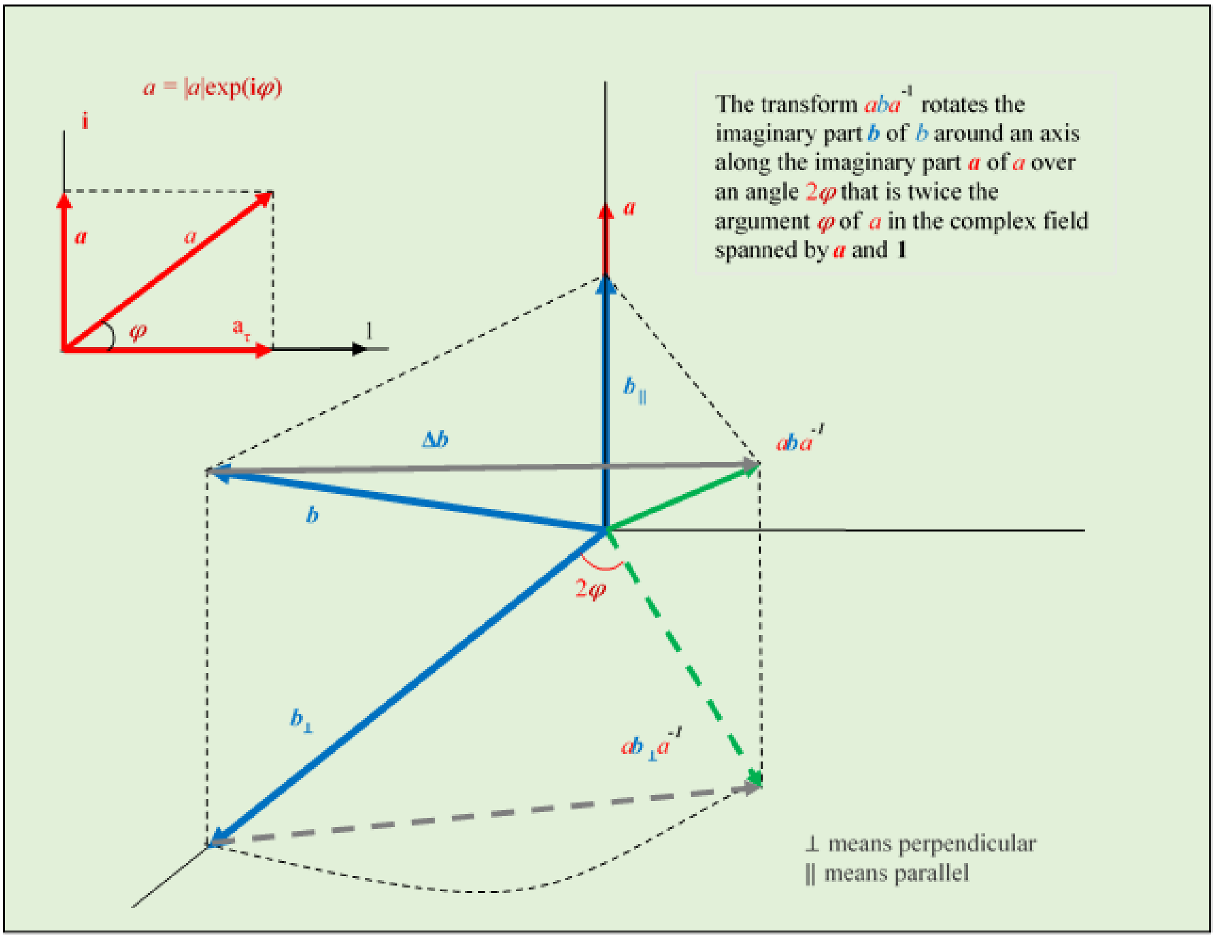
$\frac{\vec{q}}{|\vec{q}|}$ is the spatial direction of q.

A quaternion and its inverse can rotate a part of a third quaternion. The imaginary part of the rotated quaternion that is perpendicular to the imaginary part of the first quaternion is rotated over an angle that is twice the angle of the argument φ between the real part and the imaginary part of the first quaternion. This makes it possible to shift the imaginary part of the third quaternion to a different dimension. For that reason, must $\varphi = \pi / 4$.

Each quaternion *c* can be written as a product of two complex numbers *a* and *b* of which the imaginary base vectors are perpendicular

$$\begin{aligned} c &= \left(a_r + a_1\vec{i}\right)\left(b_r + b_2\vec{j}\right) \\ &= a_r b_r + \left(a_1 + b_r\right)\vec{i} + \left(a_r + b_2\right)\vec{j} + a_1 b_2 \vec{k} \\ &= c_r + c_1\vec{i} + c_2\vec{j} + c_3\vec{k} \end{aligned} \tag{11.1.7}$$

Where $\vec{k} = \vec{i} \times \vec{j}$



11.1Versions

The quaternionic number system exists in many versions that differ in the way coordinate systems rank their elements. Because all separable Hilbert spaces use the same underlying vector space, the Hilbert Book Model uses only those versions where the Cartesian coordinate systems display parallel axes. Only in this way can the model distinguish between the different symmetries of the versions of the number system

12 Quaternionic Hilbert spaces

Around the turn of the nineteenth century into the twentieth century, David Hilbert and others developed the type of vector space that later got Hilbert's name [12].

The Hilbert space is a specific vector space because it defines an inner product for every pair of its member vectors [13].

That inner product can take values of a number system for which every non-zero member owns a unique inverse [14]. This requirement brands the number system as a division ring [14].

Only three suitable division rings exist:

- The real numbers
- The complex numbers
- The quaternions

Hilbert spaces cannot cope with bi-quaternions or octonions

12.1Vector spaces

A vector space over a mathematical field F is a set V together with two operations that satisfy the eight axioms listed below. In the following, $V \times V$ denotes the Cartesian product of V with itself, and $\rightarrow$ denotes a mapping from one set to another.

- The first operation, called **vector addition** or simply **addition** $+ : V \times V \rightarrow V$, takes any two vectors $\vec{v}$ and $\vec{w}$ and assigns to them a third vector which is commonly written as $\vec{v} + \vec{w}$, and called the sum of these two vectors. (The resultant vector is also an element of the set V.)
- The second operation, called **scalar multiplication** $\cdot : F \times V \rightarrow V$, takes any scalar a and any vector $\vec{v}$ and gives another vector $a \cdot \vec{v}$. (Similarly, the vector $a \cdot \vec{v}$ is an element of the set V. Scalar multiplication is not to be confused with the scalar product, also called *inner product* or *dot product*, which is an additional structure present on some specific, but not all vector spaces. Scalar multiplication is a multiplication of a vector *by* a scalar; the other is a multiplication of two vectors *producing* a scalar.)

Elements of V are commonly called *vectors*. Elements of F are commonly called *scalars*.

12.1.1 Axioms

- *Associativity of addition* $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$
- *Commutativity of addition* $\vec{u}+\vec{v}=\vec{v}+\vec{u}$
- *Identity element of addition* There exists an element $\vec{0}\in V$, called the *zero vector*, such that $\vec{0}+\vec{v}=\vec{v}$ for all $\vec{v}\in V$.
- *Inverse elements of addition* For every $\vec{v}\in V$, there exists an element $-\vec{v}\in V$, called the *additive inverse* of $\vec{v}$, such that $-\vec{v}+\vec{v}=\vec{0}$
- *Compatibility of scalar multiplication with field multiplication* $a\cdot(b\cdot\vec{v}) = (ab)\cdot\vec{v}$
- *Identity element of scalar multiplication* $1\cdot\vec{v}=\vec{v}$, where 1 denotes the multiplicative identity in *F*
- *Distributivity of scalar multiplication with respect to vector addition* $a\cdot(\vec{u}+\vec{v}) = a\cdot\vec{u}+a\cdot\vec{v}$
- *Distributivity of scalar multiplication with respect to field multiplication* $(a+b)\cdot\vec{v}=a\cdot\vec{v}+b\cdot\vec{v}$

12.2 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket [78].

The bra $\langle\vec{f}|$ is a covariant vector, and the ket $|\vec{g}\rangle$ is a contravariant vector. The inner product $\langle\vec{f}\,|\,\vec{g}\rangle$ acts as a metric.

It has a quaternionic value. Since the product of quaternions is not commutative, care must be taken with the format of the formulas.

For bra vectors hold

$$\langle\vec{f}|+\langle\vec{g}|=\langle\vec{g}|+\langle\vec{f}|=\langle\vec{f}+\vec{g}| \quad (12.2.1)$$

$$\left(\langle\vec{f}+\vec{g}|\right)+\langle\vec{h}|=\langle\vec{f}|+\left(\langle\vec{g}+\vec{h}|\right)=\langle\vec{f}+\vec{g}+\vec{h}| \quad (12.2.2)$$

For ket vectors hold

$$\left|\vec{f}\right\rangle+\left|\vec{g}\right\rangle=\left|\vec{g}\right\rangle+\left|\vec{f}\right\rangle=\left|\vec{f}+\vec{g}\right\rangle \tag{12.2.3}$$

$$\left(\left|\vec{f}+\vec{g}\right\rangle\right)+\left|\vec{h}\right\rangle=\left|\vec{f}\right\rangle+\left(\left|\vec{g}+\vec{h}\right\rangle\right)=\left|\vec{f}+\vec{g}+\vec{h}\right\rangle \tag{12.2.4}$$

For the inner product holds

$$\left\langle \vec{f}\mid\vec{g}\right\rangle=\left\langle \vec{g}\mid\vec{f}\right\rangle^{*} \tag{12.2.5}$$

For quaternionic numbers α and β hold

$$\langle \alpha f\mid g\rangle=\langle g\mid \alpha f\rangle^{*}=\left(\langle g\mid f\rangle\alpha\right)=\alpha^{*}\langle f\mid g\rangle \tag{12.2.6}$$

$$\left\langle \alpha\vec{f}\mid\vec{g}\right\rangle=\left\langle \vec{g}\mid\alpha\vec{f}\right\rangle^{*}=\left(\left\langle \vec{g}\mid\vec{f}\right\rangle\alpha\right)=\alpha^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle \tag{12.2.7}$$

$$\left\langle \vec{f}\mid\beta\vec{g}\right\rangle=\left\langle \vec{f}\mid\vec{g}\right\rangle\beta \tag{12.2.8}$$

$$\begin{aligned}\left\langle(\alpha+\beta)\,\vec{f}\mid\vec{g}\right\rangle&=\alpha^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle+\beta^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle\\&=(\alpha+\beta)^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle\end{aligned} \tag{12.2.9}$$

Thus

$$\alpha\left|\vec{f}\right\rangle \tag{12.2.10}$$

$$\left\langle \alpha\vec{f}\right|=\alpha^{*}\left\langle \vec{f}\right| \tag{12.2.11}$$

$$\left|\alpha\vec{g}\right\rangle=\left|\vec{g}\right\rangle\alpha \tag{12.2.12}$$

We made a choice. Another possibility would be $\left\langle \alpha\vec{f}\right|=\alpha\left\langle \vec{f}\right|$ and $\left|\alpha\vec{g}\right\rangle=\alpha^{*}\left|\vec{g}\right\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\left\{\left|\vec{f}_i\right\rangle\right\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$.

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\{|\vec{f}_n\rangle\}$ exists that spans the whole space.

If $\langle \vec{f}_m | \vec{f}_n \rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\{|\vec{f}_n\rangle\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\{|\vec{k}\rangle\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $|\vec{k}\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $|\vec{f}\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\{|\vec{k}\rangle\}$.

$$|\vec{f}\rangle = \sum_k |\vec{k}\rangle\langle \vec{k} | \vec{f}\rangle \qquad (12.2.13)$$

A bra base $\{\langle\vec{b}|\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\langle\vec{b}|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\langle\vec{f}|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\{\langle\vec{b}|\}$.

$$\langle\vec{f}| = \sum_b \langle\vec{f} | \vec{b}\rangle\langle\vec{b}| \qquad (12.2.14)$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

12.3 Operators

Operators act on a subset of the elements of the Hilbert space.

An operator L is linear when for all vectors $|\vec{f}\rangle$ and $|\vec{g}\rangle$ for which L is defined and for all quaternionic numbers α and β

$$\begin{aligned} L|\alpha\vec{f}\rangle + L|\beta\vec{g}\rangle &= L|\vec{f}\rangle\alpha + L|\vec{g}\rangle\beta = L\left(|\vec{f}\rangle\alpha + |\vec{g}\rangle\beta\right) \\ &= L\left(|\alpha\vec{f}\rangle + |\beta\vec{g}\rangle\right) \end{aligned} \qquad (12.3.1)$$

The operator B is **colinear** when for all vectors $\left|\vec{f}\right\rangle$ for which Bs defined and for all quaternionic numbers α there exists a quaternionic number γ such that

$$\alpha B\left|\vec{f}\right\rangle = B\left|\vec{f}\right\rangle \gamma\alpha\gamma^{-1} \equiv B\left|\gamma\alpha\gamma^{-1}\vec{f}\right\rangle \tag{12.3.2}$$

If $|\vec{a}\rangle$ is an eigenvector of the operator A with quaternionic eigenvalue α,

$$A|\vec{a}\rangle = |\vec{a}\rangle\alpha \tag{12.3.3}$$

then $|\beta a\rangle$ is an eigenvector of A with quaternionic eigenvalue $\beta^{-1}\alpha\beta$.

$$A|\beta\vec{a}\rangle = A|\vec{a}\rangle\beta = |\vec{a}\rangle\alpha\beta = |\beta\vec{a}\rangle\beta^{-1}\alpha\beta \tag{12.3.4}$$

$A^{\dagger}$ is the **adjoint** of the **normal** operator A

$$\left\langle \vec{f} \mid A\vec{g}\right\rangle = \left\langle \vec{f}A^{\dagger} \mid \vec{g}\right\rangle = \left\langle \vec{g} \mid A^{\dagger}\vec{f}\right\rangle^{*} \tag{12.3.5}$$

$$A^{\dagger\dagger} = A \tag{12.3.6}$$

$$(A+B)^{\dagger} = A^{\dagger} + B^{\dagger} \tag{12.3.7}$$

$$(AB)^{\dagger} = B^{\dagger}A^{\dagger} \tag{12.3.8}$$

If $A = A^{\dagger}$ then A is a **self-adjoint** operator.

A linear operator L is normal if $LL^{\dagger}$ exists, and $LL^{\dagger} = L^{\dagger}L$

For the normal operator N holds

$$\left\langle N\vec{f} \mid N\vec{g}\right\rangle = \left\langle NN^{\dagger}\vec{f} \mid \vec{g}\right\rangle = \left\langle \vec{f} \mid NN^{\dagger}\vec{g}\right\rangle \tag{12.3.9}$$

Thus

$$N = N_r + \vec{N} \tag{12.3.10}$$

$$N^{\dagger} = N_r - \vec{N} \tag{12.3.11}$$

$$N_r = \frac{N + N^{\dagger}}{2} \tag{12.3.12}$$

$$\vec{N} = \frac{N - N^\dagger}{2} \qquad (12.3.13)$$

$$NN^\dagger = N^\dagger N = N_r N_r + \langle \vec{N}, \vec{N} \rangle = |N|^2 \qquad (12.3.14)$$

N_r is the Hermitian part of N.

$\vec{N}$ is the anti-Hermitian part of N.

For two normal operators A and B holds

$$AB = A_r B_r - \langle \vec{A}, \vec{B} \rangle + A_r \vec{B} + \vec{A} B_r \pm \vec{A} \times \vec{B} \qquad (12.3.15)$$

For a unitary transformation U holds

$$\langle U\vec{f} \mid U\vec{g} \rangle = \langle \vec{f} \mid \vec{g} \rangle \qquad (12.3.16)$$

The closure of separable Hilbert space $\mathfrak{H}$ means that converging rows of vectors of $\mathfrak{H}$ converge to a vector in $\mathfrak{H}$.

12.3.1 Operator construction

$|\vec{f}\rangle\langle\vec{g}|$ is a constructed operator.

$$|\vec{g}\rangle\langle\vec{f}| = \left(|\vec{f}\rangle\langle\vec{g}|\right)^\dagger \qquad (12.3.17)$$

For the orthonormal base $\{|\vec{q}_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle \vec{q}_n \mid \vec{q}_m \rangle = \delta_{nm} \qquad (12.3.18)$$

The reverse bra-ket method enables the definition of new operators that are defined by quaternionic functions.

$$\langle \vec{g} \mid F \mid \vec{h} \rangle = \sum_{i=1}^{N} \left\{ \langle \vec{g} | \vec{q}_i \rangle F(q_i) \langle \vec{q}_i | \vec{h} \rangle \right\} \qquad (12.3.19)$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |\vec{q}_i\rangle F(q_i) \langle \vec{q}_i| \qquad (12.3.20)$$

It is evident that

$$F^{\dagger} \equiv |\vec{q}_i\rangle F^{*}(q_i)\langle\vec{q}_i| \tag{12.3.21}$$

For ***reference operator*** $\mathfrak{R}$ holds

$$\mathfrak{R} = |\vec{q}_i\rangle q_i \langle\vec{q}_i| \tag{12.3.22}$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the operator F in the formula (12.3.20).

12.4 Non-separable Hilbert space

Every infinite-dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\langle\vec{g}\,|\,F\,|\,\vec{h}\rangle \equiv \int\iiint\left\{\langle\vec{g}|\vec{q}\rangle F(q)\langle\vec{q}|\vec{h}\rangle\right\}dV d\tau \tag{12.4.1}$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator F is now

$$F \equiv |\vec{q}\rangle F(q)\langle\vec{q}| \tag{12.4.2}$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as

$$F(q) = \langle\vec{q}\,|\,F\vec{q}\rangle = \int\iiint\left\{\langle\vec{q}\,|\,\vec{q}\,'\rangle F(q')\langle\vec{q}\,'\,|\,\vec{q}\rangle\right\}dV'd\tau' \tag{12.4.3}$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\langle \vec{g} \mid \mathcal{R}\vec{h} \rangle \equiv \int\iiint \{ \langle \vec{g} | \vec{q} \rangle q \langle \vec{q} | \vec{h} \rangle \} dV d\tau \qquad (12.4.4)$$

The corresponding shorthand is

$$\mathcal{R} \equiv |\vec{q}\rangle q \langle \vec{q}| \qquad (12.4.5)$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

13 Quaternionic differential calculus

The quaternionic analysis is not so well accepted as complex function analysis [29]

13.1 Field equations

Maxwell equations apply the three-dimensional nabla operator in combination with a time derivative that applies coordinate time. The Maxwell equations derive from results of experiments. For that reason, those equations contain physical units.

In this treatment, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations form a complete and self-consistent set. They use the properties of the three-dimensional spatial nabla.

The corresponding formulas are taken from Bo Thidé's EMTF book., section Appendix F4 [31].

Another online resource is Vector calculus identities [32].

The quaternionic differential equations play in a Euclidean setting that is formed by a continuum quaternionic parameter space and a quaternionic target space. The parameter space is the eigenspace of the reference operator of a quaternionic non-separable Hilbert space. The target space is eigenspace of a defined operator that resides in that same Hilbert space. The defined operator is specified by a quaternionic function that completely defines the field. Each basic field owns a private defining quaternionic function. All basic fields that are treated in this chapter are defined in this way.

Physical field theories tend to use a non-Euclidean setting, which is known as spacetime setting. This is because observers can only perceive in spacetime format. Thus, Maxwell equations use coordinate time, where the quaternionic differential equations use proper time. In both settings, the observed event is presented in Euclidean format. The hyperbolic Lorentz transform converts the Euclidean format to the perceived spacetime format. Chapter 17 treats the Lorentz transform. The Lorentz transform introduces time dilation and length contraction. Quaternionic differential calculus describes the interaction between discrete objects and the continuum at the location where events occur. Converting the results of this calculus by the Lorentz transform will describe the information that the observers perceive. Observers perceive in spacetime format. This format features a Minkowski signature. The Lorentz transform converts from the Euclidean storage format at the situation of the observed event to the perceived spacetime format. Apart from this coordinate transformation, the perceived scene is influenced by the fact that the retrieved information travels through a field that can be deformed and acts as the living space for both the observed event and the observer. Consequently, the information path deforms with its carrier field, and this affects the transferred information. In this chapter, we only treat what happens at the observed event. So, we ignore the Lorentz coordinate transform, and we are not affected by the deformations of the information path.

The Hilbert Book Model archives all dynamic geometric data of all discrete creatures that exist in the model in eigenspaces of separable Hilbert spaces whose private parameter spaces float over the background parameter space, which is the private parameter space of the non-separable Hilbert space. For example, elementary particles reside on a private floating platform that is implemented by a private separable Hilbert space.

Quantum physicists use Hilbert spaces for the modeling of their theory. However, most quantum physicists apply complex-number based Hilbert spaces. Quaternionic quantum mechanics appears to represent a natural choice. Quaternionic Hilbert spaces store the dynamic geometric data in the Euclidean format in quaternionic eigenvalues that consists of a real scalar-valued timestamp and a spatial, three-dimensional location.

In the Hilbert Book Model, the instant of storage of the event data is irrelevant if it coincides with or precedes the stored timestamp. Thus, the model can store all data at an instant, which precedes all stored timestamp values. This impersonates the Hilbert Book Model as a creator of the universe in which the observable events and the observers exist. On the other hand, it is possible to place the instant of archival of the event at the instant of the event itself. It will then coincide with the archived timestamp. In both interpretations, after sequencing the timestamps, the repository tells the life story of the discrete objects that are archived in the model. This story describes the ongoing embedding of the separable Hilbert spaces into the non-separable Hilbert space. For each floating separable Hilbert space this embedding occurs step by step and is controlled by a private stochastic process, which owns a characteristic function. The result is a stochastic hopping path that walks through the private parameter space of the platform. A coherent recurrently regenerated hop landing location swarm characterizes the corresponding elementary object.

Elementary particles are elementary modules. Together they constitute all other modules that occur in the model. Some modules constitute modular systems. A dedicated stochastic process controls the binding of the components of the module. This process owns a characteristic function that equals a dynamic superposition of the characteristic functions of the stochastic processes that control the components. Thus, superposition occurs in Fourier space. The superposition coefficients act as gauge factors that implement displacement generators, which control the internal locations of the components. In other words, the superposition coefficients may install internal oscillations of the components. These oscillations are described by differential equations.

13.2 Fields

In the Hilbert Book Model fields are eigenspaces of operators that reside in the non-separable Hilbert space. Continuous or mostly continuous functions define these operators, and apart from some discrepant regions, their eigenspaces are continuums. These regions might reduce to single discrepant point-like artifacts. The parameter space of these functions is constituted by a version of the quaternionic number system. Consequently, the real number valued coefficients of these parameters are mutually independent, and the differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (13.2.1)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ are quaternions.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \qquad (13.2.2)$$

The Hilbert Book Model assumes that the quaternionic fields are moderately changing, such that only first and second-order partial differential equations describe the model. These equations can describe fields of which the continuity gets disrupted by point-like artifacts. Spherical pulse responses, one-dimensional pulse responses, and Green's functions describe the reaction of the field on such disruptions.

13.3 Field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \qquad (13.3.1)$$

The $\pm$ indicates that quaternions exist in right-handed and left-handed versions.

The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

We define the quaternionic nabla as

$$\nabla \equiv \left\{\frac{\partial}{\partial\tau},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right\} = \nabla_r + \vec{\nabla} \tag{13.3.2}$$

$$\vec{\nabla} \equiv \left\{\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right\} \tag{13.3.3}$$

$$\nabla_r \equiv \frac{\partial}{\partial\tau} \tag{13.3.4}$$

$$\begin{aligned}\phi = \phi_r + \vec{\phi} = \nabla\psi = \left(\frac{\partial}{\partial\tau} + \vec{\nabla}\right)(\psi_r + \vec{\psi}) \\ = \nabla_r\psi_r - \left\langle\vec{\nabla},\vec{\psi}\right\rangle + \nabla_r\vec{\psi} + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi}\end{aligned} \tag{13.3.5}$$

$$\phi_r = \nabla_r\psi_r - \left\langle\vec{\nabla},\vec{\psi}\right\rangle \tag{13.3.6}$$

$$\vec{\phi} = \nabla_r\vec{\psi} + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} = -\vec{E} \pm \vec{B} \tag{13.3.7}$$

Further,

$\vec{\nabla}\psi_r$ is the gradient of ψ_r

$\left\langle\vec{\nabla},\vec{\psi}\right\rangle$ is the divergence of $\vec{\psi}$

$\vec{\nabla}\times\vec{\psi}$ is the curl of $\vec{\psi}$

The change $\nabla\psi$ divides into five terms that each has a separate meaning. That is why these terms in Maxwell equations get different names and symbols. Every basic field offers these terms!

$$\vec{E} = -\nabla_r\vec{\psi} - \vec{\nabla}\psi_r \tag{13.3.8}$$

$$\vec{B} = \vec{\nabla}\times\psi \tag{13.3.9}$$

It is also possible to construct higher-order equations. For example

$$\vec{J} = \vec{\nabla}\times\vec{B} - \nabla_r\vec{E} \tag{13.3.10}$$

The equation (13.3.6) has no equivalent in Maxwell's equations. Instead, its right part is used as a gauge.

Two special second-order partial differential equations use the terms $\frac{\partial^2\psi}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle\psi$

$$\phi=\left\{\frac{\partial^2}{\partial\tau^2}-\langle\vec{\nabla},\vec{\nabla}\rangle\right\}\psi \qquad (13.3.11)$$

$$\rho=\left\{\frac{\partial^2}{\partial\tau^2}+\langle\vec{\nabla},\vec{\nabla}\rangle\right\}\psi \qquad (13.3.12)$$

The equation (13.3.11) is the quaternionic equivalent of the wave equation [35].

The equation (13.3.12) can be divided into two first-order partial differential equations.

$$\begin{aligned}\chi&=\nabla^*\varphi=\nabla^*\nabla\psi=\nabla\nabla^*\psi=\left(\nabla_r+\vec{\nabla}\right)\left(\nabla_r-\vec{\nabla}\right)\left(\psi_r+\vec{\psi}\right)\\&=\left(\nabla_r\nabla_r+\langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi\end{aligned} \qquad (13.3.13)$$

This composes from $\chi=\nabla^*\varphi$ and $\varphi=\nabla\psi$

The prove of (13.3.13) applies the equality

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{a}\right)=\vec{\nabla}\langle\vec{\nabla},\vec{a}\rangle-\langle\vec{\nabla},\vec{\nabla}\rangle\vec{a} \qquad (13.3.14)$$

Such that

$$\begin{aligned}\vec{\nabla}\left(\vec{\nabla}a\right)&=\vec{\nabla}\left(\vec{\nabla}\times\vec{a}\right)-\langle\vec{\nabla},\vec{a}\rangle+\vec{\nabla}a_r\\&=\vec{\nabla}\times\vec{\nabla}\times\vec{a}-\vec{\nabla}\langle\vec{\nabla},\vec{a}\rangle-\langle\vec{\nabla},\vec{\nabla}\rangle a_r\\&=\vec{\nabla}\langle\vec{\nabla},\vec{a}\rangle-\langle\vec{\nabla},\vec{\nabla}\rangle\vec{a}-\vec{\nabla}\langle\vec{\nabla},\vec{a}\rangle-\langle\vec{\nabla},\vec{\nabla}\rangle a_r\\&=-\langle\vec{\nabla},\vec{\nabla}\rangle a\end{aligned} \qquad (13.3.15)$$

$\frac{\partial^2}{\partial\tau^2}-\langle\vec{\nabla},\vec{\nabla}\rangle$ is the quaternionic equivalent of d'Alembert's operator □

.

The operator $\frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle$ does not yet have an accepted name.

The Poisson equation equals

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \tag{13.3.16}$$

A very special solution of this equation is the Green's function

$\frac{1}{4\pi |\vec{q} - \vec{q'}|}$ of the affected field

$$\nabla \frac{1}{|\vec{q} - \vec{q'}|} = -\frac{(\vec{q} - \vec{q'})}{|\vec{q} - \vec{q'}|^3} \tag{13.3.17}$$

$$\begin{aligned} \langle \vec{\nabla}, \vec{\nabla} \rangle \frac{1}{|\vec{q} - \vec{q'}|} &\equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{|\vec{q} - \vec{q'}|} \right\rangle = -\left\langle \vec{\nabla}, \vec{\nabla} \frac{(\vec{q} - \vec{q'})}{|\vec{q} - \vec{q'}|^3} \right\rangle \\ &= 4\pi \delta(\vec{q} - \vec{q'}) \end{aligned} \tag{13.3.18}$$

The spatial integral over Green's function is a volume.

(13.3.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(|\vec{q} - \vec{q'}| - c(\tau - \tau')\right)}{|\vec{q} - \vec{q'}|} \tag{13.3.19}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(|\vec{q} - \vec{q'}| - c(\tau - \tau')\right) \tag{13.3.20}$$

The equation (13.3.11) is famous for its wave type solutions

$$\nabla_r \nabla_r \psi = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi = -\omega^2 \psi \tag{13.3.21}$$

Periodic harmonic actuators cause the appearance of waves,

Planar and spherical waves are the simpler wave solutions of this equation.

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \tag{13.3.22}$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \tag{13.3.23}$$

The Helmholtz equation considers the quaternionic function that defines the field separable [36].

$$\psi(q_r,\vec{q})=A(\vec{q})T(q_r) \tag{13.3.24}$$

$$\frac{\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A}{A}=\frac{\nabla_r\nabla_r T}{T}=-k^2 \tag{13.3.25}$$

$$\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A=-k^2 A \tag{13.3.26}$$

$$\nabla_r\nabla_r T=-k^2 T \tag{13.3.27}$$

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \tag{13.3.28}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules [38] [39].

A more general solution is a superposition of these basic types.

(13.3.12) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi=\frac{f\left(\vec{q}-\overrightarrow{q'}+c(\tau-\tau')\right)}{\left|\vec{q}-\overrightarrow{q'}\right|} \tag{13.3.29}$$

A one-dimensional type of shock front solution is

$$\psi=\vec{f}\left(\vec{q}-\overrightarrow{q'}+c(\tau-\tau')\right) \tag{13.3.30}$$

Equation (13.3.12) offers no waves as part of its solutions.

During travel, the amplitude and the lateral direction $\frac{\vec{f}}{|\vec{f}|}$ of the one-dimensional shock fronts are fixed. The longitudinal direction is along $\frac{\vec{q}-\vec{q'}}{|\vec{q}-\vec{q'}|}$.

The shock fronts that are triggered by point-like actuators are the tiniest field excitations that exist. The actuator must fulfill significant restricting requirements. For example, a perfectly isotropic actuator must trigger the spherical shock front. The actuator can be a quaternion that belongs to another version of the quaternionic number system than the version, which the background platform applies. The symmetry break must be isotropic. Electrons fulfill this requirement. Neutrinos do not break the symmetry but have other reasons why they cause a valid trigger. Quarks break symmetry, but not in an isotropic way.

13.4 Energy operators

In contemporary quantum physics the del operator stands for the momentum operator $\vec{p} = h\vec{\nabla}$. In quaternionic function theory this is automatically an imaginary operator.

In contemporary quantum physics T is the kinetic energy operator.

$$T = \frac{p^2}{2m} = -\frac{h^2}{2m}\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \tag{13.4.1}$$

The gravitation potential of an elementary particle is a superposition ψ_E of solutions ψ_L of equation

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi_L = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{13.4.2}$$

that are integrated over the regeneration cycle of the location density distribution of the footprint of the particle. Here the Dirac delta function and the step function represent the location and the timestamps of the hop landings.

The fact that the stochastic process, which controls the elementary particle owns a characteristic function makes that the superposition ψ_E of the solutions ψ_L is a wave package. The superposition is defined in Fourier space. ψ_E is a solution of

$$\left(\nabla_r \nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi_E = 0 \tag{13.4.3}$$

Thus, the gravitation potential ψ_E of an elementary particle is also a superposition of wave-like solutions ψ_ω of the equation

$$\nabla_r \nabla_r \psi_\omega = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi_\omega = \omega^2 \psi_\omega \tag{13.4.4}$$

The composition of composite objects that are constituted by elementary particles are also controlled by a stochastic process that owns a characteristic function. That characteristic function is a dynamic superposition of the characteristic functions of the components of the composite. So, its gravitational potential ψ is a dynamic wave package and it is a dynamic superposition of functions of type ψ_E . Again, the superposition is defined in Fourier space.

According to the Klein Gordon equation, which presents the vision of current physics, ψ is a solution of

$$\left(\nabla_r \nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right) \psi = -\left(\frac{m}{\hbar}\right)^2 \psi \tag{13.4.5}$$

Here we have taken $c^2 = 1$. This shows a significant difference between conventional physics and the Hilbert Book Model in how these theories handle the concept of mass. The HBM applies Newton's gravitation potential to relate mass to the gravitation potential of massive objects. See equation (8.4.4).

14 Line, surface and volume integrals

14.1 Line integrals

The curl can be presented as a line integral [85]

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \langle \vec{\psi}, d\vec{r} \rangle \right) \tag{14.1.1}$$

14.2 Surface integrals

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\begin{aligned} \vec{\nabla}\psi &= -\left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle \vec{n}, \vec{\psi} \rangle + \vec{n}\psi_r \pm \vec{n} \times \vec{\psi} \end{aligned} \tag{14.2.1}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems [43] [44].

$$\iiint \vec{\nabla}\psi \, dV = \oiint \vec{n}\psi \, dS \tag{14.2.2}$$

$$\iiint \left\langle \vec{\nabla}, \vec{\psi} \right\rangle dV = \oiint \langle \vec{n}, \vec{\psi} \rangle \, dS \tag{14.2.3}$$

$$\iiint \vec{\nabla} \times \vec{\psi} \, dV = \oiint \vec{n} \times \vec{\psi} \, dS \tag{14.2.4}$$

$$\iiint \vec{\nabla}\psi_r \, dV = \oiint \vec{n}\psi_r \, dS \tag{14.2.5}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\begin{aligned} \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) &= \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \\ &= \vec{n} \langle \vec{n}, \vec{\psi} \rangle - \langle \vec{n}, \vec{n} \rangle \vec{\psi} \end{aligned} \tag{14.2.6}$$

$$\iiint_V \left\{ \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \right\} dV = \oiint_S \left\{ \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \right\} dS - \oiint_S \left\{ \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \right\} dS \tag{14.2.7}$$

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle\right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{14.2.8}$$

14.3 Using volume integrals to determine the symmetry-related charges

In its simplest form in which no discontinuities occur in the integration domain Ω the generalized Stokes theorem runs as

$$\int_\Omega d\omega = \int_{\partial\Omega} \omega = \oint_\Omega \omega \tag{14.3.1}$$

We separate all point-like discontinuities from the domain Ω by encapsulating them in an extra boundary. Symmetry centers represent spherically shaped or cube-shaped closed parameter space regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries ∂H_n^x separate the regions from the domain H_n^x. The regions H_n^x are platforms for local discontinuities in basic fields. These fields are continuous in the domain $\Omega - H$.

$$H = \bigcup_{n=1}^{N} H_n^x \tag{14.3.2}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions H_n^x, and the encapsulating boundary ∂H_n^x is not part of the disconnected boundary, which encapsulates all continuous parts of the quaternionic manifold ω that exists in the quaternionic model.

$$\int_{\Omega - H} d\omega = \int_{\partial\Omega \cup \partial H} \omega = \int_{\partial\Omega} \omega - \sum_{k=1}^{N} \int_{\partial H_n^x} \omega \tag{14.3.3}$$

In fact, it is sufficient that ∂H_n^x surrounds the current location of the elementary module. We will select a boundary, which has the shape of a small cube of which the sides run through a region of the parameter spaces where the manifolds are continuous.

If we take everywhere on the boundary the unit normal to point outward, then this reverses the direction of the normal on ∂H_n^x which negates the integral. Thus, in this formula, the contributions of boundaries $\{\partial H_n^x\}$ are subtracted from the contributions of the boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\{\partial H_n^x\}$

***This fact renders the integration sensitive to the ordering of the participating domains**.*

Domain Ω corresponds to part of the background parameter space $\mathfrak{R}$. As mentioned before the symmetry centers $\mathfrak{S}_n^x$ represent encapsulated regions $\{\partial H_n^x\}$ that float on the background parameter space $\mathfrak{R}$. The Cartesian axes of $\mathfrak{S}_n^x$ are parallel to the Cartesian axes of background parameter space $\mathfrak{R}$. Only the orderings along these axes may differ.

Further, the geometric center of the symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}$.

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. With the orientation of the coordinate axes fixed, eight independent Cartesian orderings are possible.

The consequence of the differences in the symmetry flavor on the subtraction can best be comprehended when the encapsulation ∂H_n^x is performed by a ***cubic space form*** that is aligned along the Cartesian axes that act in the background parameter space. Now the six sides of the cube contribute differently to the effects of the encapsulation when the ordering of H_n^x differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}$. Each discrepant axis ordering corresponds to one-third of the surface of the cube. This effect is represented by the ***symmetry-related charge***, which includes the ***color charge*** of the symmetry center. It is easily comprehensible related to the algorithm which below is introduced for the computation of the symmetry-related charge. Also, the relation to the color charge will be clear. ***Thus, this effect couples the ordering of the local parameter spaces to the symmetry-related charge of the encapsulated elementary module.*** The differences with the ordering of the surrounding parameter space determine the value of the symmetry-related charge of the object that resides inside the encapsulation!

14.4Symmetry flavor

The Cartesian ordering of its private parameter space determines the symmetry flavor of the platform [18]. For that reason, this symmetry is compared with the reference symmetry, which is the symmetry of the background parameter space. Four arrows indicate the symmetry of the platform. The background is represented by:

⬆⬆⬆⬆

Now the symmetry-related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry of the platform with the spatial part of the symmetry of the background parameter space.
2. Switch the sign of the result for anti-particles.

Symmetrieversie					
Ordering x y z τ	Sequence	Handedness Right/Left	Color charge	Electric charge * 3	Symmetry type.
⬆⬆⬆⬆	⓪	**~~R~~**	N	+0	neutrino
⬇⬆⬆⬆	①	**L**	R	− 1	down quark
⬆⬇⬆⬆	②	**L**	G	− 1	down quark
⬇⬇⬆⬆	③	**R**	B	+2	up quark
⬆⬆⬇⬆	④	**L**	B	−1	down quark
⬇⬆⬇⬆	⑤	**R**	G	+2	up quark
⬆⬇⬇⬆	⑥	**R**	R	+2	up quark
⬇⬇⬇⬆	⑦	**L**	N	− 3	electron
⬆⬆⬆⬇	⑧	**R**	N	+3	positron
⬇⬆⬆⬇	⑨	**L**	R	− 2	anti-up quark
⬆⬇⬆⬇	⑩	**L**	G	− 2	anti-up quark
⬇⬇⬆⬇	⑪	**R**	B	+1	anti-down quark
⬆⬆⬇⬇	⑫	**L**	B	− 2	anti-up quark
⬇⬆⬇⬇	⑬	**R**	G	+1	anti-down quark
⬆⬇⬇⬇	⑭	**R**	R	+1	anti-down quark
⬇⬇⬇⬇	⑮	**~~L~~**	N	− 0	anti-neutrino

Probably, the neutrino and the antineutrino own an abnormal handedness.

The suggested particle names that indicate the symmetry type are borrowed from the Standard Model. In the table, compared to the standard model, some differences exist with the selection of the anti-predicate. All considered particles are elementary fermions. The freedom of choice in the polar coordinate system might determine the spin [19]. The azimuth range is 2π radians, and the polar angle range is π radians. Symmetry breaking means a difference between the platform symmetry and the symmetry of the background. Neutrinos do not break the symmetry. Instead, they probably may cause conflicts with the handedness of the multiplication rule.

14.5 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \tag{14.5.1}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \tag{14.5.2}$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r \vec{B} = \vec{\nabla} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \tag{14.5.3}$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \tag{14.5.4}$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v}\rho \tag{14.5.5}$$

$$\iint_S \langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \langle \vec{v}\rho + \nabla_r \vec{\varphi}, \vec{n} \rangle dS \tag{14.5.6}$$

The equations (14.5.4) and (14.5.6) enable the derivation of the ***Lorentz force*** [82].

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \tag{14.5.7}$$

$$\frac{d}{d\tau}\iint_S \left\langle \vec{B},\vec{n}\right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0),\vec{n}\right\rangle ds + \frac{d}{d\tau}\iint_{S(\tau)} \left\langle \vec{B}(\tau_0),\vec{n}\right\rangle ds \qquad (14.5.8)$$

The Leibniz integral equation states [83]

$$\begin{aligned}&\frac{d}{dt}\iint_{S(\tau)} \left\langle \vec{X}(\tau_0),\vec{n}\right\rangle dS \\ &= \iint_{S(\tau_0)} \left\langle \dot{\vec{X}}(\tau_0)+\left\langle \vec{\nabla},\vec{X}(\tau_0)\right\rangle \vec{v}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times \vec{X}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (14.5.9)$$

With $\vec{X}=\vec{B}$ and $\left\langle \vec{\nabla},\vec{B}\right\rangle = 0$ follows

$$\begin{aligned}&\frac{d\Phi_B}{d\tau} = \\ &\frac{d}{d\tau}\iint_{S(\tau)} \left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS = \iint_{S(\tau_0)} \left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times \vec{B}(\tau_0),d\vec{l}\right\rangle \\ &= -\oint_{C(\tau_0)} \left\langle E(\tau_0),d\vec{l}\right\rangle - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times \vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (14.5.10)$$

The electromotive force (EMF) ε equals [84]

$$\begin{aligned}\varepsilon &= \oint_{C(\tau_0)} \left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle = -\left.\frac{d\Phi_B}{d\tau}\right|_{\tau=\tau_0} \\ &= \oint_{C(\tau_0)} \left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle + \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times \vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (14.5.11)$$

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} \qquad (14.5.12)$$

In polar coordinates, the nabla delivers different formulas.

$$\vec{\psi} = \psi_0 + \psi_r \vec{r} + \psi_\theta \vec{\theta} + \psi_\varphi \vec{\varphi} \tag{15.1.1}$$

$$\vec{\nabla} \psi_0 = \frac{\partial \psi_0}{\partial r} \vec{r} + \frac{1}{r} \frac{\partial \psi_0}{\partial \theta} \vec{\theta} + \frac{1}{r \sin\theta} \frac{\partial \psi_0}{\partial \varphi} \vec{\varphi} \tag{15.1.2}$$

$$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle = \frac{1}{r^2} \frac{\partial \left(r^2 \psi_r \right)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial \left(\psi_\theta \sin\theta \right)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \psi_\varphi}{\partial \varphi} \tag{15.1.3}$$

$$\begin{aligned} \vec{\nabla} \times \vec{\psi} = & \frac{1}{r \sin\theta} \left(\frac{\partial \left(\psi_\theta \sin\theta \right)}{\partial \theta} - \frac{\partial \psi_\varphi}{\partial \varphi} \right) \vec{r} \\ & + \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial \psi_r}{\partial \varphi} - \frac{\partial \left(r \frac{\partial \psi_r}{\partial \varphi} \right)}{\partial r} \right) \vec{\theta} \\ & + \frac{1}{r} \left(\frac{\partial \left(r \psi_\theta \right)}{\partial r} - \frac{\partial \psi_r}{\partial \theta} \right) \vec{\varphi} \end{aligned} \tag{15.1.4}$$

$$\begin{aligned} & \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \\ & = \frac{1}{r^2} \frac{\partial \left(r^2 \frac{\partial \psi}{\partial r} \right)}{\partial r} + \frac{1}{r^2 \sin\theta} \frac{\partial \left(\sin\theta \frac{\partial \psi}{\partial \theta} \right)}{\partial \theta} + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \psi}{\partial \varphi^2} \end{aligned} \tag{15.1.5}$$

In pure spherical conditions, the Laplacian reduces to:

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) \tag{15.1.6}$$

The Green’s function blurs the location density distribution of the hop landing location swarm of an elementary particle. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green’s function. The Gaussian distribution is

$$\rho(r) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^3} \exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{15.1.7}$$

The shape of the deformation of the field for this ***example*** is given by:

$$\mathfrak{T}(r) = \frac{ERF\left(-\frac{r}{\sigma\sqrt{2}}\right)}{4\pi r} \tag{15.1.8}$$

In this function, every trace of the singularity of the Green's function has disappeared. It is due to the distribution and the huge number of participating hop locations. This shape is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The shown extra contribution is due to the local elementary module that the swarm represents. Together, a myriad of such bumps constitutes the content of the living space.

16 Lorentz transform

16.1 The transform

The shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \tag{16.1.1}$$

Swarms of spherical pulse response triggers move with lower speed v.

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \tag{16.1.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{16.1.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{16.1.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{16.1.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{16.1.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{16.1.7}$$

This is a hyperbolic transformation that relates two coordinate systems.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location P that contains a timestamp t and spatial location $\{x, y, z\}$ and platform P' that has coordinate time t and location $\{x', y', z'\}$.

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x, y, z\}$ and proper timestamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x', y', z', ct'\}$ in which t' plays the role of proper time.

The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L \sqrt{1 - \frac{v^2}{c^2}}$

16.2 Minkowski metric

Spacetime is ruled by the Minkowski metric.

In flat field conditions, proper time τ is defined by

$$\tau = \pm \frac{\sqrt{c^2 t^2 - x^2 - y^2 - z^2}}{c} \qquad (16.2.1)$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \qquad (16.2.2)$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval

16.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric. The proper time interval $d\tau$ obeys [89] [90]

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) \qquad (16.3.1)$$

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, in the environment of a black hole, the symbol r_s stands for the Schwarzschild radius.

$$r_s = \frac{2GM}{c^2} \tag{16.3.2}$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

17 Black holes

Black holes are regions from which nothing, not even photons, can escape. Consequently, no information exists about the interior of a black hole. Only something is known about the direct environment of the black hole [86]. In this section, we try to follow the findings of mainstream physics.

17.1 Geometry

Mainstream physics characterizes the simplest form of black holes by a Schwarzschild radius. [87] [88] It is supposed to be the radius where the escape speed of massive objects equals light speed. The gravitational energy U of a massive object with mass m in a gravitation field of an object with mass M is

$$U = -\frac{GMm}{r} \tag{17.1.1}$$

In non-relativistic conditions, the escape velocity follows from the initial energy $½m\, v^2$ of the object with mass m and velocity v. At the border, the kinetic energy is consumed by the gravitation energy.

$$½mv_0^2 - \frac{GMm}{r_0} = 0 \tag{17.1.2}$$

This results in escape velocity v_0

$$v_0 = \sqrt{2\frac{GM}{r_0}} \tag{17.1.3}$$

It looks as if the Schwarzschild radius can be obtained by taking the speed of light for the escape velocity. Apart from the fact that this condition can never be tested experimentally, this violates the non-relativistic conditions. If we replace $½m\, v^2$ by the energy equivalent of the rest mass $m\,c^2$, then the wrong formula for the Schwarzschild radius results.

17.2 The border of the black hole

First, we consider what happens if a spherical pulse response injects geometric volume into the region of the black hole.

Spherical shock fronts can only add volume to the black hole when their actuator hovers over the region of the black hole. The injection increases the Schwarzschild radius. The injection also increases the mass M. An increase in the radius of the black hole means an increase in the geometric volume of this sphere. This is like the injection of volume into the volume of the field that occurs via the pulses that generate the elementary modules. However, in this case, the volume stays within the black hole sphere. According to the formula of the black hole radius, the volume of the enclosed sphere is not proportional to the mass of the sphere. The mass is proportional to the radius. In both cases, the volume of the field expands, but something different happens.

The HBM postulates that the geometric center of an elementary module cannot enter the region of the black hole. This means that part of the active region of the stochastic process that produces the footprint of the elementary module can hover over the region of the black hole. In this overlap region, the pulses can inject volume into the black hole. Otherwise, the stochastic process cannot inject volume into the black hole.

According to the HBM, the black hole region contains unstructured geometric volume. No modules exist within that sphere.

17.3 An alternative explanation

The two modes in which spherical pulse responses can operate offers a second interpretation. This explanation applies the volume sucking mode of the spherical pulse response. This mode removes the volume of the Green's function from the local field that modifies the environment of the location of the pulse into a continuum, such that only the rational value of the location of the pulse results. A large series of such pulse responses will turn the local continuum into a discrete set of rational location values. Thus, within the region of the black hole, the pulses ***turn the continuum field into a sampled field***. Inside that discrete set, oscillation is no longer possible, and shock fonts do not occur. The elementary particles cannot develop in that region. However, the pulses appear to extend the black hole region not in a similar way as the volume injection pulses in empty space would do. In both cases, these pulses can extend the mass of the region. But in the black hole region, the mass increment is proportional to the radius of the sphere, while in free space the mass increment is proportional to the injected volume. Also, this second approach does not give a proper explanation for the different increase of the volume of the black hole region with the increase of its mass.

In the next chapter, a more sensible explanation is given that introduces mixed fields, which contain closed regions, which do not contain a continuum, but instead a compact discrete set of not further defined objects.

18 Mixed fields

Usually, a dynamic field is a continuum eigenspace of a normal operator that resides in a quaternionic non-separable Hilbert space. In a quaternionic separable Hilbert space, the field is countable and is a sampled field that consists only of the rational target values of the quaternionic function that defines the eigenspace of the operator. This function uses the eigenspace of the reference operator as its parameter space.

If a dense set of rational numbers in a version of the quaternionic number system is convoluted with the Green’s function of a quaternionic field, then a corresponding quaternionic continuum results. Thus, adding the geometric volume of the Green's function to a rational number converts its environment into a continuum. In reverse, sucking the volume in the surround of a rational number that is embedded in a continuum will turn the rational number into its naked value. This can only happen at a border that separates the continuum from a discrete set. It will move the rational number from the continuum to the discrete set.

It is possible to define functions that are continuous in most of the parameter space, but that takes only discrete values in one or more closed regions of the parameter space. In the non-separable Hilbert space, the closed region corresponds to a subspace that encloses a separable Hilbert space. The surface that encloses the closed region that contains discrete rational numbers must be a continuum. However, it's interior only contains a discrete set. All converging series of elements of this set must, if the limit exists, have this limit in the enclosing surface. This surface has a minimal surface that corresponds to the geometric volume of the enclosed region. We can interpret the shift of a rational number from a discrete set to a nearby continuum as the embedding of a separable Hilbert space into a non-separable Hilbert space. The reverse of this procedure is also possible.

A mechanism that injects geometric volume into this region must steal this volume from the surrounding continuum. If this mechanism applies point-sized pulses, then the injection inserts a rational number and the corresponding geometric volume increases. This inserted geometric volume relates to the volume of the Green's function of the continuum. We use the word "relates to" instead of "is proportional to" because the relation is not proportionality. This is explained by Birkhoff's theorem [89] [90].

In its simplest shape, the region is a sphere, and the radius of the sphere is proportional to the mass of the region. At a considerable distance, the gravitational potential has its simplest form and corresponds to the shape of the Green's function of the continuum.

Shock fronts and waves cannot pass the border of the enclosed region and cannot exist inside this region.

The enclosed region deforms the surrounding continuous part of the field. This deformation relates to the geometric volume of the enclosed region and thus relates to the number of injected rational numbers. The deformation corresponds to the mass property of the enclosed region. According to the equation *equation reference goes here*(8.6.1), the mass M determines the gravitational potential energy of mass m at distant r from the center of the region.

$$U(r) \approx \frac{GMm}{r} \tag{18.1.1}$$

Due to gravitation, if a photon that started from a long-distance location and approaches the region. then the contained energy reduces when the gravitational potential increases. Photons are strings of equidistant one-dimensional shock fronts. At a huge distance from the center of the black hole, the energy of the one-dimensional shock front equals a mass-energy equivalent $E_0 = mc^2$. At the border of the black hole, the gravitational potential energy reduces the total energy of the energy package to zero.

$$E = mc^2 - \frac{mMG}{r} = 0 \tag{18.1.2}$$

The equivalent mass *m* plays no role in the value of the computation of the radius of the black hole. Thus, the border of a simple black hole is given by

$$r_{bh} = \frac{GM}{c^2} \tag{18.1.3}$$

At this radius, the energy packages are supposed to be no longer capable of transferring kinetic energy.

The energy of the standard energy packages changes with distance r from the center of the black hole as

$$E = E_0\left(1 - \frac{MG}{c^2 r}\right) = E_0\left(1 - \frac{r_{bh}}{r}\right) \tag{18.1.4}$$

For photons, the initial energy is $E_0 = h\nu_0$. The photon energy changes proportionally to the energy of the one-dimensional shock fronts.

$$E = E_0\left(1 - \frac{r_{bh}}{r}\right) = h\nu_0\left(1 - \frac{r_{bh}}{r}\right) \tag{18.1.5}$$

Mainstream physics sees the border of the black hole as the Schwarzschild radius r_s. This is twice the radius r_{bh}.

$$r_s = \frac{2GM}{c^2} \tag{18.1.6}$$

18.1Open questions

The Hilbert Book Model uses a different radius for the border of a black hole than the Schwarzschild radius that mainstream physics uses. The difference is a factor 2.

Mixed fields can contain regions that only contain a set of not further defined discrete objects. The region is encapsulated by a surface that represents a continuum. This border separates the discrete region from a continuum. The encapsulated region behaves like a black hole. The continuum can contain a series of such regions. It is not clear whether and how these regions can merge.

It is possible that the continuum is surrounded by a continuous border that separates it from a discrete region. This discrete region can contain a series of regions that are surrounded by a continuous border and that contain a continuum. In this way, a multiverse can be established.

Inside the discrete regions, information transfer is blocked, and no field excitations exist.

18.2 The envelopes of black holes

The continuum envelop of the discrete regions are surfaces that can interact with point-like artifacts in a two-dimensional way. Thus, in these envelops two-dimensional pulse responses exist. The sheet transforms emitted pulses into one-dimensional vibrations in the surrounding continuum.

At the outside, the surface may be surrounded by the platforms on which elementary particles reside. The geometric centers of these platforms cannot enter the encapsulated region. The footprint of the particles is generated by a private stochastic process. Outside the border, generated hop landing locations generate pulse responses. This extends the volume of the continuum, but not the volume of the enclosed region. Inside the border, the mechanism injects the rational value of the location, but not the volume of the Green's function of the continuum. The volume of the enclosed region is increased in a different way. The radius of the encapsulated sphere is proportional with the included mass.

18.3 The Bekenstein bound

The Bekenstein bound relates the surface of the border of the black hole to its entropy.

$$S \leq \frac{\kappa ER}{hc} \Rightarrow S = \frac{\kappa ER}{hc} = \frac{2\kappa GM^2}{hc} \qquad (18.3.1)$$

This indicates that the entropy S is proportional to the area of the black hole. This only holds for the entropy at the border of the black hole.

19 Life of an elementary module

An elementary module is a complicated construct. First, the particle resides on a private quaternionic separable Hilbert space that uses a selected version of the quaternionic number system to specify the inner products of pairs of Hilbert vectors and the eigenvalues of operators. The vectors belong to an underlying vector space. All elementary modules share the same underlying vector space. The selected version of the number system determines the private parameter space, which is managed by a dedicated reference operator. The coordinate systems that sequence the elements of the parameter space determine the symmetry of the Hilbert space and the elementary module inherits this symmetry. The private parameter space floats over a background parameter space that belongs to a background platform. The background platform is a separable Hilbert space that also applies the same underlying vector space. The difference in the symmetry between the private parameter space and the background parameter space gives rise to a symmetry-related (electric) charge and a related color charge. A non-zero electric charge raises a corresponding symmetry-related field. The corresponding source or drain locates at the geometric center of the private parameter space.

The eigenspace of a dedicated footprint operator contains the dynamic geometric data that after sequencing of the timestamps form the complete life-story of the elementary module. A subspace of the underlying vector space acts as a window that scans over the private Hilbert space as a function of a progression parameter that corresponds with the archived timestamps. This subspace synchronizes all elementary modules that exist in the model.

Elementary particles are elementary modules, and together these elementary modules form all modules and modular systems that exist in the universe.

The complicated structure of elementary modules indicates that these particles never die. This does not exclude the possibility that elementary modules can zigzag over the progression parameter. Observers will perceive the progression reflection instants as pair creation and pair annihilation events. The zigzag will only become apparent in the creator's view. Thus, only the footprint of the elementary module is recurrently recreated. Its platform persists.

Probably the zigzag events correspond to an organized replacement of quaternions by two complex numbers or its reversal as is described in the Cayley-Dickson doubling [77].

A private stochastic process will recurrently regenerate the footprint of the elementary module in a cyclic fashion. During a cycle, the hopping path of the elementary module will have formed a coherent hop landing location swarm. A location density distribution describes this swarm. This location density distribution equals the Fourier transform of the characteristic function of the stochastic process that generates the hop landing locations. The location density distribution also equals the squared modulus of the wavefunction of the particle. This stochastic process mimics the mechanism that the creator applied when he created the elementary module. The stochastic process also represents the embedding of the eigenspace of the footprint operator into the continuum eigenspace of an operator that resides in the non-separable companion of the background platform. This continuum eigenspace represents the universe.

The differences between the symmetry of the private parameter space and the background parameter space give rise to symmetry-related charges that locate at the geometric center of the private parameter space. These charges give rise to symmetry-related fields. Via the geometric center of the platform, these symmetry-related fields couple to the field that represents the universe.

The kinetic energy of the platform is obtained from the effects of one-dimensional shock fronts. In many cases, these energy packages are combined in photons.

19.1 Causality

No causal reason indicates the application of a stochastic process to generate the content of the footprint operator's eigenspace because no event precedes this action. It is possible to understand the meaning of this approach. In this way, deadlock or race conditions can be prevented from stopping the progress of the dynamics. The stochastic process implements a repetitive renewal of the hop landing location swarm. This implies an automatic watchdog mechanism as it is applied in an RTOS.

19.2 Structure hierarchy

The Hilbert Book Model chooses an orthomodular lattice as its foundation. This lattice develops into the structure of a separable Hilbert space. This includes the addition of an underlying vector space and the selection of a private number system. Under the influence of the supporting mathematics, the relationship between the foundation and the resulting structure limits the choice of the applied numerical system to an associative division ring. The most versatile associative division ring is the number system of the quaternions. The foundation leads almost directly to the platform on which elementary particles live. Quaternionic number systems exist in many versions that differ in the way the coordinate system organizes the elements. This determines the symmetry of the number system and thus the symmetry of the Hilbert space and of the corresponding elementary particle. All elementary particles share the same underlying vector space. This limits the choice of the allowed versions of the number system to a small subgroup, in which the axes of the Cartesian coordinate systems run parallel. Elementary particles behave as elementary modules. Together they form all massive composite objects that occur in the universe. Some of these objects are modular systems.

Stochastic processes that own a characteristic function control the footprint of all massive objects. For composite objects, they regulate the binding of the components. The Hilbert Spaces archive the actions of these stochastic processes.

One of the separable Hilbert spaces acts as a background platform. Each used separable Hilbert space maintains a parameter space consisting of elements of the used version of the number system. The geometric center of the parameter space of the private separable Hilbert space hovers over the parameter space of the background platform. The separable Hilbert space of the background platform has infinitely many dimensions. Consequently, this separable Hilbert space possesses a non-separable companion Hilbert space that fully embeds his companion. As a result, this companion Hilbert space is also part of the background platform. Due to its non-separable nature, the companion's operators can provide continuum eigenspaces and can, therefore, archive continuous fields. One of these fields is the universe. This dynamic field dynamically describes its interaction with point-shaped artifacts that cause field excitations. Another type of field is linked to the symmetry of the private parameter spaces of the elementary particles. Each elementary particle that possesses such a symmetry-related field shows a corresponding source or sink in the geometric center of its private parameter space. On each progression step, this point-shaped artifact provides or gobbles up a stream that corresponds to a part of the present symmetry-related charge.

The platform of the elementary particle owns a real-time clock that can change its time direction. That change switches the source into a sink or the sink into the source. Also, the spin switches sign.

The second kind of stochastic process applies oscillations to bind the components of composite modules. Oscillations help to bind but are not enough. Gravitational attraction must also help to couple the components in a compact conglomerate. Gravitational attraction relies on the existence of the conservation field. On its turn, this relies on the stable recurrent regeneration of the elementary particles and on the fact that these components move together as one unit. As extra glue, the electric fields of the elementary particles can strengthen the coupling.

19.3 The holographic principle

The holographic principle states that the description of a volume of a region of the universe can be thought of as encoded on a lower-dimensional boundary to the region.

The relation between an encapsulated volume and the encapsulating surface is also treated by the Stokes theorem and the Gauss theorem. The solutions of second-order partial differential equations differ between odd and even participating dimensions.

A hologram uses the head plane of an imaging aperture or lens, which acts as a Fourier transforming surface.

If the surface of a black hole represents the densest packaging of the description of the region, then the fact that the elementary particles form a dense coverage of this surface, states that the information contained in these elementary particles together form the full description of the region.

Consequently, the elementary particles that are contained in a volume, together represent all information that is contained in that volume.

Each elementary particle or better said its platform forms an elementary patch of physical information.

This view is in correspondence with the fact that all elementary particles are elementary modules, and together they constitute all modules and modular systems that occur in the universe.

This view neglects that photons and free one-dimensional shock fronts can carry information.

Apart from the information that is carried by photons, the information about massive objects that occur in the universe is carried by the floating platforms that carry the elementary particles. These platforms may cling together into atoms or molecules or still higher-order modules, or they cling together at the surface of a black hole. Some of these floating platforms move freely in space. Also, spherical shock fronts may stay free, but they quickly fade away.

The surfaces on which the elementary particles cling together have some similarity with holograms. But the likeness is not very large.

The patch of information that is contained in an elementary particle comprises its electric charge, its color charge, its spin, its mass and the variables that describe the shape of its location density

20 Material penetrating field

20.1 Field equations

Basic fields can penetrate homogeneous regions of the material. Within these regions, the fields get crumpled. Consequently, the average speed of spherical fronts, one-dimensional fronts, and waves diminish, or these vibrations just get dampened away. The basic field that we consider here is a smoothed version $\bar{\bar{\psi}}$ of the original field ψ that penetrates the material.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \qquad (20.1.1)$$

$$\vec{\bar{\bar{\varphi}}} = \nabla_r \vec{\bar{\bar{\psi}}} + \vec{\nabla} \bar{\bar{\psi}}_r \pm \vec{\nabla} \times \vec{\bar{\bar{\psi}}} = -\vec{\mathfrak{E}} \pm \vec{\mathfrak{B}} \qquad (20.1.2)$$

The first order partial differential equation does not change much. The separate terms in the first-order differential equations must be corrected by a material-dependent factor, and extra material-dependent terms appear.

These extra terms correspond to polarization $\vec{P}$ and magnetization $\vec{M}$ of the material, and the factors concern the permittivity ε and the permeability μ of the material. This results in corrections in the $\vec{\mathfrak{E}}$ and the $\vec{\mathfrak{B}}$ field and the average speed of one-dimensional fronts and waves reduces from 1 to $\frac{1}{\sqrt{\varepsilon\mu}}$.

$$D = \varepsilon \vec{\mathfrak{E}} + \vec{P} \qquad (20.1.3)$$

$$\vec{H} = \frac{1}{\mu} \vec{\mathfrak{B}} - \vec{M} \qquad (20.1.4)$$

$$\rho_b = -\left\langle \vec{\nabla}, \vec{P} \right\rangle \qquad (20.1.5)$$

$$\rho_f = -\left\langle \vec{\nabla}, \vec{D} \right\rangle \qquad (20.1.6)$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} + \nabla_r \vec{P} \qquad (20.1.7)$$

$$\vec{J}_f = \vec{\nabla} \times \vec{H} - \nabla_r \vec{D} \tag{20.1.8}$$

$$\rho = \frac{1}{\varepsilon} \left\langle \vec{\nabla}, \vec{\mathfrak{E}} \right\rangle = \rho_b + \rho_f \tag{20.1.9}$$

$$\vec{J} = \frac{1}{\mu} \vec{\nabla} \times \vec{\mathfrak{B}} - \frac{\varepsilon}{\mu} \nabla_r \vec{\mathfrak{E}} = \vec{J}_b + \vec{J}_f \tag{20.1.10}$$

$$\vec{\phi} = \vec{\mathfrak{E}} - \vec{\mathfrak{B}} = \frac{1}{\varepsilon} \left(\vec{D} - \vec{P} \right) - \mu \left(\vec{H} + \vec{M} \right) \tag{20.1.11}$$

The subscript $_b$ signifies bounded. The subscript $_f$ signifies free.

The homogeneous second-order partial differential equations hold for the smoothed field $\overline{\psi}$.

$$\left\{ \nabla_r \nabla_r \pm v^2 \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right\} \overline{\psi} = 0 \tag{20.1.12}$$

20.2 Pointing vector

The **Poynting vector** represents the directional energy flux density (the rate of energy transfer per unit area) of a basic field. The quaternionic equivalent of the Poynting vector is defined as:

$$\vec{S} = \vec{E} \times \vec{H} \tag{20.2.1}$$

u is the electromagnetic energy density for linear, nondispersive materials, given by

$$u = \frac{\left\langle \vec{E}, \vec{B} \right\rangle + \left\langle \vec{B}, \vec{H} \right\rangle}{2} \tag{20.2.2}$$

$$\frac{\partial u}{\partial \tau} = -\left\langle \vec{\nabla}, \vec{S} \right\rangle - \left\langle \vec{J}_f, \vec{E} \right\rangle \tag{20.2.3}$$

21 Action

The set of basic fields that occur in the model form a system. These fields interact at a finite number of discrete locations. The symmetry-related $\mathfrak{A}^x$ fields always attach to the geometrical center of a dedicated symmetry center. The $\mathfrak{C}$ field attaches at a stochastically determined location somewhere in the vicinity of this geometric center. However, integrated over the regeneration cycle of the corresponding particle, the averaged attachment point coincides with the geometric center of the symmetry center. Thus, in these averaged conditions, the two fields can be considered as being superposed. In the averaged mode the $\mathfrak{C}$ field has weak extrema. The $\mathfrak{A}^x$ fields always have strong extrema. In the averaged mode the fields can be superposed into a new field $\mathfrak{F}$ that shares the symmetry center related extrema.

The path of the geometric center of the symmetry center is following the least action principle. This is not the hopping path along which the corresponding particle can be detected.

The coherent location swarm $\{a_i^x\}$ also represents a path, which is a hopping path. Its coherence means that the swarm owns a continuous location density distribution that characterizes this swarm. A more far-reaching coherence requirement is that the characterizing continuous location density distribution also has a Fourier transform. At first approximation, the swarm moves as one unit. The swarm owns a displacement generator. These facts have much impact on the hopping path and on the movement of the underlying symmetry center. The displacement generator that characterizes part of the dynamic behavior of the symmetry center is represented by the momentum operator $\vec{p}$. This displacement generator describes the movement of the swarm as one unit. It describes the movement of the platform that carries the elementary particle. On the platform, the hopping path is closed. In the embedding field, the platform moves.

21.1 Deriving the action

We suppose that momentum $\vec{p}$ is constant during the particle generation cycle. We use $\vec{n} = \vec{p}/|\vec{p}|$. Every hop gives a contribution to the path. These contributions can be divided into three steps per contributing hop:

1. Change to Fourier space. This involves inner product $\langle \vec{a}_i | \vec{p} \rangle$
2. Evolve during an infinitesimal progression step into the future.
    a. Multiply with the corresponding displacement generator $\vec{p}$
    b. The generated step in configuration space is $(\vec{a}_{i+1} - \vec{a}_i)$.
    c. The action contribution in Fourier space is $\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle$.
    d. This combines in a unitary factor $\exp(\vec{n}\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle)$
3. Change back to configuration space. This involves inner product $\langle \vec{p} | \vec{a}_{i+1} \rangle$
    a. The combined term contributes a factor $\langle \vec{a}_i | \vec{p} \rangle \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+1} \rangle$.

Two subsequent steps give:

$$\langle \vec{a}_i | \vec{p} \rangle \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+1} \rangle \langle \vec{a}_{i+1} | \vec{p} \rangle \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle \tag{21.1.1}$$

The terms in the middle turn into unity. The other terms also join.

$$\begin{aligned} &\langle \vec{a}_i | \vec{p} \rangle \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_{i+1} \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle \\ &= \langle \vec{a}_i | \vec{p} \rangle \exp(\vec{n}\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle \end{aligned} \tag{21.1.2}$$

Over a full particle generation cycle with N steps this results in:

$$\begin{aligned}
&\prod_{i=1}^{N-1}\langle\vec{a}_i|\vec{p}\rangle\exp\left(\vec{n}\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle\right)\langle\vec{p}|\vec{a}_{i+1}\rangle\\
&=\langle\vec{a}_1|\vec{p}\rangle\exp\left(\vec{n}\langle\vec{p},\vec{a}_N-\vec{a}_1\rangle\right)\langle\vec{p}|\vec{a}_N\rangle\\
&=\langle\vec{a}_1|\vec{p}\rangle\exp\left(\vec{n}\sum_{i=2}^{N}\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle\right)\langle\vec{p}|\vec{a}_N\rangle\\
&=\langle\vec{a}_1|\vec{p}\rangle\exp\left(\vec{n}L\right)\langle\vec{p}|\vec{a}_N\rangle
\end{aligned} \tag{21.1.3}$$

$$L d\tau=\sum_{i=2}^{N}\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle=\langle\vec{p},d\vec{q}\rangle \tag{21.1.4}$$

$$L=\langle\vec{p},\dot{\vec{q}}\rangle \tag{21.1.5}$$

L is known as the Lung operator.

The equation (21.1.5) holds for the special condition in which $\vec{p}$ is constant. If $\vec{p}$ is not constant, then the Hart operator H varies with location.

This procedure derives the Lung operator and the Hamilton equations from the stochastic hopping path. Each term in the series shows that the displacement generator forces the combination of terms to generate a closed hopping path on the platform that carries the elementary particle. The only term that is left is the displacement generation of the whole hop landing location swarm. That term describes the movement of the platform.

Mainstream physics applies the Lung operator as the base of the path integral. In the Hilbert Book Model, the Lung operator results from the analysis of the hopping path.

21.2 Classical mechanics

In contemporary classical mechanics, the action principle leads to the following set of equations

$$T=\frac{p^2}{2m} \tag{21.2.1}$$

$$H = T - V \tag{21.2.2}$$

$$L = T + V \tag{21.2.3}$$

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \tag{21.2.4}$$

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \tag{21.2.5}$$

$$\frac{\partial L}{\partial q_i} = \dot{p}_i \tag{21.2.6}$$

$$\frac{\partial L}{\partial \dot{q}_i} = p_i \tag{21.2.7}$$

$$\frac{\partial H}{\partial \tau} = -\frac{\partial L}{\partial \tau} \tag{21.2.8}$$

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i} \tag{21.2.9}$$

$$H + L = \sum_{i=1}^{3} \dot{q}_i p_i \tag{21.2.10}$$

Here we used proper time τ rather than coordinate time t.

22 Dirac equation

In its original form, the Dirac equation for the free electron and the free positron is formulated by using complex number-based spinors and matrices [91] [92]. That equation can be split into two equations, one for the electron and one for the positron. The matrices implement the functionality of a bi-quaternionic number system. Biquaternions do not form a division ring. Thus, Hilbert spaces cannot cope with bi-quaternionic eigenvalues. The Dirac equation plays an important role in mainstream physics.

22.1The Dirac equation in original format

In its original form, the Dirac equation is a complex equation that uses spinors, matrices, and partial derivatives.

Dirac was searching for a split of the Klein-Gordon equation into two first-order differential equations.

$$\frac{\partial^2 f}{\partial t^2}-\frac{\partial^2 f}{\partial x^2}-\frac{\partial^2 f}{\partial y^2}-\frac{\partial^2 f}{\partial z^2}=-m^2 f \tag{22.1.1}$$

$$\Box f=\left(\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)f=-m^2 f \tag{22.1.2}$$

Here $\Box=\left(\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)$ is the d'Alembert operator.

Dirac used a combination of matrices and spinors in order to reach this result. He applied the Pauli matrices in order to simulate the behavior of vector functions under differentiation [93].

The unity matrix I and the Pauli matrices $\sigma_1,\sigma_2,\sigma_3$ are given by

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\sigma_2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \tag{22.1.3}$$

Here $i=\sqrt{-1}$. For one of the potential orderings of the quaternionic number system, the Pauli matrices together with the unity matrix I relate to the quaternionic base vectors 1*, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$*

$$1\Rightarrow I,\vec{i}\Rightarrow i\sigma_1,\vec{j}\Rightarrow i\sigma_2,\vec{k}\Rightarrow i\sigma_3 \tag{22.1.4}$$

This results in the multiplication rule

$$\sigma_1\sigma_2-\sigma_2\sigma_1=2i\sigma_3,\sigma_2\sigma_3-\sigma_3\sigma_2=2i\sigma_1,\sigma_3\sigma_1-\sigma_1\sigma_3=2i\sigma_2 \tag{22.1.5}$$

$$\sigma_1\sigma_1=\sigma_2\sigma_2=\sigma_3\sigma_3=I \tag{22.1.6}$$

The different ordering possibilities of the quaternionic number system correspond to different symmetry flavors. Half of these possibilities offer a right-handed external vector product. The other half offers a left-handed external vector product.

We will regularly use:

$$i\left(\vec{\sigma},\vec{\nabla}\right)=\vec{\nabla} \tag{22.1.7}$$

With

$$p_{\mu} = -i\nabla_{\mu} \tag{22.1.8}$$

follow

$$p_{\mu}\sigma_{\mu} = -ie_{\mu}\nabla_{\mu} \tag{22.1.9}$$

$$\langle p,\sigma \rangle = -i\vec{\nabla} \tag{22.1.10}$$

22.2 Dirac's formulation

The original Dirac equation uses 4x4 matrices $\vec{\alpha}$ and β.

$\vec{\alpha}$ and β *are* matrices that implement the bi-quaternion arithmetic behavior including the possible symmetry flavors of bi-quaternionic number systems and continuums.

$$\alpha_1 = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} = -i\begin{bmatrix} 0 & \vec{i} \\ \vec{i} & 0 \end{bmatrix} \tag{22.2.1}$$

$$\alpha_2 = \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix} = -i\begin{bmatrix} 0 & \vec{j} \\ \vec{j} & 0 \end{bmatrix} \tag{22.2.2}$$

$$\alpha_3 = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix} = -i\begin{bmatrix} 0 & \vec{k} \\ \vec{k} & 0 \end{bmatrix} \tag{22.2.3}$$

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{22.2.4}$$

$$\beta\beta = I \tag{22.2.5}$$

The interpretation of the Pauli matrices as a representation of a special kind of angular momentum has led to the half-integer eigenvalue of the corresponding spin operator.

Dirac's selection leads to

$$\left(p_r - \langle \vec{\alpha}, \vec{p} \rangle - \beta mc \right)\{\varphi\} = 0 \tag{22.2.6}$$

$\{\varphi\}$ is a four-component spinor, which splits into

$$\left(p_r - \langle \vec{\alpha}, \vec{p} \rangle - \beta mc\right)\varphi_A = 0 \tag{22.2.7}$$

and

$$\left(p_r - \langle \vec{\alpha}, \vec{p} \rangle + \beta mc\right)\varphi_B = 0 \tag{22.2.8}$$

φ_A and φ_B are two component spinors. Thus, the original Dirac equation splits into:

$$\left(\nabla_r - \vec{\nabla} - imc\right)\varphi_A = 0 \tag{22.2.9}$$

$$\left(\nabla_r - \vec{\nabla} + imc\right)\varphi_B = 0 \tag{22.2.10}$$

This split does not lead easily to a second-order partial differential equation that looks like the Klein Gordon equation.

22.3 Relativistic formulation

Instead of Dirac's original formulation, usually, the relativistic formulation is used.

That formulation applies gamma matrices, instead of the alpha and beta matrices. This different choice influences the form of the equations that result in the two-component spinors.

$$\gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{i} \\ -\vec{i} & 0 \end{bmatrix} \tag{22.3.1}$$

$$\gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{j} \\ -\vec{j} & 0 \end{bmatrix} \tag{22.3.2}$$

$$\gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{k} \\ -\vec{k} & 0 \end{bmatrix} \tag{22.3.3}$$

$$\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{22.3.4}$$

Thus

$$\begin{aligned}\gamma_\mu &= \gamma_0 \alpha_\mu; \mu = 1,2,3 \\ \gamma_0 &= \beta\end{aligned} \qquad (22.3.5)$$

Further

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (22.3.6)$$

The matrix γ_5 anti-commutes with all other gamma matrices.

Several different sets of gamma matrices are possible. The choice above leads to a "Dirac equation" of the form

$$\left(i\gamma^\mu \nabla^\mu - mc\right)\{\varphi\} = 0 \qquad (22.3.7)$$

More extended:

$$\left(\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} - \frac{m}{ih}\right)\{\varphi\} = 0 \qquad (22.3.8)$$

$$\left(\gamma_0 \frac{\partial}{\partial t} + \left\langle \vec{\gamma}, \vec{\nabla} \right\rangle - \frac{m}{ih}\right)\{\varphi\} = 0 \qquad (22.3.9)$$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \left\langle \vec{\sigma}, \vec{\nabla} \right\rangle \\ -\left\langle \vec{\sigma}, \vec{\nabla} \right\rangle & 0 \end{bmatrix} - \frac{m}{ih}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \qquad (22.3.10)$$

$$\left(i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \vec{\nabla} \\ -\vec{\nabla} & 0 \end{bmatrix} + \frac{m}{h}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \qquad (22.3.11)$$

$$i\frac{\partial}{\partial t}\varphi_A + \vec{\nabla}\varphi_B + \frac{m}{h}\varphi_A = 0 \qquad (22.3.12)$$

$$i\frac{\partial}{\partial t}\varphi_B + \vec{\nabla}\varphi_A - \frac{m}{h}\varphi_B = 0 \qquad (22.3.13)$$

Also, this split does not easily lead to a second-order partial differential equation that looks like the Klein Gordon equation.

22.4A better choice

Another interpretation of the Dirac approach replaces γ_0 with γ_5:

$$\left(\gamma_5 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} - \frac{m}{ih}\right)\{\varphi\} = 0 \tag{22.4.1}$$

$$\left(\gamma_5 \frac{\partial}{\partial t} + \left\langle \vec{\gamma}, \vec{\nabla} \right\rangle - \frac{m}{ih}\right)\{\varphi\} = 0 \tag{22.4.2}$$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \left\langle \vec{\sigma}, \vec{\nabla} \right\rangle \\ -\left\langle \vec{\sigma}, \vec{\nabla} \right\rangle & 0 \end{bmatrix} - \frac{m}{ih}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \tag{22.4.3}$$

$$\left(i\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \vec{\nabla} \\ -\vec{\nabla} & 0 \end{bmatrix} + \frac{m}{h}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \tag{22.4.4}$$

$$i\frac{\partial}{\partial t}\varphi_B + \vec{\nabla}\varphi_B + \frac{m}{h}\varphi_A = 0 \tag{22.4.5}$$

$$i\frac{\partial}{\partial t}\varphi_A - \vec{\nabla}\varphi_A + \frac{m}{h}\varphi_B = 0 \tag{22.4.6}$$

This version invites splitting of the four-component spinor equation into two equations for two-component spinors:

$$\left(i\frac{\partial}{\partial t} + \vec{\nabla}\right)\varphi_B = -\frac{m}{h}\varphi_A \tag{22.4.7}$$

$$\left(i\frac{\partial}{\partial t} - \vec{\nabla}\right)\varphi_A = -\frac{m}{h}\varphi_B \tag{22.4.8}$$

This looks far more promising. We can insert the right part of the first equation into the left part of the second equation.

$$\left(i\frac{\partial}{\partial t} - \vec{\nabla}\right)\left(i\frac{\partial}{\partial t} + \vec{\nabla}\right)\varphi_A = \frac{m^2}{h^2}\varphi_A \tag{22.4.9}$$

$$\left(\frac{\partial^2}{\partial t^2} + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\varphi_A = -\frac{m^2}{h^2}\varphi_A \tag{22.4.10}$$

$$\left(i\frac{\partial}{\partial t} + \vec{\nabla}\right)\left(i\frac{\partial}{\partial t} - \vec{\nabla}\right)\varphi_B = \frac{m^2}{h^2}\varphi_B \tag{22.4.11}$$

$$\left(\frac{\partial^2}{\partial t^2}+\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\varphi_B=-\frac{m^2}{h^2}\varphi_B \qquad (22.4.12)$$

This is what Dirac wanted to achieve. The two first-order differential equations couple into a second-order differential equation, but that equation is not equivalent to the Klein Gordon equation. It is equivalent to the equation (4.2.1).

The nabla operator acts differently onto the two-component spinors φ_A and φ_B.

22.5The Dirac nabla

The Dirac nabla $\mathfrak{D}$ differs from the quaternionic nabla ∇.

$$\nabla=\left\{\frac{\partial}{\partial\tau},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right\}=\nabla_r+\vec{\nabla} \qquad (22.5.1)$$

$$\nabla^*=\nabla_r-\vec{\nabla} \qquad (22.5.2)$$

$$\nabla\nabla^*=\nabla^*\nabla=\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle \qquad (22.5.3)$$

$$\mathfrak{D}=\left\{i\frac{\partial}{\partial\tau},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right\}=i\nabla_r+\vec{\nabla} \qquad (22.5.4)$$

$$\mathfrak{D}^*=i\nabla_r-\vec{\nabla} \qquad (22.5.5)$$

$$\mathfrak{D}\mathfrak{D}^*=\mathfrak{D}^*\mathfrak{D}=-\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle=-\nabla^*\nabla \qquad (22.5.6)$$

23 Low dose rate imaging

The author started his career in the high-tech industry in the development of image intensifier devices. His job was to help to optimize the imaging quality of these image intensifier devices. This concerned both image intensifiers for night vision applications and x-ray image intensifiers that were aimed at medical applications. Both types of devices target low dose rate application conditions. These devices achieve image intensification in quite different ways. Both types can be considered to operate in a linear way. The here described qualification of the image intensifier aided recognition capability is possible because human image perception is optimized for low dose rate conditions.

At low dose rates, the author never perceived waves in the intensified images. At the utmost, he saw hail storms of impinging discrete particles, and the corresponding detection patterns can simulate interference patterns. The conclusion is that the waves that might be present in the observed image are probability waves. Individual photons are perceived as detected quanta. They are never perceived as waves.

23.1 Intensified image perception

When I entered my new job, the head of the department confronted me with a remarkable relationship that observers of intensified images had discovered and that he used in order to optimize the imaging quality of image intensifier devices. It appeared that perceptibility increases when the dose rate increases. It also increased when the surface of the observed detail increases. As expected, it increases when the object contrast increases. Temporal integration also had a positive effect on the perception of relatively static objects. Phosphors that are applied as scintillators or as electron-to-photon convertors cause a significant temporal integration. The rate at which the perceptibility increases seems to indicate that the perceived quanta are generated by spatial Poisson point processes. Thus, increasing the quantum detection capability should be the prime target of the image intensifier developers. However, for X-ray image intensifiers increasing gamma quantum detection capability usually conflicts with keeping sufficient imaging sharpness for perceiving small details. Thus, the second level target for image intensifying devices is getting the imaging sharpness at an acceptable level. The variance of the quantum intensification factor reduces the signal to noise ratio, and that effect must be compensated by increasing the dose rate. This is unwanted. Further, the quantum intensification must be high enough in order to trigger the image receiver.

In the intensification chain, also some attenuations take place. These attenuations can be represented by binomial processes. For example, photocathodes and scintillation layers do not reach the full hundred percent detection capability. In addition, input windows and input screens just absorb part of the impinging quanta. A primary Poisson process combines with one or more binomial processes in order to form a new Poisson process that offers a lower quantum production efficiency. A very important binomial process is represented by the spatial point spread function that is the result of imaging blur. Imaging blur can be characterized by the optical transfer function, which is the Fourier transform of the spatial point spread function.

Another important fact is that not only the existence of an object must be decided by the receiver. The detected object must also be recognized by the receiver. For intensified image recognition, some very complicated processes in the visual trajectory of the receiver become decisive. The recognition process occurs in stages, and at every stage, the signal to noise ratio plays a decisive role. If the level of the signal to noise ratio is too low, then the signal transfer is blocked.

24 Human perception

24.1 Information encoding

With respect to the visual perception, the human visual trajectory closely resembles the visual trajectory of all vertebrates. This was discovered by Hubel and Weisel [94]. They got a Noble price for their work.

The sensitivity of the human eye covers a huge range. The visual trajectory implements several special measures that help to extend that range. At high dose rates, the pupil of the eye acts as a diaphragm that partly closes the lens and, in this way, it increases the sharpness of the picture on the retina. At such dose rates, the cones perform the detection job. The cones are sensitive to colors and offer a quick response. In unaided conditions, the rods take over at low dose rates, and they do not differentiate between colors. In contrast to the cones, the rods apply a significant integration time. This integration diminishes the effects of quantum noise that becomes noticeable at low dose rates. The sequence of optimizations does not stop at the retina. In the trajectory from the retina to the fourth cortex of the brain, several dedicated decision centers decode the received image by applying masks that trigger on special aspects of the image. For example, a dedicated mask can decide whether the local part of the image is an edge, in which direction this edge is oriented and in which direction the edge moves. Other masks can discern circular spots. Via such masks, the image is encoded before the information reaches the fourth cortex. Somewhere in the trajectory, the information of the right eye crosses the information that is contained in the left eye. The difference is used to construct a three-dimensional vision. Quantum noise can easily disturb the delicate encoding process. That is why the decision centers do not pass their information when its signal to noise ratio is below a given level. That level is influenced by the physical and mental condition of the observer. At low dose rates, this signal to noise ratio barrier prevents a psychotic view. The higher levels of the brain thus do not receive a copy of the image that was detected at the retina. Instead, that part of the brain receives a set of quite trustworthy encoded image data that will be deciphered in an associative way. ***It is expected that other parts of the brain for a part act in a similar noise-blocking way.***

The evolution of the vertebrates must have installed this delicate visual data processing subsystem in a period in which these vertebrates lived in rather dim circumstances, where the visual perception of low dose rate images was of vital importance.

This indicates that the signal to noise ratio in the image that arrives at the eye's pupil has a significant influence on the perceptibility of the low dose image. At high dose rates, the signal to noise ratio hardly plays a role. In those conditions, the role of the spatial blur is far more important.

It is fairly easy to measure the signal to noise ratio in the visual channel by applying a DC meter and an RMS meter. However, at very low dose rates, the damping of both meters might pose problems. What quickly becomes apparent is the relation of the signal to noise ratio and the number of the quanta that participate in the signal. The measured relation is typical for stochastic quantum generation processes that are classified as Poisson processes.

It is also easy to comprehend that when the signal is spread over a spatial region, the number of quanta that participate per surface unit is diminishing. Thus, spatial blur has two influences. It lowers the local signal, and on the other hand, it increases the integration surface. Lowering the signal decreases the number of quanta. Enlarging the integration surface will increase the number of involved quanta. Thus, these two effects partly compensate each other. An optimum perceptibility condition exists that maximizes the signal to noise ratio in the visual trajectory.

24.2 Blur

The blur is caused by the Point Spread Function. This function represents a spatially varying binomial process that attenuates the efficiency of the original Poisson process. This creates a new Poisson process that features a spatially varying efficiency. Several components in the imaging chain may contribute to the Point Spread Function such that the effective Point Spread Function equals the convolution of the Point Spread Functions of the components. Mathematically it can be shown that for linear image processors the Optical Transfer Functions form an easier applicable characteristic than the Point Spread Functions because the Fourier transform that converts the Point Spread Function into the Optical Transfer Function converts the convolutions into simple multiplications.

The Optical Transfer Function is influenced by several factors. Examples are the color distribution, the angular distribution, and the phase homogeneity of the impinging radiation. Seidel aberrations and chromatic aberrations characterize the defects of imaging devices. Also, veiling glare may hamper the imaging quality.

24.3 Detective quantum efficiency

The fact that the signal to noise ratio appears to be a deciding factor in the perception process has led to a second way of characterizing the relevant influences. The Detective Quantum Efficiency (DQE) characterizes the efficiency of the usage of the available quanta. It compares the actual situation with the hypothetical situation in which all generated quanta would be used in the information channel. The measured signal to noise ratio is compared to the ideal situation in which the stochastic generator is a Poisson process, and no binomial processes will attenuate that primary Poisson process. This means that blurring and temporal integration must play no role in the determination of the idealized reference detector that is used in specifying the DQE and the measured device will be compared to quantum detectors that will capture all available quanta. It also means that intensification processes will not add extra relative variance to the signal of the idealized detector. The application of microchannel plates will certainly add extra relative variance. This effect will be accounted for as a deterioration of the detection efficiency and not as a change of the stochastic process from a Poisson process to an exponential process. Mathematically this is an odd procedure, but it is a valid approach when the measurements are used to evaluate the perceptibility objectively.

24.4Quantum Physics

The fact that the objective qualification of perceptibility can be performed by the Optical Transfer Function in combination with the Detective Quantum Efficiency indicates that the generation of the quanta is governed by a Poisson process that is coupled to a series of binomial process and secondary Poisson processes, where some of the binomial processes are implemented by spatial Point Spread Functions, and others are spatially uniform attenuators.

The processes that generate the primary quanta are considered to belong to the category of the inhomogeneous spatial Poisson point processes. These are processes that are applied by mechanisms that produce the locations of elementary particles, or they are processes that control the distribution of photons during the emission of these information messengers.

25 How the brain works

25.1 Preprocessing

A study on how the environment is observed and interpreted should start with an investigation of how the sense-organs and the brain cooperate. Between the sense-organs and the brain exists a series of pre-processors that encode and pre-interpret the incoming signals. This process also performs some noise filtering, such that later stages of the processing are not bothered by misinformation. For that reason, the pre-processors act as decision centers where the signal transfer is blocked when the signal to noise ratio stays underneath a given level, e.g., 2.3 (Crozier's law. The level may differ in different persons). In this way, the visual trajectories run via a cross-over to the cortex. The cross-over encodes and adds depth information. After a series of additional pre-processing steps, the signal arrives in the fourth cortex layer. Here about four square millimeters is devoted to the direct environment of each receptor of the fovea. In this area, a complete geometric encoding of the local geometry and dynamics of the perceived picture is presented. This includes whether the detected detail is a line or an edge or another form, in which direction it is positioned and whether the detail moves. (See the papers of Hubel and Wiesel on the visual trajectory and the visual cortex for more detailed information) [94].

25.2 Processing

Thus, the brain does not work with a pictorial copy of the picture that is received on the fovea. In further steps, the encoded map is interpreted. That part of the brain tries to associate the details of the map with remembered and recognized items. When dynamics is considered, then it must also be considered that the eyes are continuously scanning the input scene.

25.3 Image intensification

I studied visual perception because I needed this to specify useful measuring standards for night vision and X-ray imaging equipment

(~1975). Many of the known visual illusions are due to the pre-processing in the visual trajectory. The viewing chain includes lenses, image intensifier tubes, and either a camera or the human visual system. This last component includes the eyeball. The object is noisy and can be considered as a Poisson process. With respect to the noise, the optical components in the imaging chain act as binomial processes or as generalized Poisson processes. Their point spread functions act as integration area. Image intensification is usually a Poisson process, but channel plates are characterized by an exponential distribution rather than by a Poisson distribution. Chains that include Poisson processes and binomial processes can be considered as one generalized Poisson process. Imaging chains that include channel plates are more difficult to characterize.

25.4 Imaging quality characteristics

When the imaging chain can be characterized by a Poisson process, then its quantum detection efficiency can be characterized by the Detective Quantum Efficiency (DQE). Its optical imaging quality can be characterized by the Optical Transfer Function (OTF). With inhomogeneous light imaging, it is sufficient to use the modulus, the Modulation Transfer Function (MTF). The MTF of the chain is the product of the MTF's of the components of the imaging chain.

25.5 The vision of noisy images

The intensification of image intensifiers is such that at low radiation levels the output image is formed by large numbers of separate light dots that together give the impression of a snowy picture. The visual trajectory contains a sequence of pre-processors that each performs a part of the encoding of the object. At its input, the visual cortex gets an encoded image rather than an optical image of the perceived scene. This encoded image is further encoded and interpreted in channels higher in the brain. This is done by associating the elements of the encoded image that is entering the visual cortex with already existing information. The folded visual cortex offers about four square millimeters for the encoding of the environment of each separate receptor in the fovea. The pre-processors act as decision centers. When the offered signal to noise ratio is too low, then nothing is passed. This is a general principle in the encoding process and also governs the association of encoded data in other parts of the brain.

The research resulted in a significant contribution of our laboratory to the world standards for the measurement of the OTF and the DQE.

25.6Information association

The associative nature of the process is common for all kinds of objects and parts of objects. That includes objects that did not enter through one of the sense-organs. For example, a house is not stored in the brain as a complete concept. It is stored as a series of details that can be associated with the concept. If a sufficient number of these details are detected, then a decision center in the brain decides that the whole concept is present. In this way, not only a particular house can be recognized, but the process can also recognize a series of objects that resemble the original house. It classifies houses. By adding details that can be associated with it, the concept of a house can be widened. The resulting information, i.e., the information that passed the decision center, is used for further reasoning. Together with other details, the same details can also be used to detect other concepts by a different association. When the association act still produces too much noise, then the information is not produced, and further reasoning is neither disturbed nor triggered by this fact. High enough in the hierarchy, individuals can be discerned. The brain is not static. The network of communication paths and decision centers is dynamically adapted to changing needs.

25.7 Noise filter

The decision level for the signal to noise ratio may vary from person to person. If the level becomes too low, then the person may start hallucinating. Further, the level may be influenced by body owned messenger stuff, drugs, poisons, and medicines.

25.8 Reasoning

The brain is capable of performing complex reasoning. However, it must be trained to perform the reasoning in a logical way. For example, it must learn that the start from a false presumption can cause the deduction of any conclusion, just or false. When a path of reasoning is helpful, then it is stored in a similar way as an observation. Not the reasoning itself is stored, but the details that are part of the reasoning path. Also, here association of the details and a suitable noise threshold plays its role. The reasoning can be identified as a theory, and its concept can be widened. The brain can also generate new details that together with existing details, can act as a reasonable theory. Even noise can generate such signals. These details can be perceived as a dream or as a newly invented theory. It depends on whether the theory is accepted as realistic. That means that the brain must be capable of testing the realism of a theory. This testing can be improved by training. The brain can forget stored details and stored concepts. This holds for objects as well as theories. Valuable concepts are regularly refreshed and become better remembered.

25.9 Other species

Hubel and Wiesel did their experiments on several kinds of vertebrates, such as goldfishes, cats, and humans. Their main target was visual perception. Where the handling of the signals of sense organs in the brains is quite similar for all vertebrates, the handling of paths of reasoning by humans is superior in comparison to other vertebrates.

25.10 Humans

Humans have an advantage over other vertebrates. Apart from direct observation, the theories and the concepts of things can also be retrieved by communication with other parties. This occurs by education, discussion, reading books, papers or journals, seeing films or videos or surfing the internet. These media can also act as a reference medium that extends the storage capacity of the brain.

25.11 Science

Mathematics is a particularly helpful tool that extends the capability of the brain to perform reasoning in a logical and precise way. Physics extends this capability further with a focus on observables. Philosophy adds self-reflection and focuses on the why and how of existence. Every branch of science adds to the capabilities of the individuals and to the effectiveness of the community.

25.12 Physical reality

Our brain has limited storage capacity. We cannot comprehend things that have an enormous complexity. However, we can detect regularities. Our brain is optimized to detect regularities. The laws of physics appear regularly in our observations or can be deduced from regularly returning observations. More complex laws are derived using tools and in combination with other people. Nature is not only controlled by laws. It is also controlled by boundary conditions. These boundary conditions may be caused by the influence of items that lay beyond the reach of our direct observations. The number and complexity of boundary conditions far outgrow the number of recognized laws of nature. The laws of nature play a role in our theories. However, the boundary conditions play a much smaller role. This is because the laws of nature that we detect treat a simplified version of the environment. In this abstraction, the boundary conditions play no real role. This is another reason why our theories differ from physical reality.

25.13 Theories

These deliberations learn that theories are a product of our mind. They can be used as a looking glass that helps in the observation and interpretation of physical reality. However, it is false to interpret the theories as or as part of physical reality. When a theory fits, then it is congruent, to some extent, with physical reality. That does not say that we, as human beings and the environment from which we take our observations are not part of reality. It says that what our brain produces is another thing than physical reality.

25.14 Inventions of the human mind

Infinity is typically an invention by the human mind. There exist strong indications that nature does not support infinity. In the same sense, unlimited precision real numbers are prohibited in the physical universe. However, we can embed the results of our observations in a model that includes infinities and unlimited precision. For example, classical mechanics and field theories use these concepts. Quantum mechanics shows us that as soon as we introduce unlimited precision, we are immediately confronted with Heisenberg's uncertainty principle. We need infinity and unlimited precision in order to resolve the paradoxes that otherwise creep into our theories. We use theories that are in direct conflict with each other. One forbids infinity; the other theory uses and requires it. This says at least one thing; none of the theories describes physical reality correctly. Thus, none of the theories can replace the concept of physical reality. Still, it appears useful to use both views side by side. It means that great care must be taken with the interpretation of the theories.

25.15 History

Mathematical theories and physical theories tend to build upon the results of other exact theories. After some generations, a very complex building is obtained. After a while, it becomes humanly impossible to check whether the building elements are correct and whether the binding is done correctly. So, complex exact theories should be questioned.

25.16 Dreams

In this sense, only when we study our own dreams, fantasies, or theories, then we observe these items and the dreams; fantasies and theories become part of "physical reality." If the theory is congruent with a part of physical reality, it will become useful as a view on physical reality.

25.17 Addendum

I measured/calculated only up to the fourth layer of the visual cortex. Hubel and Wiesel did the pinching. We did perception experiments and developed and built equipment that worked optimally with that part of the visual tract. During that investigation, several disciplines that were considered advanced at that time (1970-1987) were used and expanded. For example, together with Wolfgang Wittenstein, I wrote most of the STANAG on the measurement of the optical transfer function (OTF and its modulus the MTF) of electron-optical appliances. Later I took this NATO standard to the ISO standardization committee that transferred it into an equivalent standard for optical equipment. Next, I was also involved in raising the corresponding IEC and DIN standards. Parallel to this, I also took part in the creation of the IEC and DIN standards for the measurement of the detective quantum efficiency (DQE). The research of the imaging channel starting from the radiation source and ending in the visual cortex resulted in a useful perception model that we used to improve our products. The standardized measuring methods enabled us to communicate the superior imaging quality of our products to our customers in a reliable and trustworthy way.

Personally, it offered me deep insight into the relationship between optics and quantum physics. I learned to handle Fourier transforms into an environment where the idealized Fourier theory does not fit. The measured multidimensional Fourier transform has a restricted validity not only due to the spatial non-uniformity of the imaging properties. The measuring result also depends on the angular and chromatic distribution of the radiation and on the homogeneity of that radiation. Part of the imaging chain consisted of glass lenses. Another part contained electron lenses and fiber plates. Intermediate imaging surfaces consist of phosphors that convert gamma quanta or electrons into light flashes. Other surfaces are covered with photocathode layers that convert detected quanta into electrons, which are sent into the electron-optical lens system until they reach the phosphor layer. The investigated appliances were image intensifier tubes for night vision purposes and X-ray image intensifier tubes that are used in medical diagnostic equipment. In this way, I got a deep insight into the behavior of quanta and experienced out of first hand that ALL information comes to us in the form of a noisy cloud of quanta. Only in masses, these quanta can be interpreted as a continuous wave of radiation.

26 Physical creation story

The fundamental consideration of physical reality quickly leads to a story of creation, in which the whole course of the creation of what occurs in the universe is told.

26.1 Motivation

More and more, my theories about the structure and the behavior of the universe became looking like a story of the creation of the universe. This is an alarming development because of scientists and especially physicists averse to religious concepts inside scientific documents. Still, I decided to write my theory in the form of a creation story. It is mixed with many mathematical and physical concepts because I wanted to generate a paper that is scientifically justified. I cannot avoid treating what can be accomplished with Hilbert spaces and number systems. They play an essential role in the theory as ways to store the dynamic geometric data that describe the life story of tiny objects. The same holds for shock fronts. These are field excitations that appear to constitute all other objects.

Already at the instant of creation, the creator appears to have archived all dynamic geometric data of all discrete objects in a read-only repository. In this repository, every elementary particle owns a private book that contains its full life story. Elementary particles behave as elementary modules, and together they constitute all other modules. Some modules constitute modular systems. My readers are intelligent modular systems. Freethinkers will be disillusioned by the fact that everything is already determined in the repository. The creator fools them by applying stochastic processes in the generation of the footprints of the elementary particles.

By installing a few ground rules, the creator generated a very complicated universe that contains intelligent creatures. This creation process took more than thirteen billion years, but the result exists in front of our nose.

By showing that he is a modular designer and a modular constructor, the creator presents his intelligent creatures an interesting example.

26.2 Justification

This story is not about religion. It concerns the creation of the universe. If a creator is mentioned, then this concerns a creating abstract object and not an individual that creates.

The universe is a field in which we live. The field can be deformed by the embedding of massive objects and is a carrier of radiation, of which a part can be observed with the naked eye.

The physical reality is represented by this field and what happens in this field.

What appears in this field is at the time of creation stored in an abstract storage medium. This storage medium is here called the Hilbert Book Base Model. The HBM consists of many separate books that each describe the history of an elementary particle and a background platform that archives the history of the universe in another way. Each part of the model describes the genesis, the past, the present, and the future of the described subject. The present is a window that runs across all the books.

26.3 Creation

This historiography gives the opportunity to speak about a story of creation. In fact, the model itself is the creator of the situation.

Elementary particles are described in a mathematical storage medium known as a quaternionic separable Hilbert space. A Hilbert space is a special vector space that provides an inner product for each pair of vectors. Quaternions are arithmetic numbers composed of a scalar and a three-dimensional vector. Therefore, they are ideally suited as a storage bin for a timestamp and a three-dimensional location. The quaternions give the number value to the inner product of the corresponding vector pair. The separable Hilbert space contains operators that describe the map of the Hilbert space onto itself and can store rational quaternions into storage bins that are attached to Hilbert vectors. The numbers are called eigenvalues, and the corresponding vectors are called eigenvectors. Together, the eigenvalues form the eigenspace of the operator.

Quaternionic number systems exist in many versions that differ in the way that Cartesian and polar coordinate systems rank their members. Each quaternionic separable Hilbert space chooses its own version of the number system and maintains that choice in the eigenspace of a special reference operator. In this way, the Hilbert space owns a private parameter space. The private separable Hilbert spaces of elementary particles hover with the geometric center of their parameter space over the parameter space of the background platform. By using this parameter space and a set of continuous quaternionic functions, a series of newly defined operators can be specified. The new defined operator reuses the eigenvectors of the reference operator and replaces the corresponding eigenvalue by the target value of the selected function by using the original eigenvalue as the parameter value. This newly-defined operator contains in its eigenspace a field that is defined by the function. The field is a continuum. The eigenspaces of the operators are countable. Thus, the eigenspace of the new operator contains the sampled values of the field. In fact, the private parameter space is also a sampled continuum. The eigenspace of the reference operator contains only the rational elements of the selected version of the number system.

Nothing prevents all applied separable Hilbert spaces from sharing the same underlying vector space. We assume that the background platform is a quaternionic separable Hilbert space which contains infinitely many dimensions. This possesses a unique non-separable partner Hilbert space that supports operators, which possess continuous eigenspaces. These eigenspaces are, therefore, complete fields. Such eigenspaces are not countable. One of these operators owns an eigenspace that contains the field, which represents the universe. This field is deformed by the embedding of the hop landings of the elementary particles. The locations of the hop landings are stored in the eigenspace of the footprint operator in the private Hilbert space of the corresponding elementary particle. After sorting the timestamps, the footprint operator's eigenspace describes the entire lifecycle of the elementary particle as one continued hopping path. That hopping path recurrently generates a swarm of hop landing locations. A location density distribution describes the swarm. Because the particle is point-shaped, this is a detection probability density distribution. This is equal to the square of the modulus of the what physicists call the wavefunction of the elementary particle. The hop landing location swarm represents the particle.

26.4 Dynamics

At the time of the creation, the creator let a private stochastic process determine the hop landing locations of each elementary particle. This process is a combination of a Poisson process and a binomial process. A point spread function controls the binomial process. The stochastic process possesses a characteristic function that causes the production of a coherent swarm. It is the Fourier transform (the spatial spectrum) of the detection probability density distribution. As a result, the point spread function is equal to the location density distribution of the produced swarm. This design ensures that when the characteristic function becomes wider, the point spread function becomes narrower. In this way, the creator gives his creatures the impression that he does not determine the hopping path. He leaves some freedom to the objects that are formed by the elementary particles. However, in the beginning, the entire lifecycle of all elementary particles is already archived in their private storage medium. After that archival, nothing changes in this storage medium. The archive can only be read. Since the timestamps are stored together with the locations, the corresponding Hilbert book contains the entire life chronicles of the elementary particle.

The version of the quaternionic number system that the private Hilbert space of the elementary particle selects determines the symmetry of the private Hilbert space and of the elementary particle. This is characterized by an electric charge that houses in the geometric center of the particle platform's parameter space. The axes of all Cartesian coordinate systems must be parallel or perpendicular to each other. The geometric center may differ and may even move. Only the ranking along the axles may differ in direction. The electrical charge turns out to be a consequence of the difference between the symmetry of the gliding platform and the symmetry of the background platform. Because only a small number of versions of the quaternionic number are allowed, there exist very few different electrical charges. As a result, electrical charges can occur in the proportions -3,-2,-1, 0, 1, 2, and 3.

The separable Hilbert space of the background platform is naturally embedded in the non-separable Hilbert space. This is because both Hilbert spaces have the same symmetry. The embedding does not cause any disruption of symmetry. This does not apply to the embedding of the footprints of the elementary particles, because their Hilbert spaces possess a deviating symmetry. When embedding, only isotropic disturbances of the symmetry can cause an isotropic disturbance. Such a disturbance may temporarily deform the embedding field.

The swarm of hop landing locations can generate a swarm of spherical pulse responses. Only an isotropic pulse causes a spherical shock front. This shock front integrates over time into the Green’s function of the field. This function has volume, and the pulse response injects this volume into the field. The shock front then spreads this volume over the field. As a result, the initial deformation of the field is rapidly flowing away. The stochastic process must continue to deliver new pulses to achieve a significant and permanent deformation. To get an impression of the deformation, we must convolute the location density distribution of the hop landing location swarm with the Green's function of the field. Convolution blurs the image of the swarm. This does not give a correct picture, because the overlap of the spherical shock fronts depends on the spatial density of the swarm and on the time that the shock fronts need to overlap sufficiently. Far from the geometrical center of the swarm, the deformation is like the shape of the Green's function. The two functions still differ in a factor. This factor indicates the strength of the deformation. The factor is proportional to the mass of the particle. In fact, this is the method by which the scholars determine the mass of an object.

26.5 Modularity

Elementary particles behave as elementary modules. Together they form all the other modules that occur in the universe. Some modules constitute modular systems.

The composite modules and the modular systems are also controlled by a stochastic process. This is a different type of process than the type of process that regulates the footprint of the elementary particle. This second type controls the composition of the object. This type of stochastic process also possesses a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the compound object. The superposition coefficients act as displacement generators. They determine the internal positions of the components. The characteristic function connects to an additional displacement generator that regulates the movement of the whole module. This means that the composed module moves as one unit. The binding of the components is reinforced by the deformation of the embedding field and by the attraction of the electrical charges of the elementary particles.

This description shows that superposition takes place in the Fourier space. So what a composite module or modular system determines, is captured in the Fourier space. Locality does not play a role In Fourier space. This sketches the phenomenon that scholars call entanglement. In principle, the binding within a composite module is, to a large extent, established in the Fourier space. The parts can, therefore, be far apart. For properties of components, for which an exclusion principle applies, this can have remarkable consequences.

All modules act as observers and can perceive phenomena. Elementary particles are very primitive observers. All observers receive their information through the field in which they are embedded. The observed event has a timestamp. For the observer that timestamp locates in the past. As a result, the information is stored in the Euclidean format in the storage medium in a storage bin that contains a timestamp and a three-dimensional location. By the observer, that information is perceived in space-time coordinates. A hyperbolic Lorentz transformation describes the conversion of the Euclidean storage coordinates into the perceived spacetime coordinates. The hyperbolic Lorentz transformation adds time interval dilatation and length compression. The deformation of the embedding field also deforms the path through which the information is transported. This also influences the transported information.

26.6Illusion

At the instant of creation, the creator fills the storage bins of the footprint operators. The contents of this store won't change any more. The later events in the embedding field also have no influence on the archive. Since the creator uses stochastic processes to fill the footprint storage, intelligent observers will get the impression that they still possess free will. The embedding of the footprints follows step by step the timestamped locations that were generated by the stochastic processes and were archived in the eigenspace of the footprint operator. The observer should not be fatalistic and think that his behavior does not matter because everything is already determined. The reverse is true. The behavior of each module has consequences because each perceived event affects the observer. This fact affects the future in an almost causal fashion. The stochastic disturbance is relatively small.

26.7 Cause

The driving force behind the dynamics of the universe is the continual embedding of the hop landing locations of the elementary particles into the field that represents the universe.

It seems as if the continuous deformation of the field seems to come out of nowhere and that the individual deformations then disappear quickly by the flooding away of the inserted volume. The expansion persists.

The shock-fronts play an essential role because the spherical fronts spread the volume into the field. The one-dimensional shock fronts carry information and also carry extra movement energy. They move the platforms on which elementary particles travel through the universe.

26.8 Begin to end

The universe is a field, and that field can be described by a quaternionic function. In the begin of its existence, the stochastic processes that produce particle hop landings that inject volume into this field had not yet done any work. Thus, the field did not yet contain any spatial volume. However, spread over the full spatial part of the parameter space, a myriad of stochastic processes immediately started to deform and expand the spatial part of the field. The deformations fade away but are quickly repeated by the recurrently regenerated hop location swarms. This produced a bumpy look of the early universe. This is not in accordance with the usual interpretation of the begin of the universe, which is sketched as a big bang at a single location.

Black holes are special phenomena. They are bordered areas in which volume can only be added by widening the border. No shock front can pass this edge. The black holes wipe elementary particles together at their border. Part of the footprint of the elementary particles hovers over the black hole region and the pulses extend the volume that is enclosed by the border into all directions. The BH is filled by the equivalent of storage capacity, which is no longer devoted to the modular design and modular construction process. Inside the BH region, these processes are impossible. Also, the elementary particles that are stitched at the border will be prevented from generating higher-order modules.

This characterizes the event horizon of the BH. It also indicates what the end of the history of the universe will be. A huge BH uniformly filled with spatial volume. The platforms of the elementary particles can still cling at the outside of the border of the final BH.

26.9Lessons

After the instant of creation, the creator does no longer care for his creatures. This creator is not a merciful God. It makes no sense to beg this creator for special benefits. For the creator, everything is determined. Due to the application of the stochastic processes, the intelligent observers still get the illusion that they have a free will. For their perception, all their actions cause a sensible result. The uncertainty that is introduced by the stochastic processes is relatively small.

The creator is a modular designer and a modular constructor. For his intelligent creatures, this makes an important example. Modular construction is very economical with its resources and provides relatively fast usable and reliable results. This method of working creates its own rules. It makes sense to have a large number and a large variety of suitable modules at hand. It even makes sense to create communities of module types and communities of modular system types.

The lessons, which the creator teaches his intelligent creatures, follow from the modular design of the creation. The lesson is not the survival of the fittest. The survival of the module type-community is more important than the survival of the individual modular system. It makes sense to care for the module type-community that the individual belongs to. It also makes sense to care for the module type-communities on which your module type-community depends. That then demands to correctly secure the module communities of which one depends. It has much sense to care for the habitat of your module type-community.

27 Story of a war against software complexity

27.1 Summary

This is the account of the course of a project that had the aim to improve the efficiency of embedded software generation with several orders of magnitude. All factors that determined the success of the project are treated honestly and in detail.

The story shows that also, human activity is affected by the way products are designed and constructed.

27.2 Prelude

In 1995 the physicist HvL was an employee of an internal software house of a large electronics company. His specialty was the creation of scientific software. Then he got the invitation from a software strategist HdV to join the semiconductor department in order to resolve a quickly emerging problem. The costs of complex embedded software were growing exponentially, and this would cause severe problems in the next future. The reasons why the costs of software generation grow exponentially are a growing size and the growing complexity of the embedded software that goes into high-tech appliances. One of the reasons for the non-linear growth of costs is the growth of complexity. But the exponential growth of costs is mainly caused by surpassing of the available resources, which on its turn required measures against expected internal and external damage claims. In many cases, software projects were stopped when the costs were expected to explode, or when they did not seem to reach the expected result.

27.3 Analysis

There exist several possible solutions to this dilemma. One is to move software development to low wage countries. Another is to apply open-source software. A third possibility is to increase the quality of the software generation process. One way to do this is to improve the control of the flow of the generation process. Another way is to improve the way that software is generated. The management of the electronics company tried all these possibilities. Improving the control of the flow of the software generation process has little sense when the generation process itself has severe defects.

27.4 Setting

The electronics company is successful in the generation of hardware. This is for a large part because the hardware is generated via a modular approach. This is one of the reasons that the research lab of the firm created a dedicated way to create modular embedded software. However, this is a rather closed system, and it is directed to the direct need, the generation of software for consumer appliances. Still, it is intuitively felt that a modular approach will improve the effectiveness of software generation. There are also many objective reasons for this point of view.

27.5 History

The electronics firm was not very successful with its software projects. Many software projects were stopped after having burned hundreds of man-years and millions of dollars, leaving the project leaders, system architects, and the software designers back in despair. For that reason, relief was sought in outsourcing of the software generation. One form of it is the use of open-source software. Parallel to it, the internal software generation was moved for a significant part to low wage countries like India. This was only a short time solution. The exponential growth of costs took its toll there as well.

Also, the switch to open source software was no smart decision. The electronics firm had no control over the way that the open-source software was evolving, and the open-source software generation suffered the same bad habits as the present-day commercial software generation process does. Commercial software generation and open source software generation are both non-modular. There exists no healthy and lively software modules market that stimulates the diversity, availability, accessibility, and favorable quality/price ratio that characterizes the hardware modules market. Thus, the high-tech hardware appliances industry is still confronted with the negative aspects of the current software generation technology. It drives their costs high, and the fragility of the software is transferred to the hardware products that include the software. The ineffectiveness of the software generation affects the affordability and the time to market of the hardware products.

27.6Strategy

A small group of experts consisting of the software specialist HvL, the software strategist HdV and a software marketing specialist WR studied the resulting possibilities and concluded that a drastic change in the way that software is generated is a promising solution of the problem. The way that hardware is generated was taken as an example. Hardware is generated mostly in a modular way. Modularization reduces the relational complexity of the design and construction process. It also enables partition and delegation of the design and construction work. It even enables a flourishing module's market.

27.7Approach

The group tried to interest vendors of embedded software generation tools to join the enterprise. It was obvious that international standards would play a crucial role. So, the group stimulated the management of the electronics firm to involve other electronics firms and the OMG. All these measures lacked enough success. The tool vendors were interested but used the opportunity to monitor whether their current way of operation was endangered. They did not really take part in the development. The other electronics companies took the role of an observer and asked for a convincing demo of the concept. OMG lets standards create by the interested parties. It is not usage that the standard is introduced by a single company.

The project did not tackle the required hiding of invested intelligence that prevents theft of ideas that are applied inside the skeleton of the modules. Possible solutions are encryption of applied software and punishing theft by hackers by excluding them from system services. The hardware platform must enable the required software encryption and decryption.

27.8 What happened

The group encountered severe resistance against their intentions from internal software development groups because it was expected that the generation of the modules would be outsourced to the suppliers of the software modules market. This fear is real. On the other hand, it became more and more clear that the internal software generation capabilities were not measured up against the task to create large and complex embedded software systems. Several costly debacles proved this. Especially managers, including the managers of software groups, showed that they lacked a proper feeling for the factors that influence complex software generation.

27.9 Attack

The group decided to create a demo version of the modular software generation system that included major parts of the envisioned system. This includes software module development tools, system configuration tools, web and local file-based repositories that act as searchable exchange places for machine and humanly readable specifications of modules and interfaces and central services that act as a marketplace for software modules. The module development tool can generate skeleton modules, and it can generate the interface definitions from specifications that are retrieved from web-based or local repositories. The tool helps to fill the skeletons with working code. The configuration tool retrieves specifications of modules and interfaces from the repositories. It can retrieve the binaries of modules from the market place or from a local store. It enables the mostly automatic assembly of modules into target systems. It adds a dedicated RTOS that consists of automatically generated modules. The RTOS provides automatic memory garbage collection. The central service collects specifications from the module developers and distributes these to the web-based repositories. The central service also collects the binaries of the modules and stores the specifications and the binaries in its banks. The central service acts as a module's market.

27.10 Set-back

The project of the group was severely hampered by the dot com crisis in 2001. This stopped all long-term research projects and brought the funding of the group to a minimum. The development of the demo continued at a low pace and stopped in 2004. At that time most planned parts of the demo worked at least for a large part.

27.11 Remnants

The central service worked partly. The development tools are functioning. Modules can be generated, and the configuration tool can assemble systems from these modules and add a service layer that consists of automatically generated dedicated modules. The service layer includes garbage collection. It uses connection schemes and scheduling schemes that dynamically control the switch between system modes. The created system does not contain a HAL, and it does not contain interrupt services. Instead, it relies on the services of a virtual machine or a POSIX OS. This is not the target to work on top of hardware, but it is good enough for most demonstration purposes. The tools generate software in C++, but as a bonus, it can deliver C# code. That code works on top of a dotNet virtual machine. The tools and central services are written in C#.

Apart from SW/SW interfaces, the modules may contain HW/SW interfaces. Streaming interfaces and the notification interfaces that handle interrupts were planned. The skeleton of the modules are modeled after Microsoft's Component Object Model (COM), but the IUnknown interface is replaced by the IAccessor interface. That interface replaces the AddRef and Release functions with a ResetInstance routine. Instead of the designer, the system is made responsible for the garbage collection. For that reason, the new module skeleton is named Robust Component Object Model (RCOM).

27.12 Goal

The demo was planned to demonstrate the generation of real-time embedded software. That goal is not reached. However, many aspects of the planned target are shown in the completed part of the demo. That part offers trust in the feasibility of the final goal.

27.13 Lessons

The project also learned many valuable lessons.

- The current suppliers of software generation tools are not interested in a drastic change in the way that software is generated.
- Even though embedded software is causing major problems, the companies that produce high-tech appliances or high-tech systems are hesitating to cooperate in improving the software generation process. Software generation is not their strength.
- This world is not good at organizing actions that are rather complex. For that reason, it is difficult to arrange standards on new subjects.
- It is difficult to motivate management to enter new inroads when the reasons are not very simple and require insight into the topic.
- Managers of these days are interested in short term low-risk solutions. They are not interested in long term solutions even when they promise high profits.

- The same holds for today’s investors.
- Although most involved people intuitively see that a modular approach provides a better effective generation process and easier support management, most of these people forget that without a suitable module market the modules are too expensive and too scarce to make the assumption true.
- A modular system generation approach has no sense when it does not include an integrated and well-functioning modules market. This also means that a system of web-based and local repositories that contain the specifications of modules and specifications must be involved as well.
- Given enough resources, even a tiny group of determined software experts can design and construct a working version of a modular software generation system that includes all essential parts.

27.14 Conclusions

This world is not good at creating new standards. However, we are good at accepting default standards. Large electronic firms seem incapable of creating a suitable software generation system. Understandably, the existing software industry appears not willing to give up the profits that they retrieve from the current deplorable way of software generation.

27.15 Way out

There still exists a possible way to get out of this misery. When a small group of enthusiastic software developers and venture capital investors start with a project that establishes a working version of a modular software generation system that includes all ingredients to get a successful result, then they may cause the seed that will extend like an oil drop and smother the current way of software generation.

In a world where such a system exists the complex software assemblies are no longer created by a genial system architect and hundreds of man-years of expensive programmers but instead by a creative modular system assembler that uses automated tools to construct his target in a fraction of the time, with a fraction of the resources and with a fraction of the costs compared to his present-day colleague. He retrieves his modules from a module market, and he may also design and produce some missing modules. In a later phase, he may decide to offer these new modules on the market.

His present-day colleague produces software systems, whose structure resembles a layered set of patchwork blankets. Even the most ingenious architect cannot oversee the details of this complex architecture. Therefore, the system cannot be completely described properly. Thus, it cannot be tested fully, and nobody can guarantee its proper functioning. Modular systems are inherently less complex. Especially its system configuration is orders of magnitude less complex. This results in better manageability of the complexity and higher robustness. On its turn, it results in a better chance to be able to guarantee its proper functioning.

27.16 Discussion

The modules market is very democratic. Everybody that owns an appropriate modules development system can participate and fill a niche of the modules market. The modules market is a good replacement of the market for open-source software. It has the advantage that the module developers can earn money for the intellectual property that they invested in the design and construction of the module. Still, the products stay very affordable. In contrast, the open-source software community is non-democratic. In many cases, the community forbids the contributing software developers to earn money from their intellectual property investments.

28 Managing the software generation process

The current software generation process is rotten. This chapter analyses why that is the case and what can be done about it.

It is no secret that the generation of complex software poses great problems for its producers. The cost is growing exponentially with the size of the software, and the time from conception to finalization grows likewise. The resulting products are fragile and force the vendors to reserve enough resources to cope with future warranty and damage claims. Buyers are aware of this situation, but without a reasonable alternative, they are ready to live with the situation. The source of the misery is the complexity of the software, and this complexity is mainly due to the relational complexity of its constituents. A radical modular approach as is applied in hardware system generation would cure the problem, but that requires a completely different way of software generation and software marketing.

28.1Introduction

First, the factors that hamper efficient system generation are treated independently of the application area. Then the solutions for eliminating these factors are given. Next, the differences between the hardware area and the software area are shown in a historical view. Finally, a possible improvement of the software case is sketched.

28.2Managing complexity

28.2.1Breaking level

Managing simple projects hardly ever poses problems. However, a situation in which complexity surpasses the boundary where a quick view no longer reveals potential problems requires special methods. These measures compensate or cure the lack of overview. The level of the boundary depends on the number of items involved in the process and on the nature of the relations between these items.

28.2.2 Measure of complexity

The number of potential relations between the items involved in the process explains a close to quadratic growth of potential complexity with the number of items involved. Between N items exist N×(N-1) potential relations. Usually, only a small percentage of the potential relations are truly relevant relations. Dynamically relevant relations are the potential carriers of communication and control signals. They carry the activity and determine the capabilities of the considered system. It takes expertise knowledge to decide whether a potential relation is dynamically relevant. Gaining this expertise takes time and other resources. This explains why all potential relations have a direct impact on manageability. For that reason, the number of potential relations may act as a rough measure of potential complexity. Similarly, the number of dynamically relevant relations may act as a rough measure of the actual complexity of the system. More precise measures will also consider the type of relations. The type of relation determines how that relation must be treated.

Procedures such as modularization of the system and categorization and grouping of the interrelations into interfaces significantly reduce the actual complexity of system design and creation. Each interface represents a well-defined group of dynamically relevant relations. Well-known interfaces contribute significantly to the reduction of complexity. They reduce a set of interrelations to a single relation. Modules can be assembled into systems by connecting them via compatible interfaces. Both the modules and the interfaces that couple these modules are of crucial importance for managing the complexity of system generation.

28.2.3 Extreme complexity

Very high degrees of complexity may introduce secondary effects that impair manageability far more severely than can be explained by the number of potential relations between the items involved in the system generation process. This occurs when it becomes humanly impossible to correctly specify the activity of all dynamically relevant relations.

The inability to specify the product implicates the inability to test it, and as a consequence, it implicates the inability to guarantee the proper functionality of the system. The implications of the lack of resources that are required to cope with complexity and the inability to specify the situation in sufficient detail can easily raise costs in an exponentially increasing way. Apart from causing unacceptably growing costs, the system generation process yields fragile results. The resulting product may even endanger the environment where it is applied. This requires reserving resources to ensure resistance against future claims.

28.3The modular approach

28.3.1 Modularization

The reasons why modularization significantly improves the manageability of the generation process are manifold. For example, it may be possible to delegate the design or the creation of modules to other parties. Potential reuse of existing modules or their design is another important reason. However, the most important reason for applying modularity is the fact that proper encapsulation of the modules and the use of well-known interfaces significantly reduce the number of dynamically relevant relations.

A simple example may explain this. A monolithic system consisting of 1000 items contains 999.000 potential relations. Its relational complexity can be characterized by this number. A comparable modular system that consists of ten modules contains far less potential relations. Let the modules be coupled by well-known interfaces and let part of the interfaces be similar. Not every module connects to every other module. Let the largest module contain 200 items and let the total number of interfaces between any pair of modules be less than 5. The largest module has a potential relational complexity of 39.800. The complexity of the other modules is less. Thus the relational complexity met by the module designers is less than 40.000, and for most modules, the relational complexity is less than 10.000. Between modules, the interfaces take the role of the relations that are the internal members of these interfaces. The system designer is confronted with a relational complexity that is less than 100. The benefits of the reuse of interfaces and the advantages of the possible reuse of modules should also be considered. Thus compared to the monolithic case, there is an increase in manageability of several orders of magnitude. Modularization of larger systems may offer benefits that are much higher. Diminishing relational complexity translates directly in lower man costs and in a shorter time to realization. Further, it has a very healthy effect on the robustness and reliability of the end product.

28.3.2 Modular system design

The system designer gets the strongest benefit from the modularization. Modularization simplifies system assembly significantly. This opens the possibility to automate the system integration process.

Modularization reaches its highest effectiveness when the design and creation process enables the assembly of modules out of other modules. In this way, the microelectronics industry reaches the exponential growth of the capabilities of integrated components that is known as Moore’s law.

28.3.3 Interfaces

In the design of a system, the introduction of an interface increments the number of potential relations. However, because the interface encapsulates a series of dynamically relevant relations, the total relational complexity will decrease. The new relation will only play a dynamic role when the corresponding modules are coupled or decoupled. This coupling can be done at system assembly time or during the operation of the system. At instances where no coupling or decoupling is performed, the new relation acts as a static relation. It relays the communication and control signals to the dynamically relevant relations that are members of the interface. In the count for complexity, a well-known interface replaces the combined contributions of its members. In that view, it can be considered as a single, dynamically relevant object.

Dynamically relevant relations are carriers of information or control signals. Depending on the direction of the control signal, the corresponding interface member belongs to the require part of the interface or to the provide part of the interface. In the first case, it acts as the sender of control signals. The require part of the interface contains members that belong to the current client module. If the interface member acts as the receiver of control signals, then the interface member belongs to the provide part of the interface. In return, the interface member causes the module to deliver corresponding services. The provide part of the interface belongs to the module that acts as the current server. In order to become active, the require part of the interface of the client module must be connected to the provide part of the interface of the module that acts as the server.

A module may act as a server at one instance, and it may act as a client in other instances. In each of its roles, it will use the appropriate provide or require interface parts. Multitasking modules may provide parallel actions.

In the assembly, the coupling of the require interface part, and a corresponding provide interface part may be stationary, or it may be temporally. The provide interface part of an interface may serve one or more other interfaces. The service may be presented in parallel or in sequential order. The specifications of the provide interface part must at least cover the requirements of each of its customers. With respect to its potential capabilities, the provide interface part may offer more than is requested by a coupled require interface part. The specification of the provide interface part must be in accordance with the specification of the require interface part, but this only holds for the part that covers the services that the require interface part may demand.

In many cases, the trigger of a provide interface member by a connected require interface member will not only result in an action of the server module. It may also cause the return of a response via the same connection. The response can be used for synchronization purposes, and it may contain requested information.

In general, an interface may contain both a provide part and a require part, and the partition may change dynamically. It is difficult to understand and handle such mixed interfaces. When manageability is strived for, then mixed interfaces must be avoided. An exception exists when the communication requires a handshaking process. Preferably pure interfaces should be used. A pure interface contains either a require part or a provide part but not both. In the simplest case, the specification of a require interface closely matches the specification of the corresponding provide interface.

28.3.4 Proper modules

Proper modules are properly encapsulated. A proper module hides its internals. Securing the intelligent property that went in its design is one of the reasons for this strict measure. Preventing unwanted access to the module is another reason. Proper modules can only be accessed through publicly known and well-specified provide interfaces. A module is a part of an actual system, or it is targeted as a part of one or more possible future systems. Proper modules take care that each access through an interface keeps the functional integrity of the module intact. An exception may be that the module signals to its environment that it is no longer in a valid state. The environment may then decide to ignore the module in future actions, or it may reset the module to a valid state.

A proper module must be able to perform one or more actions. These actions may be controlled via one or more of its provide interfaces. Purely static objects are never considered as proper modules.

28.3.5 Properties and Actions

Each proper module has a set of properties that together describe its status. Besides that, each proper module provides a series of actions. Each module interface offers indirect access in order to control the members of a well defined and ordered subset of these actions. The properties cannot be accessed directly. However, a given action may enable the reading of the value of a property, or it may enable the direct or indirect setting of one or more properties.

28.3.6 Costs of modularization

Modularization has its price. The design and generation of modules and the organization of compatible interfaces are relatively expensive. Only extensive reuse of modules may render modularization economic. Reuse of modules and the availability of compatible well-known interfaces between modules may significantly improve the manageability of the design and creation of complex systems. However, reuse implicates standardization, and it asks for actions that promote availability, accessibility, and diversity. These requirements are best provided by a healthy and lively modules market and media that publish the specification of the characteristics of available modules and interfaces. An open market may ensure a healthy price to quality ratio. It also stimulates the continuous improvement of the quality of the modules that become available. Preparing modules for an open market requires the hiding of the intellectual property that is invested in the design and creation of the module. On the other hand, the specification of provide interfaces must be publicly known. Promoting other uses of the provide interfaces, and the require interfaces that are applied in a given module will in its turn, promote the use of that module. It will increase the chance that other modules will become compatible with the considered module.

28.3.7 Abuse

Modularization can also be abused. Wrong access to a module may rupture its integrity. In that case, the module is no longer trustworthy. A proper modularization technology must prevent improper access to modules. It means that access that bypasses the official interfaces of the module must be prevented. Clients of a module may be systems or other modules. During its actions, a module may run through a sequence of states. A client of a module must only access the module while the module is in a state that is known to be safe for this access. Properly created modules will then take care that their integrity will not be impaired. If the state of a module is not known, then the client may decide to reset the module to a save known state.

Abuse of modularity is stimulated by the misuse of the terms ‘module’ and ‘component.’ It often occurs that a system part is called ‘component’ or ‘module’ while it is far from properly encapsulated. Such system parts are not designed to preserve their integrity. People that do not have sufficient expertise may fall into this trap and may think that by assembling such improper components, a similar reduction of complexity can be achieved as can be achieved with proper modules.

28.3.8 Modularization success cases

The success of modularization is widely demonstrated in the design and generation of hardware. Electronic appliances, autos, buildings, clothes, in fact, most assembled products are not affordable without the fact that they are constructed from components. Many of the constituting components are themselves assembled from components. More importantly, the price, quality, diversity, and availability of these components depend strongly on the corresponding lively components markets. The beneficial effects of the open market depend strongly on the trustable specification of the characteristics of the components and on media that report on availability and quality of these products.

Even nature relies on modularization. Most living creatures contain organs and are constituted from multiple cells. Human communities use modularity in the hierarchical structure of their organizations. This is best shown in a town hall or a post office where dedicated counters belonging to corresponding departments offer publicly known services to their customers.

The application of modularization in the software industry is far from great success. Proper software modules exist, but their application is sparse. The current software development tools do not support the assembly of systems from modules. The software components rely on the support that is offered by the operating system that embeds these components. Most software components are designed to operate as singles in a larger non-modular environment. Generally, these modules don’t couple with other modules. Currently, the software industry does not offer a technology that enables the construction of modules out of other modules.

28.3.9 Requirements for success

When applied properly, modularization may significantly improve the system design and creation process. Keywords are the standardization, the diversity and the availability of modules and interfaces, and the ease of the system integration process. The existence of a lively and effective module's market is also a very important aspect. System integration may be automated, but this requires the proper tuning of component specification, the system design tools, and the matching components market. The technology must enable the construction of modules out of simpler modules. Using these preconditions the microelectronics industry provides very complex and tremendously capable integrated circuits.

With a proper automated design and assembly organization in place, the modular system creation time will shrink to a small fraction of the time required by the manual non-modular equivalent. Where manual design and assembly of a complex monolithic target requires a genius as the system architect, a creative human operator may burn far fewer resources and achieve a similar or even better result by using an appropriate automated modular approach. Automation of the system design and creation process puts high demands on trustworthy and machine-readable specifications of modules and interfaces.

28.3.10 Difficulties posed by modularization

The requirements posed by modularization are also the reasons why modularization is never a straightforward solution.

28.3.10.1 Diversity

The requirement of a high degree of diversity is in direct conflict with the requirement of sufficient standardization. An interface has both static and dynamic aspects. Dynamic requirements may ask for different interfaces that have similar static characteristics but different dynamic behavior. Environmental requirements may ask for specially adapted interfaces. Interfaces may be replaced by other interfaces that have a wider scope or a better performance. Similar considerations hold for modules.

In order to increase market profits, to simplify component discovery and to ease system integration, the diversity of similar interfaces must be kept within sensible bounds. The same holds for modules.

28.3.10.2 Compatibility

In order to enable successful assembly, the selected modules must be mutually compatible. This translates to the requirement that the interfaces that couple the modules must be compatible. Provide interfaces must cover the demands of the coupled require interfaces. The requirements include both static and dynamic characteristics.

Real-time behavior of modules may require measures that prevent or cure deadlock and race conditions. The design tools must enable the installation of these measures. Other measures must prevent that the system runs out of essential resources. The modules must be designed to support these measures. When all relevant data of the constituting modules are known, then the system design tools can help the system designer to implement sufficient resources and to take the appropriate measures.

28.3.10.3 Platforms

Components may be designed for different application areas. For example, the software may be designed for desktop purposes, for servers or for embedding in electronic appliances. In each of these cases, there exists a choice of hardware platforms. Electronic hardware platforms require adapted software components and will certainly influence the dynamical characteristics of the interfaces of the software components. Mechanical modules may target automotive systems, avionics, nautical systems, stationary instruments, or other mechanical systems. Each application area and supporting platform may require its own range of modules and interfaces. Each application area requests an adapted components market and an adapted system assembly technology.

28.3.10.4 Hiding intellectual property

In some application areas, the hiding of the intellectual property that went into the design and the creation of modules is provided by their physical form or by market conditions such as a patent system. However, some application areas currently lack sufficient means to hide the design of the components. Without proper IP hiding, a component's creator can never make a profit in an open components market. In the past, this fact has certainly prevented that the software industry developed a healthy and lively software components market. This does not say that it is impossible to generate an effective IP hiding system for software modules.

28.3.10.5 Availability

Availability is assured when several suppliers exist for popular modules. An easily accessible publication organization must promote and enable the discovery and the selection of existing modules.

28.3.10.6 Specification

The specification must be accurate and complete. The specification must contain sufficient details such that the system integrator can determine how the considered module can be assembled with other modules into a target system. Automated assembly asks for a machine-readable and therefore well-standardized specification format. This requires a dedicated XML format. The format can be defined in an XSD document. For humans, an XML document is not easily readable. The XML document can be made readable for humans via one or more XSL documents. The specification of the statical characteristics of an interface is well established. Currently, there exists much less support for the standardized specification of the dynamical characteristics of interfaces.

28.3.11 Hardware versus software

28.3.11.1 History

The hardware industry booked far more success with the application of modularization than the software industry. Partly the volatile nature of software is responsible for this fact. However, the differences in the evolution of the corresponding design and creation technologies had more influence on the success of modularization.

Long before the birth of electronic computers, modularization took its position in the hardware industry. Computer hardware became affordable through the far-reaching application of modularization. The early computer programmers used machine code as the language to communicate with the computers. Soon the burden of inputting all these codes separately, was eased by an assembly compiler that translated assembly terms into corresponding machine code sentences. Program parts could become reusable routines. Libraries of these routines became products that could be applied in different programming projects. The next step was the introduction of the third-generation languages. These tools offered a better readable and much more flexible coding of the functionality that the programmers had to write. Powerful compilers translated the source code and combined it with the precompiled library members that were called by the written program.

28.3.11.2 Basic architecture trends

Up to so far, this was no more than easing the process of producing machine code. The growing complexity of the programs demanded software development tools that enable a better overview of the architecture of the design. At this point, two trends developed.

28.3.11.2.1 Functional analysis

The first trend phrased ‘structural analysis’ created a split between the handling of properties and the handling of the actions that influence these properties. The methodology collected properties in ‘data stores,’ actions in ‘processes,’ data messages in ‘data flows,’ and control messages in ‘control flows.’ The graphical representation of the result of the analysis was called a ‘data flow diagram.’ In advance, the approach proved very successful. It led to the introduction of several important software development items such as routine libraries, file systems, communication systems, and databases. Most third-generation programming languages and the early software development tools supported the ‘structural analysis’ approach.

28.3.11.2.2 Abstract data types

The second trend promoted the modular approach. It used ‘abstract data types’ introduced by David Parnas as its modules. In the design phase, the ‘abstract data type’ acted as an individual. It was well encapsulated and could only be accessed through one or more interfaces. In the seventies of the last century, the complexity of most software projects did not enforce a modular approach. For that reason, this modular design methodology was not well supported by programming languages and by corresponding modular software development tools.

28.3.11.2.3 Object orientation

In a later phase, the complexity of the software design increased such that a more modular approach became necessary. Instead of taking the proper modular approach of the ‘abstract data type’ the main software development turned to object orientation. Here the objects resemble ‘abstract data types,’ but the objects are not properly encapsulated. Access via interfaces is possible, but the client of the object may also access the actions of the objects more directly. More severely, often the internal properties of the object can be altered directly by external actors. The possibility to inherit functionality from an object with a simpler design was given much more attention. The result was the development of libraries of classes of objects with a deep inheritance hierarchy.

Currently, object orientation is well supported by software languages and software development tools. Pity enough, current object-oriented software development tools do not promote the use of popular interfaces.

Object orientation has some severe drawbacks. Without sufficient precautions, classes taken from different class libraries cannot be combined in programs. A class library with a deep inheritance hierarchy may become obsolete when its top classes contain services that are no longer up to date with current technology.

28.3.11.2.4 Current software components

The software industry also came with more proper software modules. Examples are Microsoft’s COM components and the Java Beans. COM components are supported by some operating systems and Java Beans are supported by the Java virtual machine.

The support for COM in software languages and in software development tools is small. The design of the architecture of the COM skeleton prevents trustworthy memory garbage collection management in cases where the module can be removed dynamically. COM is supported on some embedded systems that use UNIX or an operating system that supports POSIX.

Both Java Beans and COM components are not designed to construct components from components and need the support of an operating system or a virtual machine.

There exists a small open market for these software components. Most of them target desktop applications.

28.3.11.2.5 State of affairs

At this moment, the software industry does not apply modularization in a serious way. There exists no theoretical reason why modularization in software system generation cannot be as successful as the current modularization in hardware system generation currently is. However, effective modular software generation asks for a completely different way of software generation than is accomplished by the present software development industry.

Implementing proper modularization will offer chances to parties that are now excluded by the power of companies that control software development tools and software development processes. With the appropriate services in place, everybody who has access to a software component development environment can produce products that fill a market need. Future institutions that support software component development and component-based system assembly will help the component developer in marketing the created components. In that case, the current powers in the software industry will endanger losing market control. It is to be expected that they will battle to stay in control.

28.3.12 Coupling the market and the design and creation of software modules and interfaces

28.3.12.1 Standardization and marketing

Modularization asks for a dedicated and powerful standardization of specifications, interfaces, and coupling procedures. A globally accessible service must support the distribution of public documents. For example, dedicated web-based repositories may contain standardized and categorized specification documents that can be discovered by an appropriate search mechanism. The development tools must be able to access the specification contents contained in these documents. Another globally accessible service must support the gathering, the sale, and the delivery of the corresponding components. Both services must cooperate.

The tools and the services must intimately interact to enable the quick and efficient design of interfaces, components, and target systems. At the same time, the services must ensure that the intelligent property that is invested in the uploaded components keeps hidden from the public world. It must also be guaranteed that the component designers will get their rightful fee. It is very difficult to organize a properly controllable pay per copy of the components binary. It is suggested that the customers pay per project for each used binary.

28.3.12.2 Designing and generating components

The component designer collects the required interfaces from web-based or local repositories, or he designs one or more new interfaces. Then he designs and creates one or more components. He must test these thoroughly. When ready, he uses the components for local system design, or he packs one or more components into a package and sends this together with the appropriate documents to the institute that will market his products. The institute checks the contributions, and after a positive conclusion, the institute puts the binaries and documents in its banks. The institute will put the documents in the appropriate repositories where they become publicly accessible. Users of the components may buy the components from the institute. The institute will ensure the payment of the developer that has put the product in the bank.

28.3.12.3 Versions and diversity

Versions and diversity of components both impede and support the manageability of the system integration process. Therefore the number of versions must be limited. Diversity of components must be made manageable by reducing the number of supported platforms and by limiting the number of supported environments. Development and creation of close copies of existing components must be avoided. Breaking these rules can easily destroy the advantages of modular system design.

28.3.12.4 Hiding intelligent property

Hiding intelligent property that is invested in the design of the component is one of the most difficult points of software component technology. It can be arranged by power: excluding customers from future membership when they offense the 'rules.' Or it can be ensured by a combination of encryption en recompilation supported by hardware decryption. Every project gets its own encryption key. It must be ensured that a system designer can still use components that he himself has designed and created.

28.3.12.5 Automating system integration

The system integrator starts with collecting the required application components and with creating the necessary connection and scheduling scripts. The components are put in packages, and a project document defines the target. Because of the fact that at the start of the system integration, practically all relevant data are known, the system integration tool can automatically add a dedicated supporting operating system that includes automatic memory garbage collection. The retrieved component specifications suffice to enable the construction of skeleton systems. After linking, these skeleton components can already be tested. However, the 'empty' components do not produce much activity. During system development, the skeleton components can be replaced step by step by fully operational binaries.

28.3.12.6 Publishing

Publications related to modularization comprise specifications, market promotion media, and product quality comparison reports. The internal code of components is normally hidden. If the institution that designed the component wants this, it is possible to make this code public as part of the component specification.

28.3.13 A fully-fledged software components industry

28.3.13.1 Sketch

There exists no theoretical reason why proper modularization cannot be achieved for software as it is done for hardware. The realization of some aspects will be easier while the achievement of other aspects will be harder. It is easier to send software products over the internet. It is easy to search the document repositories of the component shops for interesting components and compatible interfaces. Using XML, it becomes feasible to automate the design and creation process that makes use of these web-based repositories, which contain machine-readable specification documents that describe components and interfaces. A local file-based equivalent of such a repository may store retrievals and new designs and serve both the system designer and the developer of the components. The repositories contain a search machine that looks for categorization terms that classify the specification documents for specific application areas. New designs can be uploaded to a central service that will check the information and store it in the worldwide accessible repositories. A webservice that acts as a dedicated web-based shop may offer the corresponding modules. In the background of the webservice, binary banks will hold the binaries of the modules. The webservice will use a dedicated money bank to support the financial part of its activity. Via the webservice, the component designers may upload their results to the central institution that will then market their products. Component development tools and system assembly tools interact with the repositories and the webservices to implement an integrated design, assembly, and marketing environment.

28.3.13.2 The demo

This is a very sketchy view of a possible implementation of an integrated software components creation and marketing system. In order to investigate the feasibility of this sketchy picture, a demonstration system is built that contains working versions of all important constituents.

The demonstration system supports:

- Embedded software and desktop software[1]
- Provide interfaces
- Require interfaces[2]
- Memory-mapped hardware interfaces
- Streaming interfaces
- Notification interfaces[3]
- Package of a coherent set of components[4].
- Components that consist of simpler components[5].
- Automatic creation of the supporting operating system from dedicated modules[6]

- Stepwise system build-up from a mix of skeleton components, partially functional components and fully functional components
- Automatic memory management
- System modes[7]

[1]In embedded software the generated system interacts directly with the hardware. The system assembly tool adds the HAL.

[2]Require interfaces are implemented as placeholders for special types that represent a reference to a provide interface.

[3]Notification interfaces accept hardware triggers.

[4]A package is a library of a coherent set of components. A component supplier will preferably deliver his products in the form of packages. A system designer will save his subsystems in the form of packages.

[5]A composed component is a dedicated package accompanied by a dedicated (fixed) connection scheme and a dedicated (fixed) scheduling scheme.

[6]In embedded software the system integration tool generates operating system modules in C++ source code. In desktop software, the system design tool generates a layer that interacts with the virtual machine. This layer is generated in source-code that corresponds with that virtual machine (C# or Java).

[7]System modes are controlled by connection schemes and scheduling schemes. Dynamic removal or creation of modules should be restricted to the instances where the system mode changes. Memory management is also restricted to these instances.

A standard RTOS schedules threads by stopping and starting routines. In a component-based environment, the real-time scheduler must stop, reset, and start modules. Eventually, the modules must be reconnected according to the currently valid connection scheme.

The demonstration system consists of the following components:

- An example of a web-based repository
    - This repository exists of a hierarchy of directories that contain
        - XML documents, which contain structured specifications. Each document contains a series of categorization tags.
        - XSD documents, which define the structure of the specifications
        - XSL documents, which help convert XML documents into humanly readable documents
    - The repository has a hierarchical structure. Components and interfaces are assembled in separate directories.

- The repository is publicly accessible. Using the XSL files the XML documents are humanly readable via a modern web browser.
- The repository contains a search machine that uses the attached category tags to find corresponding documents.

- An example of a local file-based repository
  - This repository exists of a hierarchy of directories and has the same structure as the web-based repository. This includes search capability.
  - The local repository contains a larger variety of documents than the web-based repository.
  - It acts as a local store for information that is retrieved from one or more web-based repositories.
  - It acts as a local store for documents that are prepared to be sent to a general institute that may put these documents on a web-based repository.
  - The XML documents specify:
    - Component
    - Interface
      - Require interface
      - Provide interface
      - SW/SW
      - HW/SW

        - Streaming
        - Notification
    - Types
        - Plain type
        - Enum type
        - Interface type
        - Sequence type
        - Structure type
    - Package description
    - Connection scheme
    - Scheduling scheme
    - Statechart
    - Project description

- An example of a webservice that may act as the representative of a central institute. This institution serves the community that creates or uses software components. Components may appear as packages of simpler components.
  - The institute owns a local repository that contains all specifications of interfaces that exist in the domain of the webservice.
  - The institute owns a binary database that holds the binaries of all available software components.

  - The institute owns a local repository that contains all specifications of software components that exist in the domain of the webservice.
  - The webservice uses the binary databases and the local repositories to serve the customers of the institute automatically. Customers have no direct access to these stores.
  - The webservice helps partners of the central institute to distribute documents to their specialized web-based repositories.
  - The webservice helps customers in buying software components and retrieving the corresponding binaries from the binary bank
  - The webservice helps software component developers to upload the binaries and corresponding specifications of their products.
  - The central institute takes care that the software component developers get paid for products that are downloaded via the webservice.
- A repository browser tool
  - The tool helps with searching local or web-based repositories for existing interfaces and components. Selected documents can be transferred from the web-based repository to the local repository.
- An interface and component design tool

- The tool helps with specifying new interfaces. This includes:
  - Software-software interfaces
  - Software-hardware interfaces
  - Streaming interfaces
  - Notification interfaces
- The tool helps in specifying other design documents that go into the repositories.
- The tool helps with searching local or web-based repositories for existing interfaces.
- The tool helps to design and to create the skeleton of a software component
- The tool helps with filling the skeleton with a dedicated code
- The 'internal' code is normally hidden. However, it is possible to make this code public with the rest of the specification.

- A system assembly tool
  - The tool helps with searching local or web-based repositories for existing software components. It can retrieve the corresponding binaries from web-based or local binary banks.
  - The tool can work with components that are still in skeleton form.

    - The tool can check whether components can fit together.
    - The tool assembles selected components and adds a dedicated component-based operating system.

Some hard rules must be obeyed.

- All components and all interfaces have a globally unique identifier.
- Any binary and any specification document that is uploaded to the central institute and that is accepted by this institute must never be changed or removed.
- New versions of an item are related to the previous version via a relation document that is attached to the specification document.
- The number of new versions of an item must not surpass 4.
- Close copies of items that are not new versions will not be accepted.

28.3.14 Code

The code is freely accessible at
http://www.scitech.nl/MyENHomepage.htm#software

29 References

[1] The Hilbert Book Model Project, https://en.wikiversity.org/wiki/Hilbert_Book_Model_Project

[2] ResearchGate project https://www.researchgate.net/project/The-Hilbert-Book-Model-Project

[3] ResearchGate site https://www.researchgate.net

[4] *E*-print archive http://vixra.org/author/j_a_j_van_leunen

[5] Private website http://www.e-physivs.eu

[6] Challenge http://www.e-physics.eu/#_Challenge

[7] Scientific method https://en.wikipedia.org/wiki/Scientific_method

[8] Quantum logic was introduced by Garret Birkhoff and John von Neumann in their 1936 paper.

G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Annals of Mathematics, Vol. 37, pp. 823–843 also see https://ncatlab.org/nlab/show/Hilbert+lattice

[9] Orthomodular lattice https://en.wikipedia.org/wiki/Complemented_lattice#Orthomodular_lattices

[10] projective geometry https://en.wikipedia.org/wiki/Projective_geometry

[11] Topological space, https://en.wikipedia.org/wiki/Topological_space

[12] Hilbert space https://en.wikipedia.org/wiki/Hilbert_space

[13] Vector space https://en.wikipedia.org/wiki/Vector_space

[14] Inner product https://en.wikipedia.org/wiki/Inner_product_space

[15] In the second half of the twentieth century Constantin Piron and Maria Pia Solèr,

proved that the number systems that a separable Hilbert space can use must be division rings.

See: https://golem.ph.utexas.edu/category/2010/12/solers_theorem.html

[16] “Division algebras and quantum theory” by John Baez. http://arxiv.org/abs/1101.5690

[17] Orthomodularity in infinite dimensions; a theorem of M. Solèr http://www.ams.org/journals/bull/1995-32-02/S0273-0979-1995-00593-8/

[18] Cartesian coordinates https://en.wikipedia.org/wiki/Cartesian_coordinate_system

[19] Polar coordinates https://en.wikipedia.org/wiki/Polar_coordinate_system

[20] Route https://en.wikipedia.org/wiki/Mathematical_formulation_of_quantum_mechanics

[21] Quantum field theory https://en.wikipedia.org/wiki/Quantum_field_theory

[22] Quantum electrodynamics https://en.wikipedia.org/wiki/Quantum_electrodynamics

[23] Quantum chromodynamics

https://en.wikipedia.org/wiki/Quantum_chromodynamics

also http://en.wikipedia.org/wiki/Yukawa_potential

[24] Principle of least action https://en.wikipedia.org/wiki/Principle_of_least_action

[25] Lagrangian equation https://en.wikipedia.org/wiki/Lagrangian_(field_theory)

[26] Path integral https://en.wikipedia.org/wiki/Path_integral_formulation

[27] Fock space https://en.wikipedia.org/wiki/Fock_space

[28] Tensor product of quaternionic Hilbert spaces https://arxiv.org/pdf/1101.5690.pdf

[29] A. Sudbery, Quaternionic analysis. *Math. Proc. Camb. Phil. Soc.* (1979), **85**, 199-225

[30] Del operator https://en.wikipedia.org/wiki/Del also see https://en.wikiversity.org/wiki/Hilbert_Book_Model_Project/Quaternionic_Field_Equations/Nabla_Operators

[31] Bo Thidé's EMTF book https://physics.bgu.ac.il/~gedalin/Teaching/Mater/EMFT_Book.pdf

[32] Vector calculus identities https://en.wikipedia.org/wiki/Vector_calculus_identities

[33] Dirac's function https://en.wikipedia.org/wiki/Dirac_delta_function

[34] http://www.physics.iitm.ac.in/~labs/dynamical/pedagogy/vb/3dpart2.pdf

[35] Wave equation http://en.wikipedia.org/wiki/Wave_equation#Derivation_of_the_wave_equation

[36] Helmholtz equation https://en.wikipedia.org/wiki/Helmholtz_equation

[37] https://en.wikipedia.org/wiki/Hyperbolic_partial_differential_equation#Hyperbolic_system_and_conservation_laws.

[38] Spherical Bessel functions https://en.wikipedia.org/wiki/Spherical_Bessel_Function

[39] Spherical harmonics https://en.wikipedia.org/wiki/Spherical_Harmonics

[40] Photons https://en.wikipedia.org/wiki/Photon

[41] Animation https://en.wikipedia.org/wiki/Circular_polarization

[42] Optical Transfer Function https://en.wikipedia.org/wiki/Optical_transfer_function

[43] Stokes theorem https://en.wikipedia.org/wiki/Stokes%27_theorem

[44] Gauss theorem https://en.wikipedia.org/wiki/Divergence_theorem

[45] Poisson point process https://en.wikipedia.org/wiki/Poisson_point_process

[46] Uncertainty principle https://en.wikipedia.org/wiki/Uncertainty_principle

[47] Entanglement https://en.wikipedia.org/wiki/Quantum_entanglement

[48] Atom https://en.wikipedia.org/wiki/Atom

[49] Molecule https://en.wikipedia.org/wiki/Molecule

[50] Consciousness https://en.wikipedia.org/wiki/Consciousness

[51] Intelligence https://en.wikipedia.org/wiki/Intelligence

[52] Dark energy https://en.wikipedia.org/wiki/Dark_energy

[53] Dark matter https://en.wikipedia.org/wiki/Dark_matter

[54] Space curvature https://en.wikipedia.org/wiki/Curved_space

[55] Mass https://en.wikipedia.org/wiki/Mass

[56] Higgs mechanism https://en.wikipedia.org/wiki/Higgs_mechanism

[57] Gravity https://en.wikipedia.org/wiki/Gravity

[58] Mass of a distribution of point masses https://en.wikipedia.org/wiki/Center_of_mass

[59] Gravitational potential https://en.wikipedia.org/wiki/Gravitational_potential

[60] Potential of a Gaussian charge density:

http://en.wikipedia.org/wiki/Poisson%27s_equation#Potential_of_a_Gaussian_charge_density

[61] Generations https://en.wikipedia.org/wiki/Generation_(particle_physics)

[62] “Neutrino Oscillations”;

http://www2.warwick.ac.uk/fac/sci/physics/current/teach/module_home/px435/lec_oscillations.pdf .

[63] “On Radical Ph-Solution of Number 3 Puzzle and Universal Pattern of SM Large Hierarchies“;

http://arxiv.org/abs/1212.1417

[64] https://en.wikipedia.org/wiki/Inertia

[65] Denis Sciama. (1953), On the Origin of Inertia.

http://adsabs.harvard.edu/abs/1953MNRAS.113...34S

[66] Lattice theory https://en.wikipedia.org/wiki/Lattice_(order)

[67] Complemented lattice https://en.wikipedia.org/wiki/Complemented_lattice

[68] Orthocomplemented lattice

https://en.wikipedia.org/wiki/Complemented_lattice#Orthocomplementation

[69] Distributive lattice https://en.wikipedia.org/wiki/Distributive_lattice

[70] Modular lattice https://en.wikipedia.org/wiki/Modular_lattice

[71] Orthomodular lattice https://en.wikipedia.org/wiki/Complemented_lattice#Orthomodular_lattices

[72] Atomic lattice https://en.wikipedia.org/wiki/Atom_(order_theory)

[73] Boolean algebra https://en.wikipedia.org/wiki/Boolean_algebra

[74] Propositional calculus https://en.wikipedia.org/wiki/Propositional_calculus

[75] Quantum logic https://en.wikipedia.org/wiki/Quantum_logic#Differences_with_classical_logic

[76] In 1843 quaternions were discovered by Rowan Hamilton.

http://en.wikipedia.org/wiki/History_of_quaternions

[77] Different number systems and their arithmetic capabilities are treated in

http://www.scorevoting.net/WarrenSmithPages/homepage/nce2.pdf.

[78] Paul Dirac introduced the bra-ket notation, which popularized the usage of Hilbert spaces.

Dirac also introduced its delta function, which is a generalized function.

Spaces of generalized functions offered continuums before the Gelfand triple arrived.

Dirac, P.A.M. (1982) [19[61]. Principles of Quantum Mechanics.

International Series of Monographs on Physics (4th ed.). Oxford University Press. p. 255.

ISBN 978-0-19-852011-5.

[79] Separable https://en.wikipedia.org/wiki/Separable_space

[80] Continuous function https://en.wikipedia.org/wiki/Continuous_function

[81] In the sixties Israel Gelfand and Georgyi Shilov introduced a way to model continuums,

via an extension of the separable Hilbert space into a so-called Gelfand triple.

The Gelfand triple often gets the name rigged Hilbert space. It is a non-separable Hilbert space.

http://www.encyclopediaofmath.org/index.php?title=Rigged_Hilbert_space

[82] Derivation of the Lorentz force,

https://en.wikipedia.org/wiki/Lorentz_force#Lorentz_force_and_Faraday's_law_of_induction

[83] Leibniz integral equation,

https://en.wikipedia.org/wiki/Leibniz_integral_rule#Threedimensional.2C_time-dependent_case

[84] Electromotive force, https://en.wikipedia.org/wiki/Electromotive_force

[85] Shaw, Justin (2014). Invariant Vector Calculus.

http://www.math.uwaterloo.ca/~j9shaw/Invariant%20Vector%20Calculus.pdf

[86] Black hole https://en.wikipedia.org/wiki/Black_hole

[87] Schwarzschild radius http://jila.colorado.edu/~ajsh/bh/schwp.html

[88] Schwarzschild radius https://en.wikipedia.org/wiki/Schwarzschild_radius

[89] Schwarzschild metric https://en.wikipedia.org/wiki/Schwarzschild_metric

[90] https://en.wikipedia.org/wiki/Birkhoff%27s_theorem_(relativity)

[91] Dirac equation Mathematical formulation

http://en.wikipedia.org/wiki/Dirac_equation#Mathematical_formulation

[92] http://www.mathpages.com/home/kmath654/kmath654.htm; equation (6)

[93] http://en.wikipedia.org/wiki/Pauli_matrices

[94] https://nl.wikipedia.org/wiki/David_H._Hubel ;
https://nl.wikipedia.org/wiki/Torsten_Wiesel

The book acts as a survey of the Hilbert Book Model Project. The project concerns a well-founded, purely mathematical model of physical reality. The project relies on the conviction that physical reality owns its own kind of mathematics and that this mathematics guides and restricts the extension of the foundation to more complicated levels of the structure and the behavior of physical reality. This results in a model that more and more resembles the physical reality that humans can observe.

The book is written by a retired physicist.

Hans van Leunen MSc

He started the Hilbert Book Model Project when he was 70 years.

To feed his curiosity, Hans dived deep into the crypts of physical reality. He detected that more than eighty years ago, two scholars already discovered a suitable foundation of a mathematical model of the structure and behavior of physical reality. They called their discovery quantum logic. The Hilbert Book Model Project explores this foundation by extending this structure to higher levels of the structure of the model and adds dynamics to the Hilbert Book Base Model.

This approach is unorthodox and unconventional. It enters an area where many aspects cannot be verified by experiments and must be deduced by trustworthy mathematical methods. In this way, the project discovered new mathematics and new physics.

The Hilbert Book Base Model appears to offer a very powerful and flexible modeling environment for physical theories.

The model extensively applies quaternionic Hilbert space technology, and quaternionic integral and differential calculus. The project extensively exploits the capabilities of the existing versions of the quaternionic number system.

The project explores the obvious modular design of the objects that exist in the universe.

In contrast to mainstream physics the Hilbert Book Model applies stochastic processes instead of forces and force carriers to control the coherence and the binding of modules.