Causality Between Events with Space-Like Separation

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Abstract

There was a young lady named Bright
Whose speed was much faster than light.
She left one day, in a relative way,
And returned the previous night.
– attributed to A. H. R. Buller, 1923

Since the first part of the twentieth century, it has been maintained that faster-than-light movement could produce time travel into the past with its accompanying causality-violating paradoxes. This paper demonstrates that this assumption is false because of the completely unsubstantiated belief that the past is "back there somewhere." The Lorentz transformation (LT) and the Minkowski diagram based upon it presume that time is isotropic, whereas entropy and the arrow of time govern in the real world.

1 Introduction

G. Feinberg coined the name "tachyon"[1] for a particle that always travels faster than light, satisfies the principle of relativity and is Lorentz-invariant. The limiting value is c, but, as Feinberg points out, a limit has two sides. Recent measurements of tritium decay[2] offer some evidence that $m^2 = -0.6\ eV^2/c^4$ for neutrinos, indicating that they may be tachyons. Substantial error bars in the measurement, however, provide only weak affirmation for tachyonic neutrinos, but other possibilities exist for getting from point A to point B faster than light can do it. The purpose of this paper is to investigate whether or not such processes would violate known physics or causality.

In 1907 A. Einstein considered it to be "sufficiently proven" that any velocity greater than that of light is an impossibility[3] by analysis of relativistic velocity composition and the Lorentz transformation equation for time. Given an inertial frame moving at velocity v with respect to a "stationary" frame, the time differential over a distance Δx is

$$\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2}) \tag{1}$$

He concluded that for Δt less than $v\Delta x/c^2$, $\Delta t'$ would be negative, implying that any such speedy object would arrive at its destination before it departed from its origination point. Similarly, R. C. Tolman pointed out in 1917 that ve-

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[1] G.Feinberg, Physical Review, 159, (5): 1089-1105 (1967)

[2] $C.Krauset\ al, Euro.Phys.J.C, 40 : 4(2005, Pp447 -$ 468

[3] A. Einstein, Jaharb. Radioakt. Elektron. 4, 411 (1907)

locities greater than the speed of light presented the possibility that effect could precede cause.[4]

The assertion that causality can be violated by faster-than-light travel is also mainstream thought in this century. N. D. Mermin[5] wrote, "In the [moving] frame the object is in two different places at the same time! This is such a bizarre situation that ones suspicion is strengthened that the difficulty we have already encountered in producing an object moving faster than light must be a reflection of the impossibility of such motion." This is another aspect of a causality violation, but perhaps the "impossibility" is not in the movement of such an object but, rather, in insisting that the LT in its temporally isotropic form is superior to the reality of our world, which is governed by entropy and the "arrow of time."

The purpose of this paper is to demonstrate that the minus sign in Equation (1) should be interpreted as setting a limit on speeds observed in relatively-moving inertial frames. When $\Delta t = v\Delta x/c^2$, $\Delta t' = 0$, thus the velocity, $u' = \Delta x'/\Delta t'$, of an object so described will be infinity in one frame but c^2/v in a different frame, where v is the velocity difference between the two frames. This prevents the bizarre absurdities of going backward in time and bringing multiple objects into existence which are purported to occur with superluminal movement.

The Minkowski diagram will be discussed in Section 2, Section 3 will present the case for causality at superluminal speeds and Section 4 will address relativistic velocity composition and how it prevents, or at least warns against, causality violation.

2 The Minkowski Diagram

The Minkowski diagram is a simple time-position representation of a stationary frame with a moving frame, determined from the Lorentz transformation equations, superimposed upon it. Consequently, all time and position values are viewed from the stationary frame, not the moving frame. "Moving" and "stationary" are completely arbitrary, but we will call the "stationary" frame the one in which observers A and B are at rest and the "moving" frame the one in which observers C and D are at rest.

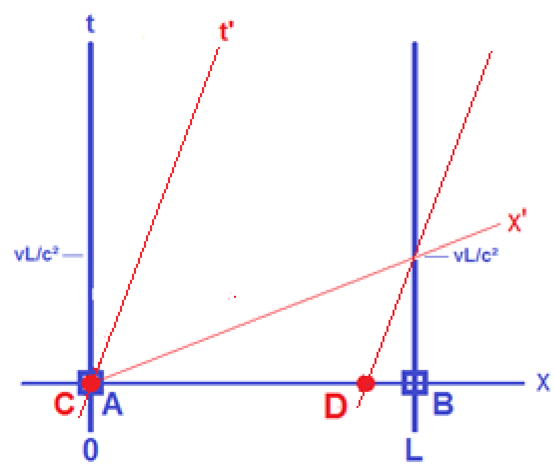

Figure 1: Typical Minkowski Diagram with Positions of Observers A, B, C and D Shown at time t = 0.

Figure 1 shows a typical Minkowski diagram, a graphical representation of the Lorentz transform. The x = 0 and x = L vertical lines represent the trajectories, or "world lines" of A and

[4] $R.C.Tolman, The Theory of Relativity of Motion (Berkeley, California, 1917), p.54$

[5] N. D. Mermin, Its About Time, (2005), pp. 53-54.

B, stationary observers in the stationary frame (shown in blue). Note that A and B lie along the x-axis, which defines that the time at both A and B is zero. Clocks at A and B will both read zero if they have been synchronized and are in agreement with the diagram.

Observers C and D are moving at some velocity, v, with respect to A and B, where v is less than c. The axes of the moving frame, x′ and t′, are tilted with respect to the stationary frame, the t′ axis of the moving frame being defined by $t = vt$ and the x‘ axis being defined by $t = vx/c^2$, where t and x are coordinates of the stationary frame. We may assume that clocks at C and D have been synchronized in the moving frame, whose t′ = 0 points lie along the x′ axis, but they are *not* synchronized with respect to the stationary frame.

The trajectories of C and D as shown in the stationary frame are defined by $x = vt$ and $x = L(1 - v^2/c^2) + vt$, respectively, and where, once again, x and t are coordinates of the stationary frame. The positions of A, B, C and D at t = 0 are shown in Figure 1. All observers advance along their trajectories as t advances.

According to the conventional view, observers A and C are also (still) back at t = 0, as depicted in Figure 2. This assumes that the past is somehow real and accessible. According to this view, B originates a signal and transfers it to D at time $t = vL/c^2$ when they are adjacent at Event E1, then D transfers the signal to C instantaneously in their moving frame, as shown by the downward-sloping black arrow. Since $t_D{}' = 0$ when $t = vL/c^2$, it should arrive when $t_C{}' = 0$, which is when t = 0 and x = 0 at Event E2. Thus when A sends the signal back to B in zero time, it arrives there at t = 0 (Event E3), before B sends it at Event E1. This means that B at $t = vL/c^2$ could not have originated the signal in the first place, hence, a causality violation.

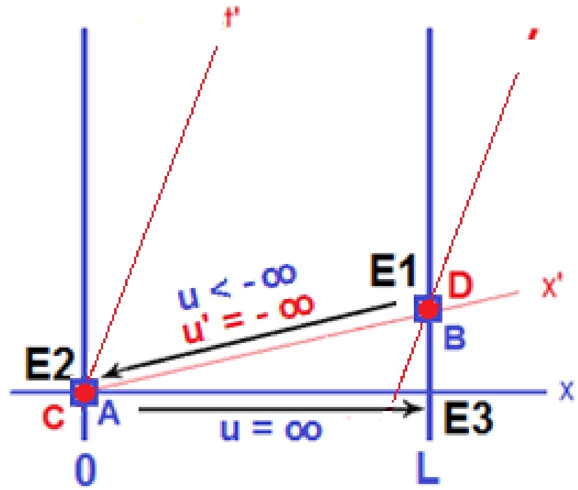

Figure 2: Typical Minkowski Diagram with observers A and C at t = 0 and B and D at t = vL/c2, Showing Purported Causality Violation. A, B, C and D are assumed to have some technology that allows instantaneous communication.

The arrow in Figure 2 labeled "$u < -\infty$" is not an error. In Minkowski diagrams, velocity is represented by an angle. Figure 3 depicts angles for speeds varying from small to large as the angle increases counterclockwise. Infinite speed is represented by a horizontal line (a distance displacement in zero time). The relationship of the speed of an object to the angle is represented as Θ = arctan(u/c), which is greater than π radians when both u and c are in the negative direction. The mathematics of this is somewhat misleading because u/c = tan(Θ) results in a positive value when u and c are both negative. If a signal arrives at its destination in some positive time, its speed is less than infinite, and if it arrives in zero time, its speed is infinite, but if it arrives before it is sent from its source, its speed is greater than infinite! That is illogical since infinity is greater than any number; hence, going backward in time is also illogical.

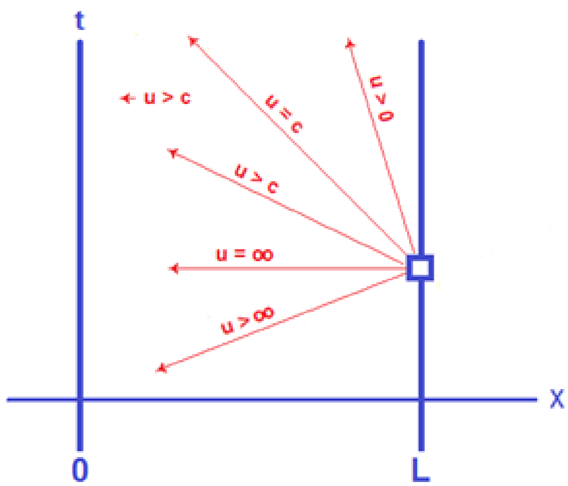

Figure 3: Minkowski Diagram Showing Directions of Various Speeds.

All experimental evidence says that only the present exists: no one has sent a signal or other object into the past. All claims to the contrary are in the domain of science fiction fantasies. Thus D cannot send a signal back to x = 0, t = 0 in the stationary frame at a speed greater than infinity in any frame.

In the mainstream view of Figure 2, advancement in time of A and C has been suppressed while B and D have been advanced, That model makes the unwarranted assumption that the past actually exists, whereas there is absolutely no evidence that it does. In fact, all evidence points to the past only existing in memory of one kind or another (rocks, tree rings, neurons, tablets, silicon, etc.). This agrees with the philosophy of the ontology of time called presentism[6] and is adamantly opposed to the block universe concept.[7] The philosophy of presentism will be regarded as a theory for the analysis of causality and faster-than-light motion in this paper. Furthermore, the clock at A reads t = 0, but the clock at B reads $t = vL/c^2$ in Figure 2. This violates the premise discussed previously that clocks in the same frame have been synchronized. B cannot advance from t = 0 unless A also advances, and by the same amount.

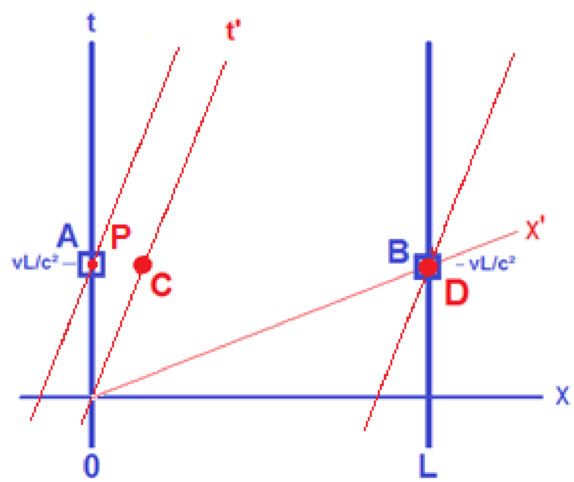

Figure 4: Proper Minkowski Diagram of Observers in Figure 1 at t = vL/c2 According to the Theory of Presentism.

According to the theory of presentism, the proper Minkowski diagram is shown in Figure 4. Observers A and C are no longer at the

[6] $https://plato.stanford.edu/entries/presentism/$
[7] $https://en.wikipedia.org/wiki/Growing_block_universe$

origin since they should also have advanced to $t = vL/c^2$ along with B and D.

3 The Case for Causality

Special relativity is an excellent model of reality within its proper domain, which is in the absence of significant gravitational effects. However, causality appears to be violated when superluminal phenomena are addressed, and the conventional view is that this strongly supports the position that such phenomena are not part of reality, that faster than light communication is impossible. However, "impossible" has been claimed in the past for phenomena that were later confirmed by experiment, and this may well be the case for superluminal phenomena. The purported causality violation produced by such phenomena appears to be an artifact of the chosen philosophical model of time, as demonstrated in the previous section.

As Figure 4 shows, The clocks of observers A and C are no longer at t = 0 when the clocks of observers B and D have advanced to $t = vL/c^2$. The claim that the clocks of A and C are still at t = 0 when B and D have advanced is responsible for the conclusions Mermin found so bizarre, not superluminal motion. In the frame where A and B are stationary, which is the perspective from which Figures 1, 2 and 4 are viewed, when D is at $t = vL/c^2$, C is also, as shown in Figure 4.

Another small problem with Figure 4 is also apparent: At $t = vL/c^2$, A and C are no longer adjacent. This is minor, however, since another observer can be presumed, call it P, in the moving frame to the left of C such that it will intersect x = 0 at point $t = vL/c^2$ (as shown in Figure 4.

4 Relativistic Velocity Composition

When a frame moving at velocity v with respect to a stationary frame sends out a signal or object at velocity u'(with respect to the moving frame), the velocity of said signal or object with respect to the stationary frame is[8]

$$u = \lim_{u' \to \infty} \frac{u' + v}{(1 + \frac{uv}{c^2})} = \frac{c^2}{v} \tag{2}$$

This equation demonstrates the invariance of c, presumed to be the speed of light. If either u'or v are equal to c, u will also be equal to c. What is not always recognized, however, is that although the derivation of Equation (2) limits the relative velocity between inertial frames of observers to those less than c, the speed of an observed phenomenon may be faster than that of light, and that what is infinite velocity in one frame is not infinite in a different frame. As Feinberg said, a limit has two sides.[1] Suppose, for example, $u'$is allowed to go to infinity. Then

$$u = \frac{c^2}{v} \tag{3}$$

This may have a significant impact on purported demonstrations of causality violation in the literature.[3,4,5,9] These scenarios involve signals exceeding the speed of light in both directions along the spatial axes, and they assume that the value of any faster-than-light speed can exceed infinity. Equations (2) and (3) weaken this assumption. These equations are derived from $u'$and v both being in the positive direction.

[8] J. D. Jackson, Classical Electrodynamics, (1965), p.361

[9] $http : //www1.phys.vt.edu/\ takeuchi/rela-tivity/notes/section10.html$

Because space is homogeneous and isotropic, we know that when u' and v are both in the negative direction, Equation (3) is also correct when u, u' and v are replaced by their negative values, and inspection shows that this is true.

Besides the scenario where both u' and v are in the same direction, designated Case I, there is Case II where u' and v are in opposite directions. Equation (2) can be converted to Case II by replacing u and u' by their negative values:

$$-u = \frac{-u' + v}{(1 - u'v/c^2)} \tag{4}$$

Positive values for u and u' in Equation (4) mean velocities going to the left in standard graphical formats.

Since the denominator of the right side of Equation (4) represents time, it must not be negative else it disagrees with presentism and violates causality. As it stands, Equation (4) says that u becomes infinite when $u' = c^2/v$. The scenario under discussion in this paper asks for the value of u when u' goes to infinity, but u goes beyond infinity according to Equation 4. When this equation is rearranged, the fallacy of such a possibility is demonstrated:

$$u' = \frac{u + v}{(1 + uv/c^2)} \tag{5}$$

Equation (5) clearly shows that when u grows without limit,

$$u' = \frac{c^2}{v} \tag{6}$$

Recall that positive values of u and u' represent negative velocities and are consistent with the scenario under consideration, which is analyzed in detail in Appendix A.

The complete problem is depicted in Figure 5. The signal is transferred from B to D at $t = vL/c^2$ and D transfers it to P in the moving frame, arriving when P is at $t = vL/c^2$ in the stationary frame, as depicted by the leftward-going horizontal arrow. The signal is transferred to A, which is adjacent to P, and then from A to B instantaneously. Thus the signal arrives back at B at the same time it was originally sent and there is no backward-in-time anomaly and no causality violation.

Figure 5 describes the situation when the signal is instantaneous from the perspective of the stationary frame.

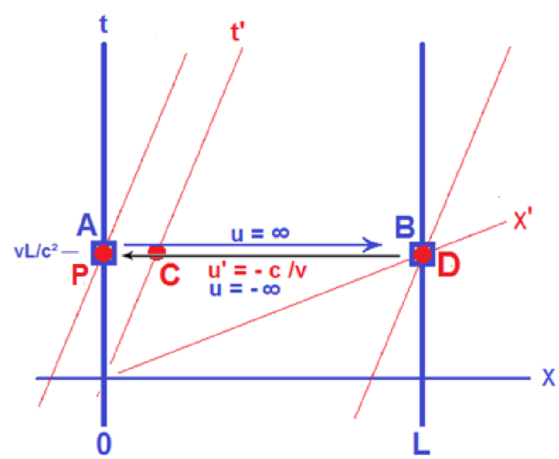

Figure 5: Minkowski Diagram without the Canard of Negative Time.

The relativistic velocity composition equation for the signal transfer from D to C has two portrayals. One is where u' is the dependent variable, as shown in Equations (5) and (6) and the other is with u as the dependent variable, as shown in Equation (4). In Equation (4) there is the temptation to allow u' to go to infinity, which leads to the unphysical anomaly of time reversal and its illogical consequence of observing velocities beyond infinity from the stationary frame. Equation (5) does not encourage this fallacious impulse and thus represents a more rational perspective.

5 Issues

Application of relativistic velocity addition and rejection of time reversal phenomena to superluminal communication leads to some potential problems. One such is the $u = c^2/v$ equation, when turned around to $v = c^2/u$, seems to indicate that there is only one inertial frame where a given superluminal speed can be observed. What it actually says is that there is only one frame where an infinite superluminal velocity can be observed. The velocity u will be less than that in all other frames.

Suppose that a superluminal velocity of u = 10c has been confirmed in laboratory experiments. When viewed in the light of presentism, the equation

$$u' = \frac{10c - v}{(1 - 10v/c)} \tag{7}$$

seems to imply that inertial frames moving at v greater than 0.1c are impossible, but this is not so. It actually says that there is one frame where that 10c is observed to be infinity, and from that frame, the speed in all other frames is less than infinity. Using this frame, the superluminal speed in all other frames can be calculated, even for those that are moving faster that 0.1c relative to the laboratory frame.

Consider now that an infinitely fast signal is sent from A at x = 0, t = 0 and arrives at B at x = L, t = 0. An observer C moving at velocity v passes A as the signal leaves and measures that signal as $u' = c^2/v$, so it seems that it would arrive at L at a later time, possibly setting up a conflict with other events; however, this is not the case. A moving observer D (stationary with respect to C) passes B as the signal arrives:

$$t = \gamma(t' - vL/c^2) \tag{8}$$

Since $t' = L/u'$, then Equation (8) yields t = 0 (i.e., D arrives at L when the clock at B reads zero), so there is no conflict even though the clock with D does not read zero.

6 Conclusion

Relativistic velocity composition is a valid consequence of the Lorentz transformation equations. Equation (2) clearly speaks to the fact that if a speed, u', in the moving frame grows without limit, the speed in the stationary frame is observed to be c^2/v. However, simply rearranging Equation (2) so that u' is the dependent variable

$$u' = \frac{u - v}{(1 - uv/c^2)} \tag{9}$$

seems to allow the possibility of causality violation, but this is false by the theory of presentism. Clearly, limiting u to c^2/v retains the context of Equation (2), and this is consistent with the theory of presentism and the rejection of speeds greater than infinity, and demonstrates that superluminal signals cannot result in causality violations.

Consequently, causality violation as a disproof of faster-than-light speeds is a canard. The rule is that when time (that is, the denominator of the relativistic velocity composition equation) becomes negative, one must resist the impulse to press further in that direction. This is because, contrary to the symmetry of time in the Lorentz transform, time in the real world is not symmetrical. Rather, it is anisotropic, since it is restricited by entropy and the arrow of time. This results in the unusual situation that infinite speeds are possible only under certain conditions. Of course, there is no solid experimental evidence at present for faster-than-light phenomena, but if and when it becomes reality, we need not worry that our past histories can be altered or erased.

7 Appendix

The parameters for Case II are

$$
\begin{aligned}
&x_A = 0\\
&t_A = L/(-u) + vL/c^2\\
&x_B = L\\
&t_B = vL/c^2\\
&x_P{}' = \gamma(0 - vt_A)\\
&t_P{}' = \gamma[L/(-u) + vL/c^2]\\
&x_D{}' = \gamma(Lvt_B)\\
&t_D{}' = \gamma(t_B vL/c^2) = 0
\end{aligned}
$$

where the moving frame moves to the right and u is the velocity of an object as observed in the "stationary" frame moving to the left (represented by a negative number). The velocity of the object in the moving frame is

$$u' = \Delta x'/\Delta t' \qquad \text{(A-1)}$$

$$u' = (x_P{}' - x_D{}')/(t_P{}' - t_D{}') \qquad \text{(A-2)}$$

$$\Delta x' = \gamma[-vL/(-u)L] \qquad \text{(A-3)}$$

$$\Delta t' = \gamma[L/(-u) + vL/c^2] \qquad \text{(A-4)}$$

$$u' = [-vL/(-u)L]/[L/(-u) + vL/c^2] \qquad \text{(A-5)}$$

$$u' = -[(-u) + v]/[1 + (-u)v/c^2] \qquad \text{(A-6)}$$

where (-u) will be a positive number. Letting (-u) grow without limit,

$$u' = -c^2/v \qquad \text{(A-7)}$$

8 Acknowledgments

The author thanks R.T. Longo for helpful discussions and members of the Google sci.physics.relativity discussion group for ardently presenting the mainstream view.