# An Event Graph in Special Relativity

(v3, 2022-12-22)

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**Abstract.** We present a graph to illustrate a chain of events within the theory of special relativity. This spacetime diagram provides both the so-called *proper time* and *calendar time,* in addition to one space parameter. All parameters of the graph are given in time units, and we also relate the graph to a time vector in two dimensions. The graph depicts objects (events) having a piecewise constant velocity. It is exemplified by various versions of the 'traveling twin paradox'.

*Keywords*: Spacetime diagram, event graph, time vector, internal clock, traveling twin, simultaneity, μ-mesons.

## 1 Introduction

Previously we have introduced a time vector in the theory of special relativity (TSR); *cf.* Hokstad (2018, 2019). Here we build on this work and present a graph to describe the movements of an object with piecewise constant velocity. In its simplest form, this was also suggested by Epstein (1987).

This 'event graph' is a spacetime diagram, analogous *e.g.* to the 'worldline' of Minkowski; *cf.* Petkov (2012). However, we use the so-called 'proper time' and the position as the two main parameters of the diagram; also providing the 'calendar time'. Starting out from the time vector, we give all these parameters in time units.

We apply the approach to various versions of the 'traveling twin' example, providing the basis for a short discussion on the simultaneity of events. The case of the μ-mesons is also used as an illustration, leading to a comment on the interpretation of this case.

## 2 Basic notation and the time vector

We start out from the following basic situation: There is a Reference Frame (RF), denoted *K*, having one space coordinate, *x*. This has synchronized, stationary clocks located at virtually any position. Further, there is an object moving at velocity, *w* relative to *K,* along the *x*-axis. The object starts out from the origin of *K,* when the clock at this position on *K* reads 0. All events are given relative to this 'point of initiation' (*p.o.i.*).

Also (we imagine that) this moving object brings with it a clock, and when the object passes the origin at time 0, this clock is synchronized with the clock on *K* at this position.

We introduce three fundamental parameters related to the movement of this object. First:

$\tau$ = Clock reading of the clock following the moving object. We would say that this is the 'internal time' of the object/event, but usually, this is referred to as the 'proper time'.

Further, we have two parameters (*t, x*), which specify an event on the chosen RF, *K*:

$x$ = Position of the moving object relative to *K,* (when the passing clock reads $\tau$);

$t$ = Clock reading of the clock permanently located at position $x$ on *K*, when the moving clock reads $\tau$; (this *t* is usually referred to as 'calendar time').

It is well-known that the proper time, $\tau$ is independent of which RF we choose as the basis for our observations. So we consider this clock reading, $\tau$ of the embedded ('internal') clock as the most fundamental parameter.

In contrast, the $(t, x)$ parameters, will of course depend on which RF we have chosen for describing the observations. Now, we also have the relation

$w = x/\, t$, *i.e.* the velocity of the object relative to the RF, *K*.

Further,

$c$ = speed of light,

Then - according to the so-called *time dilation* in TSR, we have (*e.g.* Mermin (2005)) that:

$$\tau = \sqrt{t^2 - (x/c)^2} = t\sqrt{1-(w/c)^2} \tag{1}$$

We proceed to introduce a time vector related to an event ($t$, $x$):

$$\vec{t} = \begin{pmatrix} \tau \\ x/c \end{pmatrix} = \begin{pmatrix} w/c \\ \sqrt{1-(w/c)^2} \end{pmatrix} t \tag{2}$$

The absolute value of this vector equals

$$\left|\vec{t}\right| = t = \sqrt{\tau^2 + (x/c)^2} \tag{3}$$

We note that all the three parameters τ, $t$ and $x/c$ represent time. In particular, $x/c$ equals the time required for light to traverse the distance $x$ from the origin to the location of the event. We can also see $t$ as the event's total 'distance' (in time) from the given *p.o.i.*, where *eq.* (3) represents a decomposition of this $t$ into its two components, $\tau$ and $x/c$.

This time vector, (2), actually comprises the information of all the above four fundamental parameters of an event. Obviously, we can formulate the time vector $\vec{t}$ as a complex variable, *e.g.* Hokstad (2019). Here we just point out that we may also define an angle, $\varphi_w \in [-\pi/2,\ \pi/2]$ by

$$\sin\varphi_w = \frac{w}{c} = \frac{x/c}{t} \tag{4}$$

## 3 A simple event graph

We now introduce a graphical approach to describe the series of events, defined by this moving object.

### 3.1 Specification

Fig. 1 provides a graphical illustration of the four parameters: $\tau$, $x/c$, $t$ and $w$, related to a specific event. The absolute value (length) of the time vector, $\vec{t}$, provides $t$. The slope of the line provides $\varphi_w$ and, thus, also the velocity, $w$. Now the two components (abscissa and ordinate) of the time vector are given as see *eqs*. (1), (4):

$$\tau = t\sqrt{1-(w/c)^2} = t\cos\varphi_w$$

$$\frac{x}{c} = \frac{wt}{c} = t\sin\varphi_w$$

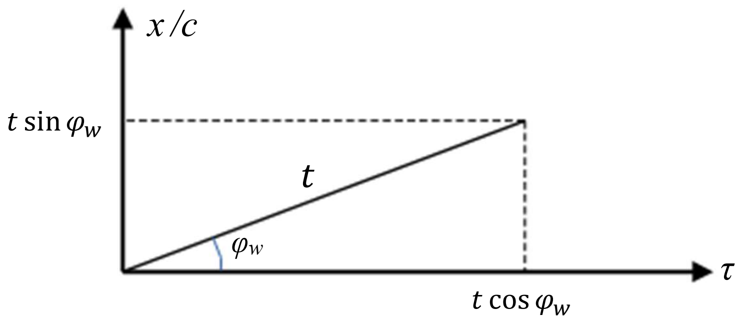


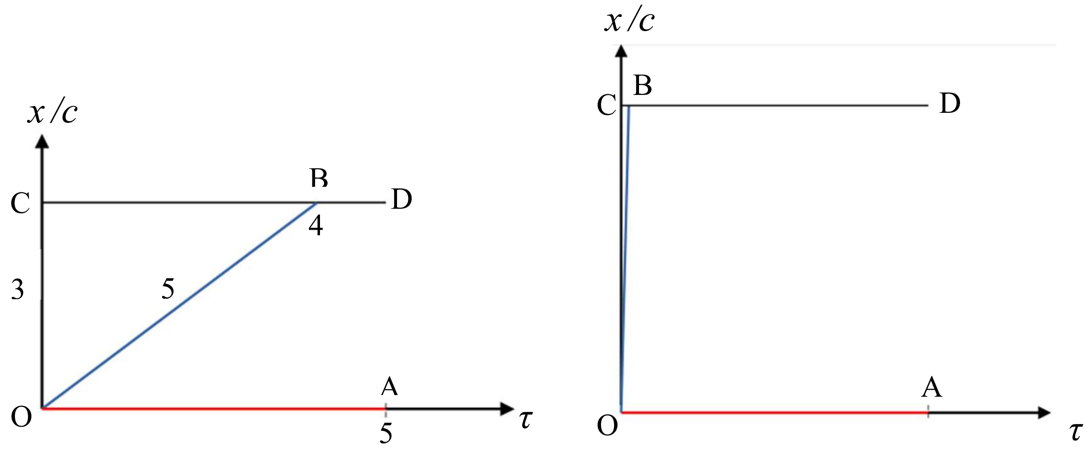

**Figure 1. An 'event graph': A chain of events, $(t, x)_{\tau\geq 0}$ on a specific RF; describing an object of a constant velocity, $w$ relative to this RF.**

Thus, by specifying the object's velocity, $w$ (relative to our RF) and its proper time, $\tau$, the graph will directly give the corresponding event parameters, $(t, x)$ relative to the chosen RF.

Now, in addition to illustrate a specific time vector, this line also illustrates the whole chain of events, representing the object's movement from time 0. We denote this an '*event graph*'.

### 3.2 Examples

We illustrate the above graph with a couple of standard examples.




a) Travelling twin (three event graphs)   b) My-meson (three event graphs)

**Figure 2. Simple examples of the event graph.**

*Example* 1. *The traveling twin.*

We apply a standard numerical example, as given for instance in Mermin (2005). Here the travelling twin leaves the earth in a rocket of velocity, $w = 0.6c$. This gives $\sqrt{1-(w/c)^2} = 0.8$. He travels to a 'star' located 3 light years from the earth. This means that $x/c = 3$ years. Further, the RF of the earth has a synchronized clock located at the star, and by the arrival of the twin to the star, this clock will now read,

$$t = x/w = 3c/0.6c = 5 \text{ years.}$$

At this instant the travelling twin's clock will - according to the TSR, (see equation (1)) – read,

$$\tau = t\sqrt{1-(w/c)^2} \ = \ 5 \cdot 0.8 = 4 \text{ years.}$$

Thus, by the arrival of the rocket to the star we have $\tau = 4$ years, $x/c = 3$ years, $t = 5$ years. Or, expressed by the time vector related to this even, (see the blue line *OB* of Fig. 2. A)):

$$\overrightarrow{t_{OB}} = \begin{pmatrix} \tau \\ x/c \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \text{ with } |\vec{t}| = t = 5.$$

Here *O* represents the twins' start of the travel, and *B* the event of his arrival to the star. Thus, the line OB also represents the twins' travel *to* the star.

In addition, we have in this figure inserted two other event graphs. First, the red line *OA*, illustrating the the clock of the brother who remains on the earth, (thus, having $x/c \equiv 0$); and finally, the line *CD,* illustrating the (imagined) clock permanently situated on the star.

So, in this illustration, it is essential that all the three event graphs are related to the same RF, (that of the earth). In this perspective, all three event chains start 'simultaneously' with $t = 0$, (at events *O* and *C* .respectively).

So, in the perspective of this RF, it is fair to say that the figure gives a snapshot of three time vectors, $\overrightarrow{t_{OA}}$, $\overrightarrow{t_{OB}}$ and $\overrightarrow{t_{CD}}$ representing three 'simultaneous' (chains of) events.

*Example* 2. *The My-mesons.*

For instance, Mermin (2005) refers to this standard example essentially as follows: The μ-mesons are produced by cosmic rays in the upper atmosphere. When 'at rest' they have a lifetime of about 2 microseconds. So if their internal clocks ran at a rate independent of their speed, about half of them would be gone after they had traveled 2.000 feet. Yet about half of the μ-mesons produced in the upper atmosphere (about 100.000 feet up) manage to make it all the way down to the ground. This is because they travel at speed so close to the speed of light that they can survive for 50 times as long as when they are stationary.

Describing this phenomenon in the perspective of (the RF of) the earth, the situation is quite similar to that of *Example* 1; and is illustrated in Fig. 2 b). The creation of the μ-mesons in the upper atmosphere is event O, ($x/c = \tau = 0$). The blue line *OB* represents the movements of the my-mesons, and by the arrival to the earth (*B*), the τ value of the my-mesons equals (*cf. CB*) $\tau = 2 \cdot 10^{-6}$ sec.

Further, the line *CD* is the event graph of the clock located on the earth, having the duration $t \approx 2 \cdot 10^{-6} \cdot 50 = 10^{-4}$ sec. This equals the distance *OB*. Actually, also the distance traveled by the μ-mesons, (*OC*), equals $x/c \approx 10^{-4}$ sec.

I think this case is interesting, as there are different ways to describe/interpret these results. Mermin (2005) says: "The atomic particles can go much further because their internal clocks that govern when they decay are running much more slowly in the frame in which they rush along at speed close to *c*. This is a real effect. .... ". When he refers to the 'internal clock', I assume this should be interpreted as the imagined clock, following the mesons, and reading the 'proper time', τ.

And obviously, the lifetime given by this τ-value is much less than the one given as the *t* – value observed by the clocks of the earth's RF. However, the τ-value is independent of the RF being applied by the observer, (*cf.* Example 3 below)! So, why say that the 'internal clock' is running more slowly in this situation? Actually, various observers will get different results for the life time of the mesons, simply because they move at different speeds relative to the phenomenon. The inner clock, however, is not affected by the velocity. For instance, Serret (2018) is in favor of this way to interpret the phenomenon. The question is rather, shall we describe this as a true physical phenomenon, or is it rather an 'observational phenomenon'.

*Example* 3. *Equivalent events. The Lorentz Transformation.*

Contrary to all the other examples, here we do *not* take the perspective of a specific RF. We consider *one* specific event, and illustrate how this is specified by different RFs, moving relative to each other. In other words, we illustrate the Lorentz Transformation (LT), but involving an arbitrary number of RFs.

We start with specifying an event having an 'internal clock' reading $\tau$; (in previous works we have denoted these clocks for 'Basic Clocks'). For simplicity, we let this clock be located at the origin of an RF, *K*, having velocity 0. Further we have, for any *w,* an RF, $K_w$, which has velocity *w* relative to *K*. Furter, let ($t_w$, $x_w$) be the time and space coordinates on $K_w$ for the specified event. Thus $x_w = w\, t_w$, and according to standard theory, (see *eq*. (1)):

$$\tau = \sqrt{t_w^2 - (x_w/c)^2} = t_w\sqrt{1 - (w/c)^2}$$

This relation is valid for all *w*, and is illustrated in Fig. 3. Here *OA* represents the time vector of the 'internal clock'; (located at the origin of *K,* having velocity 0 and $t = \tau$). Further, the blue vectors represent the time vectors of the same event, relative to some arbitrary RFs. So, *OB* exemplifies the time vector for this event, as specified on an RF moving with velocity *w* relative to *K* (with the 'internal clock' of the event).

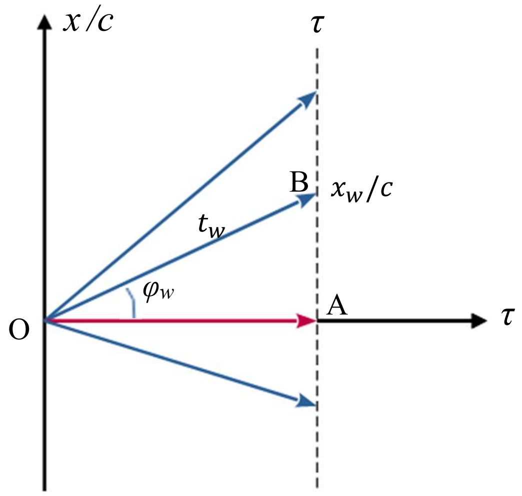



**Figure 3. Diagram illustrating the Lorentz transformation (for any number of RFs).**

The point is that – given the $\tau$ of the event – we just determine $\varphi_w$ from (*eq.* (4)), that is $\sin\varphi_w = w/c$, and then directly observe $x_w$ and $t_w$ from the figure. So, this is similar to Fig. 1, In Fig 3, however, $w$ is the velocity of the RF relative to the 'internal clock' of the event, and corresponds to -$w$ in Fig. 1.

Note that if we have two events $(t_{w_i}, x_{w_i})$, $i$ = 1, 2, corresponding to this $\tau$, it follows that

$$\tau = t_{w_i}\sqrt{1-(w_i\,/c)^2} = t\cos\varphi_w\,,\ i = 1, 2$$

$$\frac{x_{w_i}}{c} = \frac{w_i t_{w_i}}{c} = t\sin\varphi_w,\ i = 1, 2$$

We note that this gives the following relation between $(t_{w_1}, x_{w_1})$ and $(t_{w_2}, x_{w_2})$

$$t_{w_2} = \frac{\sqrt{1-(w_1\,/c)^2}}{\sqrt{1-(w_2\,/c)^2}}t_{w_1}$$

$$x_{w_2} = \frac{w_2\sqrt{1-(w_1\,/c)^2}}{w_1\sqrt{1-(w_2\,/c)^2}}x_{w_1}$$

So, this represents an alternative formulation of the LT. Now the velocity, $v$ between the two RFs are given by a standard formula in TSR, *e.g.* see Mermin (2005):

$$v = \frac{w_1 - w_2}{1 - w_1 w_2/c^2}$$

## 4 Generalizations

We now look at a couple of generalizations of the above approach.

### 4.1 An event graph of piecewise constant velocity

The event graph of Ch. 3 is based on having an event of constant velocity, *w, cf.* Fig. 1. But we can similarly illustrate a chain of events for an object having a *piecewise* constant *w*, see the example given in Fig. 4. In this figure we start with a constant $w > 0$, see the line *OA*. After a certain time, the object is brought to rest, that is $w = \varphi_w = 0$, giving a line (*AB*) parallel to the abscissa, τ. Next, the object approaches the origin; *i.e.* having a negative *w,* (line *BC*) Thereafter the 'object', (or now rather 'event'), has velocity, $w = c$, ($\varphi_w = \pi/2$), and the line (*CD*) is orthogonal to the τ-axis, *etc.* (Note that the object/clock all the time moves in the same direction; that is, the *x*-axis of *K*.)

Strictly speaking, TSR is said to apply for objects having a constant $w$. So, at the instants where the magnitude of the velocity, $w$ changes, we also imagine a change of the 'internal' clock. We may assume that a clock with the new velocity is passing the previous clock at exactly the right moment, and this is then synchronized with the old clock. Whether this is physically feasible is not really the question. We just imagine that these internal clocks are present, and the time, $\tau$ is what we would observe *if* they were present.

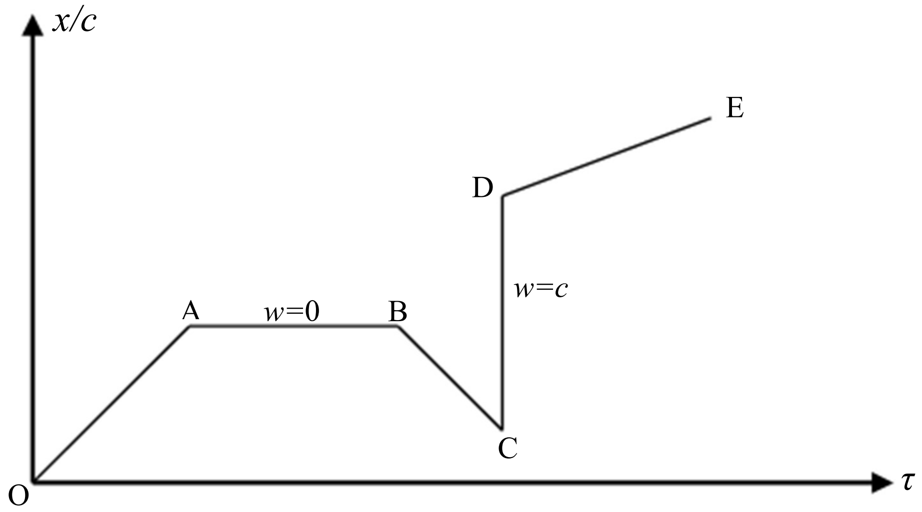



**Figure 4. An event graph illustrating an object's movement, when the velocity, *w* is piecewise constant. The length of the graph equals the elapsed (calendar) time, *t* on the RF.**

It is rather trivial to see that the length of the graph for a chain of events $(t,\ x)_{\tau\geq 0}$ up to a certain $\tau$-value will still be equal to the calendar time, $t$ of the end event. But this length is not defined by the time vector, $\vec{t}$ of the end event. So, in Fig. 4, the calendar time, $t$ will depend on the past history, and for the end event, *E,* this will equal, (hopefully, with a self-explanatory notation)

$$t = t_{O-E} = \left|\overrightarrow{t_{OA}}\right| + \left|\overrightarrow{t_{AB}}\right| + \left|\overrightarrow{t_{BC}}\right| + \left|\overrightarrow{t_{CD}}\right| + \left|\overrightarrow{t_{DE}}\right|.$$

### 4.2 Extension to three space coordinates

In the approach, described so far, there is a single space coordinate, $x$, only. We could, however, introduce three space coordinates $(x, y, z)$ with corresponding velocity coordinates, $(w_x,\ w_y,\ w_z)$. For simplicity we just consider the case of having a constant velocity throughout; now with a velocity vector

$$\vec{w} = (w_x,\ w_y,\ w_z)^{\mathrm{T}}$$

So, when we have the calendar time, $t$ at the position $(x, y, z)$ on *K,* it follows that

$$(x,\ y,\ z)^{\mathrm{T}} = t \cdot (w_x,\ w_y,\ w_z)^{\mathrm{T}}$$

At this instant the spatial distance from the origin of *K* equals

$$l = \sqrt{x^2 + y^2 + z^2} = t \cdot \sqrt{w_x^2 + w_y^2 + w_z^2}$$

The absolute value of the velocity vector, $\vec{w} = (w_x,\ w_y,\ w_z)^{\mathrm{T}}$ equals $w = \sqrt{w_x^2 + w_y^2 + w_z^2}$ . Thus, $w=l/t$.

Now we can apply the results of Section 3.1, by just replacing $x$ by $l$. In analogy with *eq.* (4) the event graph is still defined by $\varphi_w$, now having

$$\sin\varphi_w = w/c = l/ct.$$

Finally, we note that this graphical approach is somewhat similar to Minkowski's spacetime diagram with three space coordinates, *x, y, z*, and an imaginary time coordinate, *t.* The coordinates of his space are $(x,\ y,\ z,\ i{\cdot}ct)$. As opposed to this, we have transformed the spatial parameters to time, and we have $\tau$

rather than $t$ as the time coordinate. In his paper, Minkowski (1909) also introduced the spacetime distance, given as $\sqrt{c^2t^2 - x^2 - y^2 - z^2}$ in his four-dimensional space, while we in the current approach refer to $t = \sqrt{\tau^2 + (x/c)^2}$ as the total 'distance in time' (from the 'point of initiation'). As pointed out for instance by Petkov (2012), Minkowski introduced the term *proper time* for $\tau = \sqrt{t^2 - (x/c)^2}$ and *coordinate time* for $t$; which are the commonly used terms for these parameters.

# 5 Example: the travelling twin

We now apply the results of Section 4.1, and will discuss Example 1 of Section 3.2 in much more detail. This travelling twin paradox, goes back to Langevin (1911), and shall illustrate that if one twin leaves the earth at a speed close to $c$, and then returns to the earth, he will by the return have aged less than his brother. Or in terms of clocks: If we initially have two synchronized clocks at the same place, then they can end up with different readings if they move apart and then are brought together again. This example is frequently discussed, both in text books and papers, for instance see Debs and Redhead (1996) and Schuler and Robert (2014). It is very well suited to illustrate the graphical approach of the present work.

We apply the same numerical values as given in Example 1, and wil now consider the entire travel. First, we take the perspective of the twin remaining on the earth. His RF is denoted $K$, and has its origin at the earth. Next, we apply the perspective of the travelling twin on his travel *from* the earth, (RF, $K_T$). Finally, we take the perspective of an RF, $K_S$, demonstrating complete symmetry between the two twins.

Note that (we imagine) both twins carry their own 'internal' clock, and this clock reading, τ represent the twin's age.

## 5.1 The perspective of the earth-based RF, *K*

Fig. 5 presents two graphs, illustrating the total chain of events, both that of the travelling twin, and also that of the twin remaining on the earth. Both graphs describe the events in the perspective of $K$, (*i.e.,* the earth's RF). As seen in Example 1 the travelling twin's travel *to* the star is described by the blue line, *OB* at an angle $\varphi_w$, where $\sin\varphi_w = w/c = 0.6$.

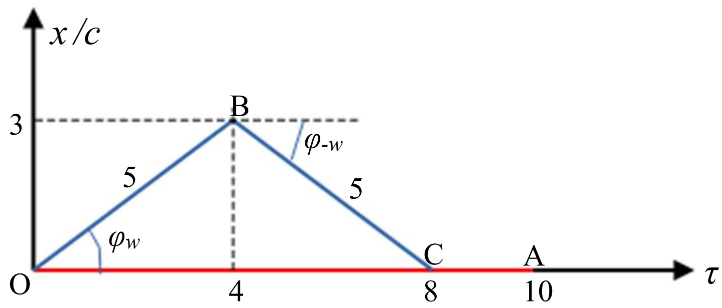



**Figure 5. Event graphs of the two twins in the perspective of the earth-based RF, *K*. The blue graph describes the travelling twin's journey. The red line follows the twin on the earth, having $x$/c ≡ 0.**

At the turning point, *B,* of the traveling twin we found that $\tau = 4,\ x/c = 3,\ t = 5$. At this point, the velocity abruptly changes to -*w.* As we pointed out in Section 3.2, this actually implies that – strictly within the framework of TSR - we should rather introduce a second rocket bringing a clock back to the earth; and by passing the star at the right moment, this returning clock is also set to time $\tau = 4$.[1] So strictly speaking it is not the travelling twin's clock that is returning.

Now the return travel is given by *BC*, and the total blue graph illustrates the well-known results that by the return to the earth (where again $x$ /$c$ = 0), the returning clock shows τ = 8 years (*OC*). So at this

[1] So this presentation corresponds to the 'three brothers' approach', suggested by Lords Halsbury, being referred in Debs and Redhead (1996).

moment it has elapsed a time $t$ = 5 + 5 = 10 years on $K$, which equals the total length of the blue graph, ($OB + BC$).

The red line, $OA$ illustrates the very simple 'chain of events' related to the twin at the earth. He is permanently stationed at the origin, $O$ of $K$. So, now we have $w \equiv 0$, and thus also $\varphi_w \equiv 0$, giving $x/c \equiv 0$ and $t \equiv \tau$, (all $\tau$). As we just found by identifying the blue graph: by the return of the twin to the earth it has elapsed a time $t$ = 10 years on $K$, and the red event graph ends with $t = \tau = 10$ years.

### 5.2 The perspective of the travelling twin $K_T$

As an exercise we also consider the same two chains of events from the perspective of the RF moving away from the earth at velocity, $w$., The rocket is located at the origin of this RF, which we denote $K_T$, (with time, $t_S$ and position, $x_S$). When the travelling twin arrives to the star, this $K_T$ continues its journey away from the earth; (as we consider RFs of constant velocity, only).

Fig. 6 provides an illustration. Again, the red line refers to the event chain for the earthbound twin. This twin is moving along the negative $x_T/c$-axis of $K_T$ at velocity $-w$, Thus, this graph has an angle $\varphi_{-w}$ relative to the $\tau$-axis, where $\sin\varphi_{-w} = -w/c = -0.6$.

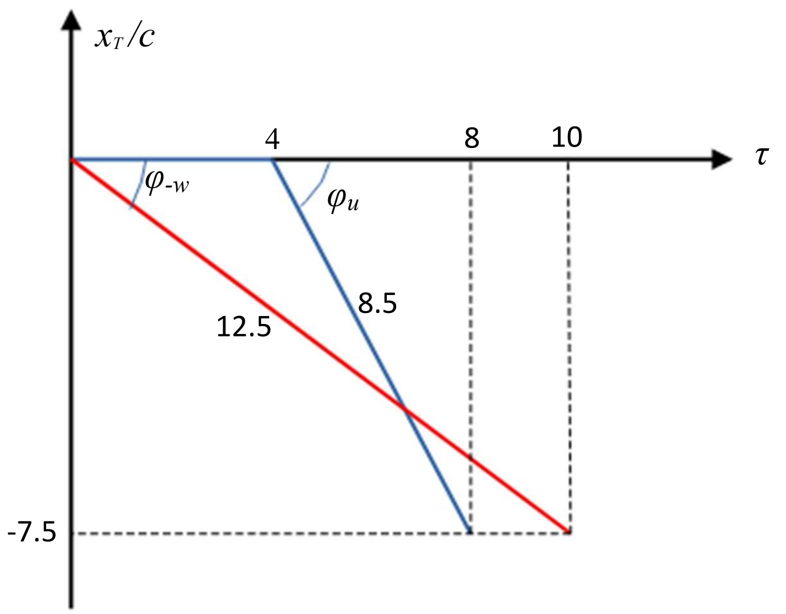



**Figure 6. Event graphs of the two twins in the perspective of $K_T$, which is the RF of the travelling twin on his travel *away* from earth.**

This line is valid for the entire event chain. So, at the instant when his internal clock reads $\tau = 10$ years, it follows that the parameters on $K_T$ of this event equals $x_T/c$ = -7.5 years and $t_T$ = 12.5 years. Thus, the red line gives a full description of the 'journey' for the earthbound twin, as described in $K_T$.

To follow the travelling twin, we look at the blue line in Fig. 6. For the first four years he remains at the origin of $K_T$. When $\tau = 4$, (and $x_T = 0$, $t_T = 4$) there is an abrupt change in his velocity. For $\tau \geq 4$ the blue line illustrates the *returning* twin (or rather clock). This shall have a velocity $w$ relative to $K$. Further, the speed between $K$ and $K_T$ also equals $w$. So now we must apply the rule for adding velocities in TSR, *e.g.* see Mermin (2005). According to this the returning clock it will have the following velocity relative to $K_T$:

$$u = \frac{-w-w}{1+(-w/c)^2} = -\frac{2w}{1+(w/c)^2} \quad (5)$$

Now inserting $w/c$ = 0.6 here, we get $u/c = -15/17$. So, for the return travel ($\tau \geq 4$) the blue graph has an angle, $\varphi_u$ given by $\sin\varphi_u = u/c = -15/17$, (and further, $\cos\varphi_u = \sqrt{1-(u/c)^2} = 8/17$. This fully describes the direction of the blue graph for $\tau \geq 4$.

Further, we found that the two event chains end when $x_T/c$ = -7.5 years on $K_T$. This is the location on $K_T$ where the two twins are reunited. So, now we have fully determined also the blue graph. Fig. 6 demonstrates that when $x_T/c$ = -7.5 years the returning clock reads $\tau = 8$ years, while that of the twin on the earth reads $\tau = 10$ years; (which we already know from Section 5.1, as $\tau$ does not depend on the RF). We also see that the clock reading on $K_T$ when the blue graph terminates, equals $t_T = 4 + 8.5 = 12.5$ years; which of course is equal to the length of the red graph.

### 5.3 A symmetric approach

Finally, we choose an approach of complete symmetry between the two twins. We observe both twins in the perspective of an auxiliary RF, $K_S$, (where time is denoted $t_S$ and position denoted $x_S$). During the travel away from the earth the origin of this RF is located at the midpoint between the two twins. Now the travelling twin has a velocity $w^*$ relative to $K_S$. and the earthbound twin a velocity $-w^*$, relative to the same RF, where the two $w^*$ 'sum up' to $w$. In TSR this means that (in analogy with *eq*. (5)):

$$\frac{2w^*/c}{1+(w^*/c)^2} = \frac{w}{c} = 0.6$$

From this relation we derive that

$$w^*/c = \frac{w/c}{1+\sqrt{1-(w/c)^2}} = 1/3$$

In other words, if the two twins move away from the origin of $K_S$ at relative speed $\pm c/3$, respectively, then the velocity between the twins will equal the specified value, $w = 0.6c$.

Now we will obtain the graph for the traveling twin's travel to the star, relative to this RF, $K_S$. Again, we know that by the arrival to the star the clock reading for the clocks following the travelling twin reads $\tau = 4$, (as it does not depend on the RF). Further, his velocity in the RF, $K_S$, equals $w^* = c/3$, giving $\sin\varphi_{w^*} = w^*/c = 1/3$. From these results it is now quite easy to derive $t_S$ and $x_S$, and we find that in this RF, the traveling twin's journey to the star is given by the line *OB* of Fig 7, where

$$\overrightarrow{t_{OB}} = \begin{pmatrix}\tau \\ x_S/c\end{pmatrix} = \begin{pmatrix}4 \\ \sqrt{2}\end{pmatrix}, \text{ and } \left|\overrightarrow{t_{OB}}\right| = t_S = 3\sqrt{2} \text{ years}$$

That is, by the travelling twin's arrival to the star, the local clock on $K_S$ reads $t_S = 3\sqrt{2}$ years, and the position on $K_S$ equals $x_S = \sqrt{2}$ light years. Now exactly the same argument applies for the earthbound twin, and the line *OA* represents the first part of his journey with respect to $K_S$.

To finalize this example, we must decide how the two twins shall unite again. The standard answer is that the traveling twin is turning, and the twin on the earth remains 'at rest', (as was demonstrated in Figs. 5 and 6). But various options exist, and in this example we will maintain the symmetry throughout.

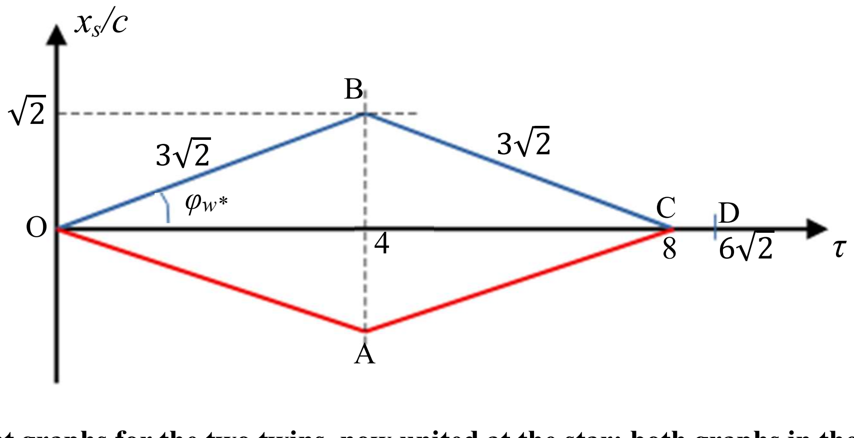



**Figure 7 Event graphs for the two twins, now united at the star; both graphs in the perspective of the auxiliary RF, $K_S$.**

So now we let the twins return to the origin of $K_S$ in a symmetric way. Thus, both twins turn and move in the opposite direction (at speed $\pm w^*$) until they meet again at the origin of $K_S$. In Fig. 7 the graphs *OBC* and *OAC* represent the total travels of the two twins in this set-up. Both graphs have length $t_S = 6\sqrt{2} \approx 8.5$ years. Thus, the clock at the origin of $K_S$ reads $6\sqrt{2} \approx 8.5$ years at this instant of return; *OD* represents the event graph of this clock. (This would be the age of a person permanently located here.)

Further, we see that when the twins meet on the star, (at the origin of $K_S$) the internal clocks of the twins both read $\tau = 8$ years; (*i.e.* their age at this instant). Note that we can sum up the travels as follows:

- $K_S$ moves at a constant velocity $w^*$ towards the star in the whole period.
- The travelling twin, after having arrived at the star, actually remains there, giving a velocity $-w^*$ relative to $K_S$.
- The 'earthbound twin' first remains on the earth (*OA*). Next he moves at velocity $v$ with respect to the earth's RF, *K,* giving velocity $w^*$ relative to $K_S$.

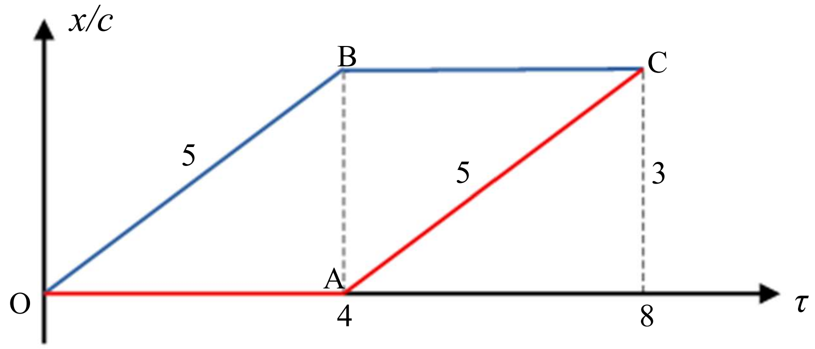



**Figure 8 Event graphs for the two twins; same events as in Fig. 7, but now in the perspective of the earth, *K*.**

So, what really happens here is more 'easily' described in the perspective of the earth (*K*); see. Fig. 8. The travelling twin (blue graph) travels to the star and arrives there after 4 years (according to his own clock); the clock of *K* reading 5 years. And he remains there until the brother arrives 4 years later. The other twin (red graph) first remains on the earth in 4 years. Then he starts out to the star, arriving there 4 years later; (but according to the stationary clocks on *K* it has passed 5 years). So, by the reunion, the twins have a position given by $x/c = 3$ light years, and the time on *K* equals $t = 4 + 5 = 9$ years. Finally, we note that the first parts of Fig. 8 (of course) is identical to Fig. 5.

### 5.4 Some observations related to these examples

The above event graphs well describe some basic features in special relativity. The first part of the travel is identical in all the Figs. 5-8, and they illustrate the basic fact that the reading, $\tau$, of any *specific* ('internal') clock does *not* depend on the chosen RF, (*i.e.* the 'perspective'). In particular, by the arrival to the star, the clock of the travelling twin reads $\tau = 4$ whatever RF we may choose.

Obviously, also the strategy for how the twins shall reunite is essential for the resulting outcome on the age difference. Figs. 5 and 6 illustrate the standard strategy. Then the twins' clocks will read $\tau = 8$ and $\tau = 10$, respectively, by the reunion, (any RF). Figs. 7 and 8 illustrate the more symmetric approach, and then the clocks of *both* twins will read $\tau = 8$ by the reunion. So, it is the strategy for uniting the twins that determines the age difference when they come together.

Further, can the above examples say something about simultaneity? Of course Figs. 5-8 present event chains that we must consider to be simultaneous: they start at both the same time and location, and also end at the same time and location, (when the twins are reunited). Quite another question is simultaneity 'at a distance' which is not properly defined in TSR. We know that the travelling twin has aged $\tau = 4$

years when he arrives at the star, but does it make sense to ask about the age of the twin brother at this moment?

There are two candidates for indicating such a simultaneity: First, the $t$-value of the chosen RF, and secondly, the $\tau$-value of twins' clock. Now consider the four cases of Figs. 5-8. In Fig. 5 the two criteria give different results; In the perspective of $K$, we get have $t$ = 5 for the travelling twin, and so the earthbound twin has aged 5 years, (as $t = \tau$ for him). But based the twins' internal clocks ($\tau$), the arrival to the star is simultaneous with the earthbound twin also having aged 4 years.

In the rather strange case of Fig. 6 we still have that the travelling twin has aged $\tau = t_T = 4$ years by the arrival to the star. Thus, according to the $\tau$-criteria, also the earthbound twin has aged 4 years. But according to the $t_T$-value he has aged $t_T \cos \varphi_w = 4 \cdot 0.8 = 3.2$ years; (by interchanging the RF, it is the earthbound twin that apparently ages more slowly).

The last two cases (Figs. 7 and 8) are special as they are symmetric with respect to the two twins. Fig. 7 presents a completely symmetric set-up where $\tau = 4$ and $t_S = 3\sqrt{2}$ when the travelling twin arrives to the star, and the symmetry makes it completely clear that this is 'simultaneous' with the earthbound twin experiencing the same values. That is, the earthbound twin has also aged 4 years at this instant. In the case of Fig. 8, $K$ is again the chosen RF, and the first part is identical to that of Fig, 5; discussed above.

So, as we know, there is no unique conclusion here. However, we must point out that the symmetry of Fig. 7 is rather convincing. Here we can define simultaneity of the two events, by having *both* the same $\tau$ *and* the same $t$-values, (and thus also the same $|x/c|$ of the relevant $K$). One may object that in Fig. 7 we do not apply the 'correct' strategy for the reunion, (as in Fig. 5). However, what happens *after* the arrival to the star should not affect how we judge simultaneity of the event $B$.

In total, these cases confirm my view that, when it comes to specifying some sort of 'simultaneity' of events, the proper time $\tau$ of the 'internal' clocks is more fundamental than the calendar time, $t$, (which varies wildly from one RF to another).

Of course, we can choose the perspective of a specific RF, and then the time $t$ of that RF gives a sensible criterion. So, if a specific perspective (RF) for some reason should be preferred, we could base a 'simultaneity' criterion on the $t$ – value.

So, even if a proper definition of 'absolute simultaneity' is not possible in TSR, there should be sensible ways to specify weaker versions of, say, a 'conditional simultaneity'; (meaning that we restrict the number of events to consider); somewhat analogous to the way we define 'conditional probability'. Some further investigation here could be interesting.

## 6 Summary

We present a graphical approach to describe and investigate the movement of an object, as observed within a specific reference frame (RF). The object has a piecewise constant velocity, and the event graph illustrates all parameters of an event:

1. The proper time, $\tau$ of the moving object, *i.e.* the reading of its imagined 'internal clock'.
2. The calendar time, $t$ as read on the chosen RF, (at the relevant location).
3. The position $x$ - divided by the speed of light, $c$; (representing the time required for the light to traverse this distance).

The graph illustrates how the time, $t$ is decomposed into its two orthogonal components, $\tau$ and $x/c$. We also observe that the event $(t, x)$ relative to the specified RF is derived from the proper time, $\tau$, together with the object's relative velocity, $w$, (which we observe in the graph by the angle $\varphi_w$).. We note that the approach is somewhat related to the work of Minkowski, *cf.* his 'world line'.

As an example, we make extensive use of the travelling twin case; illustrating different ‘perspectives’, *i.e.* choice of RF, and also different strategies for the reunion of the twins.

Based on this, we include a short discussion on ‘simultaneity’, and suggest that it could make sense to refer to a ‘conditional simultaneity’; in particular, when we restrict to a certain symmetry of events.

Contrary to the parameters ($t$, $x$), the proper time, $\tau$ is independent of the RF where we have chosen to perform the observations. As it is the phenomenon as such that should be of main interest - irrespective of the chosen RF – I rather see $\tau$ as the primary ‘event parameter’; being more fundamental than $t$ and $x$. I argue that this should affect the way we talk about/interpret time within the STR, and should be relevant *e.g.* when we discuss the case of the μ-mesons.

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