$\zeta(5), \zeta(7)..........\zeta(331), \zeta(333)$ are irrational number

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Abstract

Using the fact that $\zeta(3)$ is an irrational number, I prove that $\zeta(5)$, $\zeta(7)$..........$\zeta(331)$ and $\zeta(333)$ are irrational numbers.

$\zeta(5)$, $\zeta(7)$..........$\zeta(331)$ and $\zeta(333)$ are confirmed that they were in perfect numerical agreement.

This is because I created an odd-number formula for ζ, and the formula was created by dividing the odd-number for ζ itself into odd and even numbers.

key words

irrational number, $\zeta(3)$, odd-number formula for ζ, $\zeta(5)$, $\zeta(7)$, $\zeta(331)$, $\zeta(333)$

1 Introduction

Write the formula I finally got in advance.

$$\zeta(2m+1) = \frac{(2^{2m+1}-4)}{(2^{2m+1}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}} \tag{1}$$

m is a positive integer.

and

$$\zeta(2m+1) = \zeta(2m-1)\frac{(2^{2m+1}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}} \tag{2}$$

m is a positive integer.
and

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Eq.(1) and Eq.(2) are equation these are modification of Eq.(3).

$$\zeta(2m-1)=\frac{2^{2m-1}}{2^{2m-1}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}} \tag{3}$$

m is a positive integer.

In detail

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}=\frac{1}{2^3}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{1}{2^3}\zeta(3)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{4}$$

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{5}$$

do the same

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}=\frac{1}{2^5}\sum_{n=1}^{\infty}\frac{1}{n^5}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{1}{2^5}\zeta(5)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{6}$$

$$\zeta(5)=\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{7}$$

do the same

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}+\sum_{n=1}^{\infty}\frac{1}{(2n)^7}=\frac{1}{2^7}\sum_{n=1}^{\infty}\frac{1}{n^7}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{1}{2^7}\zeta(7)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{8}$$

$$\zeta(7)=\frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{9}$$

Use these for $\zeta(9), \zeta(11), \zeta(13)$ etc.

and
Detailed description

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{10}$$

$$\zeta(5)=\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{11}$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\zeta(3)\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{12}$$

$$\zeta(5)\frac{8}{7}=\zeta(3)\frac{32}{31}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{13}$$

$$\zeta(5)=\zeta(3)\frac{28}{31}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\frac{28}{31}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{14}$$

2 Discussion

Example 1

from Eq.(3)
if m=3

$$\zeta(5)=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{15}$$

if m=4

$$\zeta(7)=\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{16}$$

Multiply $\zeta(5)$ and $\zeta(7)$.

$$\zeta(7)\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\zeta(5)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\zeta(5)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{17}$$

$$\zeta(7)\frac{32}{31}=\zeta(5)\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{18}$$

$$\zeta(7)=\zeta(5)\frac{31}{32}\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{19}$$

$$\zeta(7)=\zeta(5)\frac{124}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}=\frac{124}{127}\zeta(5)\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{20}$$

$$=\frac{124}{127}\Big[\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{124}{127}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{21}$$

The new formula Eq.(1) has been followed.

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

(Proof 1)
If $\zeta(5)$ is assumed to be rational number. $\zeta(5) = \frac{s}{t}$, s and t are integer.

from formula Eq.(1).
$\zeta(5) = \zeta(3)\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}$ $\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \frac{o}{p}$ o, p are assumed to be integer.

$\zeta(5) = \zeta(3)\frac{o}{p}$ it equal $\zeta(3) = \zeta(5)\frac{p}{o} = \frac{sp}{to}$ But, $\zeta(3) \neq \frac{sp}{to}$

This is because $\zeta(3)$ is known to be an irrational number.
This contradicts.

$\zeta(5)$ is irrational number.

(Proof end)

Do the same, sequentially prove that $\zeta(7), \zeta(9), \zeta(11)$ etc. are irrational numbers.

Assuming that this formula Eq.(1) holds even when $\zeta(3)$

$$\zeta(3) = \zeta(1)\frac{4}{7}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} = \frac{4}{7}\zeta(1)\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} \tag{22}$$

$$= \frac{4}{7}\Big[\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty}\frac{1}{(2n)^1}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{23}$$

$$= \frac{4}{7}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^1}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{24}$$

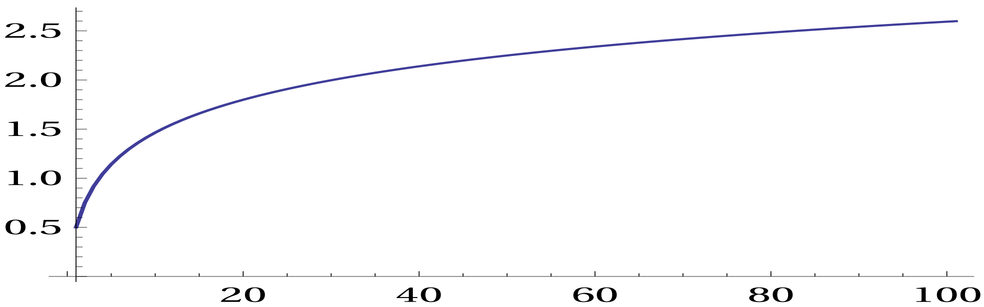
If $\infty = 10000$
The calculator gives the following results:

$$\zeta(3) \approx \zeta(1)\frac{4}{7}\frac{\sum_{n=1}^{10000}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{10000}\frac{1}{(2n-1)^1}} = \infty \tag{25}$$

$$\zeta(3) \approx \frac{4}{7}\Big[1 + \frac{\sum_{n=1}^{10000}\frac{1}{(2n)^1}}{\sum_{n=1}^{10000}\frac{1}{(2n-1)^1}}\Big]\sum_{n=1}^{10000}\frac{1}{(2n-1)^3} = \texttt{can not calculate}. \tag{26}$$

$$\zeta(1)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}+\sum_{n=1}^{\infty}\frac{1}{(2n)^1}=\infty \qquad \frac{\sum_{n=1}^{10000}\frac{1}{(2n)^1}}{\sum_{n=1}^{10000}\frac{1}{(2n-1)^1}}=\texttt{can not calculate.} \tag{27}$$

But
when assuming $\infty = 10^{2199}$
The calculator could not calculate any more. It seems to be the upper limit.
The value increases as shown below.
Below is a graph of $\sum_{n=1}^{10^{2199}}\frac{1}{(2n)^1}$.



$$\sum_{n=1}^{10^{2199}}\frac{1}{(2n)^1} \tag{28}$$

$$=2531.98091757940399600808486047049964747293216641530584894 8.... \tag{29}$$

$$\sum_{n=1}^{10^{2199}}\frac{1}{(2n-1)^1} \tag{30}$$

$$=2532.67406475996394131750209259195782404100766654966610420 2... \tag{31}$$

$$\sum_{n=1}^{10^{2199}}\frac{1}{(2n-1)^3} \tag{32}$$

$$=1.05179979026464499972477089132251874191936300579793652156 8... \tag{33}$$

$\zeta(3)\approx\frac{4}{7}[1+\frac{2531.980917579403996}{2532.6740647599639413175}]\times 1.0517997902646449997247 7$

=1.2018924125200581910912951040422079156206003630923703481 04...

$\zeta(3)=1.2020569031595942853997381615114499 9...$

The formula Eq.(1) seems to hold.

My first formula Eq.(3)
when m=1 and from Eq.(66)Eq.(67)

$$\zeta(1)=\frac{2^1}{2^1-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}\approx 2\sum_{n=1}^{10^{2199}}\frac{1}{(2n-1)^1} \tag{34}$$

$$= 2 \times 2532.6740647599639413175020925919578240410076665496661042 02... \tag{35}$$

This is an error because $\infty = 10^{2199}$.

If I transform Eq.(2)
Use $\infty = 10^{2199}$.

$$\zeta(1) = \zeta(3)\frac{7}{4}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \approx \zeta(3)\frac{7}{4} \times \frac{2532.674064759963941317502092591957824041007666549666104202...}{1.051799790264644999724770891322518741919363005797936521568...} \tag{36}$$

$$\approx \zeta(3)\frac{7}{4} \times \frac{2532.6740647599639413175}{1.0517997902646449997247 7} = 5603.7075056254558602670... \tag{37}$$

Even at this time, this is an error because n=2199.

$$\zeta(3) = \zeta(1)\frac{(2^3-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{(2^3-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} = \frac{(2^3-4)}{(2^3-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^1}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{38}$$

$$\zeta(5) = \zeta(3)\frac{(2^5-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{(2^5-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \frac{(2^5-4)}{(2^5-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{39}$$

$$\zeta(7) = \zeta(5)\frac{(2^7-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{(2^7-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} = \frac{(2^7-4)}{(2^7-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{40}$$

$$\zeta(9) = \zeta(7)\frac{(2^9-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{(2^9-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}} = \frac{(2^9-4)}{(2^9-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{41}$$

$$\zeta(11) = \zeta(9)\frac{(2^{11}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{(2^{11}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}} = \frac{(2^{11}-4)}{(2^{11}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{42}$$

$$\zeta(13) = \zeta(11)\frac{(2^{13}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{(2^{13}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}} = \frac{(2^{13}-4)}{(2^{13}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{11}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{43}$$

$$\zeta(15) = \zeta(13)\frac{(2^{15}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{(2^{15}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}} = \frac{(2^{15}-4)}{(2^{15}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{44}$$

$$\zeta(17) = \zeta(15)\frac{(2^{17}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{(2^{17}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}} = \frac{(2^{17}-4)}{(2^{17}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{15}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{45}$$

$$\zeta(19) = \zeta(17)\frac{(2^{19}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{(2^{19}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}} = \frac{(2^{19}-4)}{(2^{19}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{17}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{46}$$

$$\zeta(21) = \zeta(19)\frac{(2^{21}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{(2^{17}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}} = \frac{(2^{21}-4)}{(2^{21}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{19}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}} \tag{47}$$

$$\zeta(23) = \zeta(21)\frac{(2^{23}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{(2^{23}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}} = \frac{(2^{23}-4)}{(2^{23}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{21}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}} \tag{48}$$

$$\zeta(25) = \zeta(23)\frac{(2^{25}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{(2^{25}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}} = \frac{(2^{25}-4)}{(2^{25}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{23}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}} \tag{49}$$

$$\zeta(27) = \zeta(25)\frac{(2^{27}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{(2^{27}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}} = \frac{(2^{27}-4)}{(2^{27}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{25}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}} \tag{50}$$

$$\zeta(29) = \zeta(27)\frac{(2^{29}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{(2^{29}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}} = \frac{(2^{29}-4)}{(2^{29}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{27}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}} \tag{51}$$

$$\zeta(31) = \zeta(29)\frac{(2^{31}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{(2^{31}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}} = \frac{(2^{31}-4)}{(2^{31}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{29}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}} \tag{52}$$

$$\zeta(33) = \zeta(31)\frac{(2^{33}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{(2^{33}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}} = \frac{(2^{33}-4)}{(2^{33}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{31}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}} \tag{53}$$

$$\zeta(35) = \zeta(33)\frac{(2^{35}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{(2^{35}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}} = \frac{(2^{35}-4)}{(2^{35}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{33}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}} \tag{54}$$

$$\zeta(37) = \zeta(35)\frac{(2^{37}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{(2^{37}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}} = \frac{(2^{37}-4)}{(2^{37}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{35}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}} \tag{55}$$

$$\zeta(39) = \zeta(37)\frac{(2^{39}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{(2^{39}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}} = \frac{(2^{39}-4)}{(2^{39}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{37}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}} \tag{56}$$

$$\zeta(41) = \zeta(39)\frac{(2^{41}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{(2^{41}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}} = \frac{(2^{41}-4)}{(2^{41}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{39}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}} \tag{57}$$

$$\zeta(43)=\zeta(41)\frac{(2^{43}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{(2^{43}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}=\frac{(2^{43}-4)}{(2^{43}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{41}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}} \tag{58}$$

$$\zeta(45)=\zeta(43)\frac{(2^{45}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{(2^{45}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}=\frac{(2^{45}-4)}{(2^{45}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{43}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}} \tag{59}$$

$$\zeta(47)=\zeta(45)\frac{(2^{47}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{(2^{47}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}=\frac{(2^{47}-4)}{(2^{47}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{45}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}} \tag{60}$$

$$\zeta(49)=\zeta(47)\frac{(2^{49}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{(2^{49}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}=\frac{(2^{49}-4)}{(2^{49}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{47}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}} \tag{61}$$

$$\zeta(51)=\zeta(49)\frac{(2^{51}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{(2^{51}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}=\frac{(2^{51}-4)}{(2^{51}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{49}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}} \tag{62}$$

$$\zeta(53)=\zeta(51)\frac{(2^{53}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{(2^{53}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}=\frac{(2^{53}-4)}{(2^{53}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{51}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}} \tag{63}$$

$$\zeta(55)=\zeta(53)\frac{(2^{55}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{(2^{55}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}=\frac{(2^{55}-4)}{(2^{55}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{53}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}} \tag{64}$$

$$\zeta(57)=\zeta(55)\frac{(2^{57}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{(2^{57}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}=\frac{(2^{57}-4)}{(2^{57}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{55}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}} \tag{65}$$

$$\zeta(59)=\zeta(57)\frac{(2^{59}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{(2^{59}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}=\frac{(2^{59}-4)}{(2^{59}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{57}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}} \tag{66}$$

$$\zeta(61)=\zeta(59)\frac{(2^{61}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{(2^{61}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}=\frac{(2^{61}-4)}{(2^{61}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{59}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}} \tag{67}$$

$$\zeta(63)=\zeta(61)\frac{(2^{63}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{(2^{63}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}=\frac{(2^{63}-4)}{(2^{63}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{61}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}} \tag{68}$$

$$\zeta(65)=\zeta(63)\frac{(2^{65}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{(2^{65}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}=\frac{(2^{65}-4)}{(2^{65}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{63}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}} \tag{69}$$

$$\zeta(67)=\zeta(65)\frac{(2^{67}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{(2^{67}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}=\frac{(2^{67}-4)}{(2^{67}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{65}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}} \tag{70}$$

$$\zeta(69)=\zeta(67)\frac{(2^{69}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{(2^{69}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}=\frac{(2^{69}-4)}{(2^{69}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{67}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}} \tag{71}$$

$$\zeta(71)=\zeta(69)\frac{(2^{71}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{(2^{71}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}=\frac{(2^{71}-4)}{(2^{71}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{69}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}} \tag{72}$$

$$\zeta(73)=\zeta(71)\frac{(2^{73}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{(2^{73}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}=\frac{(2^{73}-4)}{(2^{73}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{71}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}} \tag{73}$$

$$\zeta(75)=\zeta(73)\frac{(2^{75}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{(2^{75}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}=\frac{(2^{75}-4)}{(2^{75}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{73}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}} \tag{74}$$

$$\zeta(77)=\zeta(75)\frac{(2^{77}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{(2^{77}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}=\frac{(2^{77}-4)}{(2^{77}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{75}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}} \tag{75}$$

$$\zeta(79)=\zeta(77)\frac{(2^{79}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{(2^{79}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}=\frac{(2^{79}-4)}{(2^{79}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{77}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}} \tag{76}$$

$$\zeta(81)=\zeta(79)\frac{(2^{81}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{(2^{81}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}=\frac{(2^{81}-4)}{(2^{81}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{79}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}} \tag{77}$$

$$\zeta(83)=\zeta(81)\frac{(2^{83}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{(2^{83}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}=\frac{(2^{83}-4)}{(2^{83}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{81}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}} \tag{78}$$

$$\zeta(85)=\zeta(83)\frac{(2^{85}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{(2^{85}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}=\frac{(2^{85}-4)}{(2^{85}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{83}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}} \tag{79}$$

$$\zeta(87)=\zeta(85)\frac{(2^{87}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{(2^{87}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}=\frac{(2^{87}-4)}{(2^{87}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{85}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}} \tag{80}$$

$$\zeta(89)=\zeta(87)\frac{(2^{89}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{(2^{89}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}=\frac{(2^{89}-4)}{(2^{89}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{87}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}} \tag{81}$$

$$\zeta(91)=\zeta(89)\frac{(2^{91}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{(2^{91}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}=\frac{(2^{91}-4)}{(2^{91}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{89}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}} \tag{82}$$

$$\zeta(93)=\zeta(91)\frac{(2^{93}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{(2^{93}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}=\frac{(2^{93}-4)}{(2^{93}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{91}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}} \tag{83}$$

$$\zeta(95)=\zeta(93)\frac{(2^{95}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{(2^{95}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}=\frac{(2^{95}-4)}{(2^{95}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{93}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}} \tag{84}$$

$$\zeta(97)=\zeta(95)\frac{(2^{97}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{(2^{97}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}=\frac{(2^{97}-4)}{(2^{97}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{95}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}} \tag{85}$$

$$\zeta(99)=\zeta(97)\frac{(2^{99}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{(2^{99}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}=\frac{(2^{99}-4)}{(2^{99}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{97}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}} \tag{86}$$

$$\zeta(101)=\zeta(99)\frac{(2^{101}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{(2^{101}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}=\frac{(2^{101}-4)}{(2^{101}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{99}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}} \tag{87}$$

$$\zeta(103)=\zeta(101)\frac{(2^{103}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{(2^{103}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}=\frac{(2^{103}-4)}{(2^{103}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{101}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}} \tag{88}$$

$$\zeta(105)=\zeta(103)\frac{(2^{105}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{(2^{105}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}=\frac{(2^{105}-4)}{(2^{105}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{103}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}} \tag{89}$$

$$\zeta(107)=\zeta(105)\frac{(2^{107}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{(2^{107}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}=\frac{(2^{107}-4)}{(2^{107}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{105}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}} \tag{90}$$

$$\zeta(109)=\zeta(107)\frac{(2^{109}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{(2^{109}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}=\frac{(2^{109}-4)}{(2^{109}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{107}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}} \tag{91}$$

$$\zeta(111)=\zeta(109)\frac{(2^{111}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{(2^{111}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}=\frac{(2^{111}-4)}{(2^{111}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{109}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}} \tag{92}$$

$$\zeta(113)=\zeta(111)\frac{(2^{113}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{(2^{113}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}=\frac{(2^{113}-4)}{(2^{113}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{111}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}} \tag{93}$$

$$\zeta(115) = \zeta(113)\frac{(2^{115}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{(2^{115}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}} = \frac{(2^{115}-4)}{(2^{115}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{113}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}} \tag{94}$$

$$\zeta(117) = \zeta(115)\frac{(2^{117}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{(2^{117}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}} = \frac{(2^{117}-4)}{(2^{117}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{115}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}} \tag{95}$$

$$\zeta(119) = \zeta(117)\frac{(2^{119}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{(2^{119}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}} = \frac{(2^{117}-4)}{(2^{117}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{117}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}} \tag{96}$$

$$\zeta(121) = \zeta(119)\frac{(2^{121}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{(2^{121}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}} = \frac{(2^{121}-4)}{(2^{121}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{119}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}} \tag{97}$$

$$\zeta(123) = \zeta(121)\frac{(2^{123}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{(2^{123}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}} = \frac{(2^{123}-4)}{(2^{123}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{121}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}} \tag{98}$$

$$\zeta(125) = \zeta(123)\frac{(2^{125}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{(2^{125}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}} = \frac{(2^{125}-4)}{(2^{125}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{123}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}} \tag{99}$$

$$\zeta(127) = \zeta(125)\frac{(2^{127}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{(2^{127}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}} = \frac{(2^{127}-4)}{(2^{127}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{125}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}} \tag{100}$$

$$\zeta(129) = \zeta(127)\frac{(2^{129}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{(2^{129}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}} = \frac{(2^{129}-4)}{(2^{129}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{127}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}} \tag{101}$$

$$\zeta(131) = \zeta(129)\frac{(2^{131}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{(2^{131}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}} = \frac{(2^{131}-4)}{(2^{131}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{129}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}} \tag{102}$$

$$\zeta(133) = \zeta(131)\frac{(2^{133}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{(2^{133}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}} = \frac{(2^{133}-4)}{(2^{133}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{131}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}} \tag{103}$$

$$\zeta(135) = \zeta(133)\frac{(2^{135}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{(2^{135}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}} = \frac{(2^{135}-4)}{(2^{135}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{133}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}} \tag{104}$$

$$\zeta(137) = \zeta(135)\frac{(2^{137}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{(2^{137}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}} = \frac{(2^{137}-4)}{(2^{137}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{135}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}} \tag{105}$$

$$\zeta(139)=\zeta(137)\frac{(2^{139}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{(2^{139}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}=\frac{(2^{139}-4)}{(2^{139}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{137}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}} \tag{106}$$

$$\zeta(141)=\zeta(139)\frac{(2^{141}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{(2^{141}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}=\frac{(2^{141}-4)}{(2^{141}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{139}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}} \tag{107}$$

$$\zeta(143)=\zeta(141)\frac{(2^{143}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{(2^{143}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}=\frac{(2^{143}-4)}{(2^{143}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{141}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}} \tag{108}$$

$$\zeta(145)=\zeta(143)\frac{(2^{145}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{(2^{145}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}=\frac{(2^{145}-4)}{(2^{145}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{143}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}} \tag{109}$$

$$\zeta(147)=\zeta(145)\frac{(2^{147}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{(2^{147}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}=\frac{(2^{147}-4)}{(2^{147}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{145}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}} \tag{110}$$

$$\zeta(149)=\zeta(147)\frac{(2^{149}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{(2^{149}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}=\frac{(2^{149}-4)}{(2^{149}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{147}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}} \tag{111}$$

$$\zeta(151)=\zeta(149)\frac{(2^{151}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{(2^{151}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}=\frac{(2^{151}-4)}{(2^{151}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{149}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}} \tag{112}$$

$$\zeta(153)=\zeta(151)\frac{(2^{153}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{(2^{153}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}=\frac{(2^{153}-4)}{(2^{153}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{151}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}} \tag{113}$$

$$\zeta(155)=\zeta(153)\frac{(2^{155}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{(2^{155}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}=\frac{(2^{155}-4)}{(2^{155}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{153}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}} \tag{114}$$

$$\zeta(157)=\zeta(155)\frac{(2^{157}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{(2^{157}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}=\frac{(2^{157}-4)}{(2^{157}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{155}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}} \tag{115}$$

$$\zeta(159)=\zeta(157)\frac{(2^{159}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{(2^{159}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}=\frac{(2^{159}-4)}{(2^{159}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{157}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}} \tag{116}$$

$$\zeta(161)=\zeta(159)\frac{(2^{161}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}{(2^{161}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}=\frac{(2^{161}-4)}{(2^{161}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{159}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}} \tag{117}$$

$$\zeta(163)=\zeta(161)\frac{(2^{163}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}{(2^{163}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}=\frac{(2^{163}-4)}{(2^{163}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{161}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}} \tag{118}$$

$$\zeta(165)=\zeta(163)\frac{(2^{165}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}{(2^{165}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}=\frac{(2^{165}-4)}{(2^{165}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{163}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}} \tag{119}$$

$$\zeta(167)=\zeta(165)\frac{(2^{167}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}{(2^{167}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}=\frac{(2^{167}-4)}{(2^{167}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{165}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}} \tag{120}$$

$$\zeta(169)=\zeta(167)\frac{(2^{169}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}{(2^{169}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}=\frac{(2^{169}-4)}{(2^{169}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{167}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}} \tag{121}$$

$$\zeta(171)=\zeta(169)\frac{(2^{171}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}{(2^{171}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}=\frac{(2^{171}-4)}{(2^{171}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{169}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}} \tag{122}$$

$$\zeta(173)=\zeta(171)\frac{(2^{173}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}{(2^{173}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}=\frac{(2^{173}-4)}{(2^{173}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{171}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}} \tag{123}$$

$$\zeta(175)=\zeta(173)\frac{(2^{175}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}{(2^{175}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}=\frac{(2^{175}-4)}{(2^{175}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{173}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}} \tag{124}$$

$$\zeta(177)=\zeta(175)\frac{(2^{177}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}{(2^{177}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}=\frac{(2^{177}-4)}{(2^{177}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{175}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}} \tag{125}$$

$$\zeta(179)=\zeta(177)\frac{(2^{179}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}{(2^{179}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}=\frac{(2^{179}-4)}{(2^{179}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{177}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}} \tag{126}$$

$$\zeta(181)=\zeta(179)\frac{(2^{181}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}{(2^{181}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}=\frac{(2^{181}-4)}{(2^{181}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{179}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}} \tag{127}$$

$$\zeta(183)=\zeta(181)\frac{(2^{183}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}{(2^{183}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}=\frac{(2^{183}-4)}{(2^{183}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{181}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}} \tag{128}$$

$$\zeta(185)=\zeta(183)\frac{(2^{185}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{(2^{185}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}=\frac{(2^{185}-4)}{(2^{185}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{183}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}} \tag{129}$$

$$\zeta(187) = \zeta(185)\frac{(2^{187}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{(2^{187}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}} = \frac{(2^{187}-4)}{(2^{187}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{185}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}} \tag{130}$$

$$\zeta(189) = \zeta(187)\frac{(2^{189}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{(2^{189}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}} = \frac{(2^{189}-4)}{(2^{189}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{187}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}} \tag{131}$$

$$\zeta(191) = \zeta(189)\frac{(2^{191}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{(2^{191}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}} = \frac{(2^{191}-4)}{(2^{191}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{189}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}} \tag{132}$$

$$\zeta(193) = \zeta(191)\frac{(2^{193}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{(2^{193}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}} = \frac{(2^{193}-4)}{(2^{193}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{191}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}} \tag{133}$$

$$\zeta(195) = \zeta(193)\frac{(2^{195}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{(2^{195}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}} = \frac{(2^{195}-4)}{(2^{195}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{193}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}} \tag{134}$$

$$\zeta(197) = \zeta(195)\frac{(2^{197}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{(2^{197}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}} = \frac{(2^{197}-4)}{(2^{197}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{195}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}} \tag{135}$$

$$\zeta(199) = \zeta(197)\frac{(2^{199}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{(2^{199}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}} = \frac{(2^{199}-4)}{(2^{199}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{197}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}} \tag{136}$$

$$\zeta(201) = \zeta(199)\frac{(2^{201}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}{(2^{201}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}} = \frac{(2^{201}-4)}{(2^{201}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{199}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}} \tag{137}$$

$$\zeta(203) = \zeta(201)\frac{(2^{203}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}{(2^{203}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}} = \frac{(2^{203}-4)}{(2^{203}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{201}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}} \tag{138}$$

$$\zeta(205) = \zeta(203)\frac{(2^{205}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}{(2^{205}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}} = \frac{(2^{205}-4)}{(2^{205}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{203}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}} \tag{139}$$

$$\zeta(207) = \zeta(205)\frac{(2^{207}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}{(2^{207}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}} = \frac{(2^{207}-4)}{(2^{207}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{205}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}} \tag{140}$$

$$\zeta(209) = \zeta(207)\frac{(2^{209}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}{(2^{209}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}} = \frac{(2^{209}-4)}{(2^{209}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{207}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}} \tag{141}$$

$$\zeta(211)=\zeta(209)\frac{(2^{211}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}{(2^{211}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}=\frac{(2^{211}-4)}{(2^{211}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{209}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}} \tag{142}$$

$$\zeta(213)=\zeta(211)\frac{(2^{213}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}{(2^{213}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}=\frac{(2^{213}-4)}{(2^{213}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{211}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}} \tag{143}$$

$$\zeta(215)=\zeta(213)\frac{(2^{215}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}{(2^{215}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}=\frac{(2^{215}-4)}{(2^{215}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{213}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}} \tag{144}$$

$$\zeta(217)=\zeta(215)\frac{(2^{217}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}{(2^{217}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}=\frac{(2^{217}-4)}{(2^{217}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{215}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}} \tag{145}$$

$$\zeta(219)=\zeta(217)\frac{(2^{219}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}{(2^{219}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}=\frac{(2^{219}-4)}{(2^{219}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{217}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}} \tag{146}$$

$$\zeta(221)=\zeta(219)\frac{(2^{221}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}{(2^{221}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}=\frac{(2^{221}-4)}{(2^{221}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{219}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}} \tag{147}$$

$$\zeta(223)=\zeta(221)\frac{(2^{223}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}{(2^{223}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}=\frac{(2^{223}-4)}{(2^{223}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{221}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}} \tag{148}$$

$$\zeta(225)=\zeta(223)\frac{(2^{225}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}{(2^{225}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}=\frac{(2^{225}-4)}{(2^{225}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{223}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}} \tag{149}$$

$$\zeta(227)=\zeta(225)\frac{(2^{227}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}{(2^{227}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}=\frac{(2^{227}-4)}{(2^{227}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{225}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}} \tag{150}$$

$$\zeta(229)=\zeta(227)\frac{(2^{229}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}{(2^{229}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}=\frac{(2^{229}-4)}{(2^{229}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{227}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}} \tag{151}$$

$$\zeta(231)=\zeta(229)\frac{(2^{231}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}{(2^{231}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}=\frac{(2^{231}-4)}{(2^{231}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{229}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}} \tag{152}$$

$$\zeta(233)=\zeta(231)\frac{(2^{233}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}{(2^{233}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}=\frac{(2^{233}-4)}{(2^{233}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{231}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}} \tag{153}$$

$$\zeta(235)=\zeta(233)\frac{(2^{235}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}{(2^{235}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}=\frac{(2^{235}-4)}{(2^{235}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{233}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}} \tag{154}$$

$$\zeta(237)=\zeta(235)\frac{(2^{237}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}{(2^{237}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}=\frac{(2^{237}-4)}{(2^{237}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{235}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}} \tag{155}$$

$$\zeta(239)=\zeta(237)\frac{(2^{239}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}{(2^{239}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}=\frac{(2^{239}-4)}{(2^{239}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{237}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}} \tag{156}$$

$$\zeta(241)=\zeta(239)\frac{(2^{241}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}{(2^{241}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}=\frac{(2^{241}-4)}{(2^{241}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{239}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}} \tag{157}$$

$$\zeta(243)=\zeta(241)\frac{(2^{243}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}{(2^{243}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}=\frac{(2^{243}-4)}{(2^{243}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{241}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}} \tag{158}$$

$$\zeta(245)=\zeta(243)\frac{(2^{245}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}{(2^{245}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}=\frac{(2^{245}-4)}{(2^{245}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{243}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}} \tag{159}$$

$$\zeta(247)=\zeta(245)\frac{(2^{247}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}{(2^{247}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}=\frac{(2^{247}-4)}{(2^{247}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{245}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}} \tag{160}$$

$$\zeta(249)=\zeta(247)\frac{(2^{249}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}{(2^{249}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}=\frac{(2^{249}-4)}{(2^{249}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{247}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}} \tag{161}$$

$$\zeta(251)=\zeta(249)\frac{(2^{251}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}{(2^{251}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}=\frac{(2^{251}-4)}{(2^{251}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{249}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}} \tag{162}$$

$$\zeta(253)=\zeta(251)\frac{(2^{253}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}{(2^{253}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}=\frac{(2^{253}-4)}{(2^{253}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{251}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}} \tag{163}$$

$$\zeta(255)=\zeta(253)\frac{(2^{255}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}{(2^{255}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}=\frac{(2^{255}-4)}{(2^{255}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{253}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}} \tag{164}$$

$$\zeta(257)=\zeta(255)\frac{(2^{257}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}{(2^{257}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}=\frac{(2^{257}-4)}{(2^{257}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{255}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}} \tag{165}$$

$$\zeta(259)=\zeta(257)\frac{(2^{259}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}{(2^{259}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}=\frac{(2^{259}-4)}{(2^{259}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{257}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}} \tag{166}$$

$$\zeta(261)=\zeta(259)\frac{(2^{261}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}{(2^{261}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}=\frac{(2^{261}-4)}{(2^{261}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{259}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}} \tag{167}$$

$$\zeta(263)=\zeta(261)\frac{(2^{263}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}{(2^{263}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}=\frac{(2^{263}-4)}{(2^{263}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{261}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}} \tag{168}$$

$$\zeta(265)=\zeta(263)\frac{(2^{265}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}{(2^{265}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}=\frac{(2^{265}-4)}{(2^{265}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{263}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}} \tag{169}$$

$$\zeta(267)=\zeta(265)\frac{(2^{267}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}{(2^{267}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}=\frac{(2^{267}-4)}{(2^{267}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{265}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}} \tag{170}$$

$$\zeta(269)=\zeta(267)\frac{(2^{269}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}{(2^{269}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}=\frac{(2^{269}-4)}{(2^{269}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{267}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}} \tag{171}$$

$$\zeta(271)=\zeta(269)\frac{(2^{271}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}{(2^{271}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}=\frac{(2^{271}-4)}{(2^{271}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{269}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}} \tag{172}$$

$$\zeta(273)=\zeta(271)\frac{(2^{273}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}{(2^{273}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}=\frac{(2^{273}-4)}{(2^{273}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{271}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}} \tag{173}$$

$$\zeta(275)=\zeta(273)\frac{(2^{275}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}{(2^{275}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}=\frac{(2^{275}-4)}{(2^{275}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{273}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}} \tag{174}$$

$$\zeta(277)=\zeta(275)\frac{(2^{277}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}{(2^{277}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}=\frac{(2^{277}-4)}{(2^{277}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{275}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}} \tag{175}$$

$$\zeta(279)=\zeta(277)\frac{(2^{279}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}{(2^{279}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}=\frac{(2^{279}-4)}{(2^{279}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{277}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}} \tag{176}$$

$$\zeta(281)=\zeta(279)\frac{(2^{281}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}{(2^{281}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}=\frac{(2^{281}-4)}{(2^{281}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{279}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}} \tag{177}$$

$$\zeta(283)=\zeta(281)\frac{(2^{283}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}{(2^{283}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}=\frac{(2^{283}-4)}{(2^{283}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{281}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}} \tag{178}$$

$$\zeta(285)=\zeta(283)\frac{(2^{285}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}{(2^{285}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}=\frac{(2^{285}-4)}{(2^{285}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{283}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}} \tag{179}$$

$$\zeta(287)=\zeta(285)\frac{(2^{287}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}{(2^{287}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}=\frac{(2^{287}-4)}{(2^{287}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{285}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}} \tag{180}$$

$$\zeta(289)=\zeta(287)\frac{(2^{289}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}{(2^{289}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}=\frac{(2^{289}-4)}{(2^{289}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{287}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}} \tag{181}$$

$$\zeta(291)=\zeta(289)\frac{(2^{291}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}{(2^{291}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}=\frac{(2^{291}-4)}{(2^{291}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{289}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}} \tag{182}$$

$$\zeta(293)=\zeta(291)\frac{(2^{293}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}{(2^{293}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}=\frac{(2^{293}-4)}{(2^{293}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{291}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}} \tag{183}$$

$$\zeta(295)=\zeta(293)\frac{(2^{295}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}{(2^{295}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}=\frac{(2^{295}-4)}{(2^{295}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{293}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}} \tag{184}$$

$$\zeta(297)=\zeta(295)\frac{(2^{297}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}{(2^{297}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}=\frac{(2^{297}-4)}{(2^{297}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{295}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}} \tag{185}$$

$$\zeta(299)=\zeta(297)\frac{(2^{299}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}{(2^{299}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}=\frac{(2^{299}-4)}{(2^{299}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{297}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}} \tag{186}$$

$$\zeta(301)==\zeta(299)\frac{(2^{301}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}{(2^{301}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}=\frac{(2^{301}-4)}{(2^{301}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{299}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}} \tag{187}$$

$$\zeta(303)=\zeta(301)\frac{(2^{303}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}{(2^{303}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}=\frac{(2^{303}-4)}{(2^{303}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{301}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}} \tag{188}$$

$$\zeta(305)=\zeta(303)\frac{(2^{305}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}{(2^{305}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}=\frac{(2^{305}-4)}{(2^{305}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{303}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}} \tag{189}$$

$$\zeta(307)=\zeta(305)\frac{(2^{307}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}{(2^{307}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}=\frac{(2^{307}-4)}{(2^{307}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{305}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}} \tag{190}$$

$$\zeta(309)=\zeta(307)\frac{(2^{309}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}{(2^{309}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}=\frac{(2^{309}-4)}{(2^{309}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{307}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}} \tag{191}$$

$$\zeta(311)=\zeta(309)\frac{(2^{311}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}{(2^{311}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}=\frac{(2^{311}-4)}{(2^{311}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{309}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}} \tag{192}$$

$$\zeta(313)=\zeta(311)\frac{(2^{313}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}{(2^{313}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}=\frac{(2^{313}-4)}{(2^{313}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{311}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}} \tag{193}$$

$$\zeta(315)=\zeta(313)\frac{(2^{315}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}{(2^{315}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}=\frac{(2^{315}-4)}{(2^{315}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{313}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}} \tag{194}$$

$$\zeta(317)=\zeta(315)\frac{(2^{317}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}{(2^{317}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}=\frac{(2^{317}-4)}{(2^{317}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{315}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}} \tag{195}$$

$$\zeta(319)=\zeta(317)\frac{(2^{319}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}{(2^{319}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}=\frac{(2^{319}-4)}{(2^{319}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{317}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}} \tag{196}$$

$$\zeta(321)=\zeta(319)\frac{(2^{321}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}{(2^{321}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}=\frac{(2^{321}-4)}{(2^{321}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{319}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}} \tag{197}$$

$$\zeta(323)=\zeta(321)\frac{(2^{323}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}{(2^{323}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}=\frac{(2^{323}-4)}{(2^{323}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{321}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}} \tag{198}$$

$$\zeta(325)=\zeta(323)\frac{(2^{325}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}{(2^{325}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}=\frac{(2^{325}-4)}{(2^{325}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{323}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}} \tag{199}$$

$$\zeta(327)=\zeta(325)\frac{(2^{327}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}{(2^{327}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}=\frac{(2^{327}-4)}{(2^{327}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{325}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}} \tag{200}$$

$$\zeta(329)=\zeta(327)\frac{(2^{329}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}{(2^{329}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}=\frac{(2^{329}-4)}{(2^{329}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{327}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}} \tag{201}$$

$$\zeta(331)=\zeta(329)\frac{(2^{331}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}{(2^{331}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}=\frac{(2^{331}-4)}{(2^{331}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{329}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}} \tag{202}$$

$$\zeta(333)=\zeta(331)\frac{(2^{333}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}}}{(2^{333}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}=\frac{(2^{333}-4)}{(2^{333}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{331}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}} \tag{203}$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations.

$\zeta(5), \zeta(7)$........$\zeta(331), \zeta(333)$ are irrational numbers.

Example 2

That $\zeta(5)$ is an irrational number is already proven at **Example 1** (proof 1).

(Proof 2)
If $\zeta(7)$ is assumed to be rational number. $\zeta(7)=\frac{s}{t}$, s and t are integer.

$\zeta(7)=\zeta(3)\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}$ $\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\frac{o}{p}$ o, p are assumed to be integer.

$\zeta(7)=\zeta(3)\frac{o}{p}$ it equal $\zeta(3)=\zeta(7)\frac{p}{o}=\frac{sp}{to}$ But, $\zeta(3)\neq\frac{sp}{to}$
This is because $\zeta(3)$ is known to be an irrational number.
This contradicts.

$\zeta(7)$ is irrational number.

(Proof end)

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc. prove that $\zeta(9), \zeta(11), \zeta(13)$ etc. are an irrational numbers.

and
Detailed description

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{204}$$

$$\zeta(7) = \frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} = \frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{205}$$

Multiply $\zeta(3)$ and $\zeta(7)$

$$\zeta(7)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \zeta(3)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{206}$$

$$\zeta(7)\frac{8}{7} = \zeta(3)\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{207}$$

$$\zeta(7) = \zeta(3)\frac{128}{127}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{112}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{208}$$

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

$$\zeta(3) = \zeta(1)\frac{4}{7}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} = \zeta(1)\frac{4}{(2^3-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} \tag{209}$$

$$\zeta(5) = \zeta(3)\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{(2^5-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{210}$$

$$\zeta(7) = \zeta(3)\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{(2^7-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{211}$$

$$\zeta(9) = \zeta(3)\frac{2^6\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{73\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{212}$$

$$\zeta(11) = \zeta(3)\frac{1792\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{2047\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{1792\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{(2^{11}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{213}$$

$$\zeta(13) = \zeta(3)\frac{7168\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{8191\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{7168\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{(2^{13}-1)}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{214}$$

$$\zeta(15) = \zeta(3)\frac{2^{12}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{4681\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{215}$$

$$\zeta(17) = \zeta(3)\frac{114688\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{131072\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{114688\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{(2^{17}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{216}$$

$$\zeta(19)=\zeta(3)\frac{458752\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{524287\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{458752\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{(2^{19}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{217}$$

$$\zeta(21)=\zeta(3)\frac{2^{18}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{299593\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{218}$$

$$\zeta(23)=\zeta(3)\frac{7340032\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{8388607\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{7340032\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{(2^{23}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{219}$$

$$\zeta(25)=\zeta(3)\frac{29360128\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{33554431\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{29360128\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{(2^{25}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{220}$$

$$\zeta(27)=\zeta(3)\frac{2^{24}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{19173961\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{221}$$

$$\zeta(29)=\zeta(3)\frac{469762048\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{536870911\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{469762048\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{(2^{29}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{222}$$

$$\zeta(31)=\zeta(3)\frac{1879048192\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{2147483647\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{1879048192\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{(2^{31}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{223}$$

$$\zeta(33)=\zeta(3)\frac{2^{30}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{1227133513\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{224}$$

$$\zeta(35)=\zeta(3)\frac{30064771072\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{34359738367\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{30064771072\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{(2^{35}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{225}$$

$$\zeta(37)=\zeta(3)\frac{120259084288\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{137438953471\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{120259084288\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{(2^{31}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{226}$$

$$\zeta(39)=\zeta(3)\frac{2^{36}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{78536544841\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{227}$$

$$\zeta(41)=\zeta(3)\frac{1924145348608\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{2199023255551\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{1924145348608\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{(2^{41}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{228}$$

$$\zeta(43)=\zeta(3)\frac{7696581394432\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{8796093022207\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{7696581394432\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{(2^{43}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{229}$$

$$\zeta(45)=\zeta(3)\frac{2^{42}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{5026338869833\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{230}$$

$$\zeta(47)=\zeta(3)\frac{123145302310912\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{140737488355327\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{123145302310912\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{(2^{47}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{231}$$

$$\zeta(49)=\zeta(3)\frac{492581209243648\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{562949953421311\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{492581209243648\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{(2^{49}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{232}$$

$$\zeta(51)=\zeta(3)\frac{2^{48}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{321685687669321\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{233}$$

$$\zeta(53)=\zeta(3)\frac{7881299347898368\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{9007199254740991\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{7881299347898368\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{(2^{53}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{234}$$

$$\zeta(55)=\zeta(3)\frac{31525197391593472\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{36028797018963967\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{31525197391593472\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{(2^{55}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{235}$$

$$\zeta(57)=\zeta(3)\frac{2^{54}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{20587884010836553\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{236}$$

$$\zeta(59)=\zeta(3)\frac{504403158265495552\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{576460752303423487\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{504403158265495552\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{(2^{59}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{237}$$

$$\zeta(61)=\zeta(3)\frac{2017612633061982208\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{2305843009213693951\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2017612633061982208\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{(2^{61}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{238}$$

$$\zeta(63)=\zeta(3)\frac{2^{60}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{1317624576693539401\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{239}$$

$$\zeta(65)=\zeta(3)\frac{32281802128991715328\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{36893488147419103231\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{32281802128991715328\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{(2^{65}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{240}$$

$$\zeta(67)=\zeta(3)\frac{129127208515966861312\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{147573952589676412927\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{129127208515966861312\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{(2^{67}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{241}$$

$$\zeta(69)=\zeta(3)\frac{2^{66}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{843279729083865216 73\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{242}$$

$$\zeta(71)=\zeta(3)\frac{2066035336255469780992\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{2361183241434822606847\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2066035336255469780992\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{(2^{71}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{243}$$

$$\zeta(73)=\zeta(3)\frac{8264141345021879123968\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{9444732965739290427391\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{8264141345021879123968\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{(2^{73}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{244}$$

$$\zeta(75)=\zeta(3)\frac{2^{72}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{5396990266136737387081\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{245}$$

$$\zeta(77)=\zeta(3)\frac{132226261520350065983488\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{151115727451828646838271\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{132226261520350065983488\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{(2^{77}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{246}$$

$$\zeta(79)=\zeta(3)\frac{528905046081400263933952\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{604462909807314587353087\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{528905046081400263933952\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{(2^{79}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{247}$$

$$\zeta(81)=\zeta(3)\frac{2^{78}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{345407377032751192773193\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{248}$$

$$\zeta(83)=\zeta(3)\frac{8462480737302404222943232\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{9671406556917033397649407\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{8462480737302404222943232\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{(2^{83}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{249}$$

$$\zeta(85)=\zeta(3)\frac{33849922949209616891772928\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{38685626227668133590597631\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{33849922949209616891772928\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{(2^{85}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{250}$$

$$\zeta(87)=\zeta(3)\frac{2^{84}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{22106072130096076337484361\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{251}$$

$$\zeta(89) = \zeta(3)\frac{5415987671873538702683668 48\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{(2^{89}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{252}$$

$$\zeta(91) = \zeta(3)\frac{21663950687494154810734673 92\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{(2^{91}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{253}$$

$$\zeta(93) = \zeta(3)\frac{2^{90}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{14147886163261488855989991 13\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{254}$$

$$\zeta(95) = \zeta(3)\frac{34662321099990647697175478272\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{(2^{95}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{255}$$

$$\zeta(97) = \zeta(3)\frac{138649284399962590788701913088\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{(2^{97}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{256}$$

$$\zeta(99) = \zeta(3)\frac{2^{96}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{90546471444873528678335943241\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{257}$$

$$\zeta(101) = \zeta(3)\frac{2218388550399401452619230609408\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{(2^{101}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{258}$$

$$\zeta(103) = \zeta(3)\frac{8873554201597605810476922437632\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{(2^{103}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{259}$$

$$\zeta(105) = \zeta(3)\frac{2^{102}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{5794974172471905835413500367433\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{260}$$

$$\zeta(107) = \zeta(3)\frac{141976867225561692967630759002112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{(2^{107}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{261}$$

$$\zeta(109) = \zeta(3)\frac{567907468902246771870523036008448\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{(2^{109}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{262}$$

$$\zeta(111) = \zeta(3)\frac{2^{108}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{370878347038201973466464023515721\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{263}$$

$$\zeta(113)=\zeta(3)\frac{908651950243594834992836857613516 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{(2^{113}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{264}$$

$$\zeta(115)=\zeta(3)\frac{36346078009743793399713474304540672\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{(2^{115}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{265}$$

$$\zeta(117)=\zeta(3)\frac{2^{114}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{23736214210444926301853697505006153\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{266}$$

$$\zeta(119)=\zeta(3)\frac{581537248155900694395415588872650752\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{(2^{119}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{267}$$

$$\zeta(121)=\zeta(3)\frac{2326148992623602777581662355490603008\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{(2^{121}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{268}$$

$$\zeta(123)=\zeta(3)\frac{2^{120}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{151911770946847528331863664032039380 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{269}$$

$$\zeta(125)=\zeta(3)\frac{37218383881977644441306597687849648128\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{(2^{125}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{270}$$

$$\zeta(127)=\zeta(3)\frac{148873535527910577765226390751398592512\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{(2^{127}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{271}$$

$$\zeta(129)=\zeta(3)\frac{2^{126}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{97223533405982418132392744980505203273\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{272}$$

$$\zeta(131)=\zeta(3)\frac{2381976568446569244243622252022377480192\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{(2^{131}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{273}$$

$$\zeta(133)=\zeta(3)\frac{9527906273786276976974489008089509920768\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{(2^{133}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{274}$$

$$\zeta(135)=\zeta(3)\frac{2^{132}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{6222306137982874760473135678752333009481\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{275}$$

$$\zeta(137) = \zeta(3)\frac{152446500380580431631591824129432158732288\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{(2^{137}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{276}$$

$$\zeta(139) = \zeta(3)\frac{609786001522321726526367296517728634929152\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{(2^{139}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{277}$$

$$\zeta(141) = \zeta(3)\frac{2^{138}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{398227592830903984670280683440149312606793\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{278}$$

$$\zeta(143) = \zeta(3)\frac{9756576024357147624421876744283658158866432\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{(2^{143}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{279}$$

$$\zeta(145) = \zeta(3)\frac{39026304097428590497687506977134632635465728\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{(2^{145}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{280}$$

$$\zeta(147) = \zeta(3)\frac{2^{144}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{25486565941177855018897963740169556006834761\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{281}$$

$$\zeta(149) = \zeta(3)\frac{624420865558857447963000111634154122167451648\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{(2^{149}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{282}$$

$$\zeta(151) = \zeta(3)\frac{2497683462235429791852000446536616488669806592\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{(2^{151}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{283}$$

$$\zeta(153) = \zeta(3)\frac{2^{150}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{1631140220235382721209469679370851584437424713\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{284}$$

$$\zeta(155) = \zeta(3)\frac{39962935395766876669632007144585863818716905472\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{(2^{155}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{285}$$

$$\zeta(157) = \zeta(3)\frac{159851741583067506678528028578343455274867621888\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{(2^{157}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{286}$$

$$\zeta(159) = \zeta(3)\frac{2^{156}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{104392974095064494157406059479734501403995181641\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{287}$$

$$\zeta(161) = \zeta(3)\frac{2557627865329080106856448457253495284397881950208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}}{(2^{161}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{288}$$

$$\zeta(163) = \zeta(3)\frac{10230511461316320427425793829013981137591527800832 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}}{(2^{163}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{289}$$

$$\zeta(165) = \zeta(3)\frac{2^{162} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}}{6681150342084127626073987806703008089855691625033 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{290}$$

$$\zeta(167) = \zeta(3)\frac{163688183381061126838812701264223698201464444813312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}}{(2^{167}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{291}$$

$$\zeta(169) = \zeta(3)\frac{654752733524244507355250805056894792805857779253248 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}}{(2^{169}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{292}$$

$$\zeta(171) = \zeta(3)\frac{2^{168} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}}{427593621893384168068735219628992517750764264002121 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{293}$$

$$\zeta(173) = \zeta(3)\frac{10476043736387912117684012880910316684893724468051968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}}{(2^{173}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{294}$$

$$\zeta(175) = \zeta(3)\frac{41904174945551648470736051523641266739574897872207872 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}}{(2^{175}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{295}$$

$$\zeta(177) = \zeta(3)\frac{2^{174} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}}{27365991801176586756399054056255521136048912896135753 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{296}$$

$$\zeta(179) = \zeta(3)\frac{670466799128826375531776824378260267833198365955325952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}}{(2^{179}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{297}$$

$$\zeta(181) = \zeta(3)\frac{2681867196515305502127107297513041071332793463821303808 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}}{(2^{181}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{298}$$

$$\zeta(183) = \zeta(3)\frac{2^{180} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{1751423475275301552409539459600353352707130425352688201 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{299}$$

$$\zeta(185) = \zeta(3)\frac{429098751442448880340337167602086571413246954211408609 28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{(2^{185}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{300}$$

$$\zeta(187) = \zeta(3)\frac{1716395005769795521361348670408346285652987816845634437 12\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{(2^{187}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{301}$$

$$\zeta(189) = \zeta(3)\frac{2^{186}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{112091102417619299354210525414422614573256347222572044873\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{302}$$

$$\zeta(191) = \zeta(3)\frac{274623200923167283417815787265335405704478050695301509 9392\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{(2^{191}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{303}$$

$$\zeta(193) = \zeta(3)\frac{109849280369266913367126314906134162281791220278120603 97568\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{(2^{193}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{304}$$

$$\zeta(195) = \zeta(3)\frac{2^{192}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{7173830554727635158669473626523047332688406222244610871881\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{305}$$

$$\zeta(197) = \zeta(3)\frac{1757588485908270613874021038498146596508659524449929663 61088\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{(2^{197}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{306}$$

$$\zeta(199) = \zeta(3)\frac{7030353943633082455496084153992586386034638097799718654 44352\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{(2^{199}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{307}$$

$$\zeta(201) = \zeta(3)\frac{2^{198}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}{459125155502568650154846312097475029292057998223655095800393\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{308}$$

$$\zeta(203) = \zeta(3)\frac{1124856630981293192879373464638813821765542095647954984 7109632\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}{(2^{203}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{309}$$

$$\zeta(205) = \zeta(3)\frac{4499426523925172771517493858555255287062168382591819938 8438528\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}{(2^{205}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{310}$$

$$\zeta(207) = \zeta(3)\frac{2^{204}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}{29384009952164393609910163974238401874691711886313926131225161\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{311}$$

$$\zeta(209) = \zeta(3)\frac{719908243828027643442799017368840845929946941214691190215016448 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{312}$$

$$\zeta(211) = \zeta(3)\frac{2879632975312110573771196069475363383719787764858764760860065792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{313}$$

$$\zeta(213) = \zeta(3)\frac{2^{210} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{188057663693852119103425049435125771998026956072409127239841031 3\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{314}$$

$$\zeta(215) = \zeta(3)\frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{315}$$

$$\zeta(217) = \zeta(3)\frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217}-1)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{316}$$

$$\zeta(219) = \zeta(3)\frac{2^{216} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{12035690476406535622619203163848049407873725188634184143349826004 1\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{317}$$

$$\zeta(221) = \zeta(3)\frac{294874416671960122754170477514277210492906267121537511512070737 1008}{(2^{221}-1)} \tag{318}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{319}$$

$$\zeta(223) = \zeta(3)\frac{1179497666687840491016681910057108841971625068486150046048282948 4032}{(2^{223}-1)} \tag{320}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{321}$$

$$\zeta(225) = \zeta(3)\frac{2^{222}}{770284190490018279847629002486275162103918412072587785174388864 2633} \tag{322}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{323}$$

$$\zeta(227) = \zeta(3)\frac{18871962667005447856266910560913741471546001095778400736772527174451 2}{(2^{227}-1)} \tag{324}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{325}$$

$$\zeta(229) = \zeta(3)\frac{75487850668021791425067642243654965886184004383113602947090108697804 8}{(2^{229}-1)} \tag{326}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{327}$$

$$\zeta(231) = \zeta(3)\frac{2^{228}}{49298188191361169910248256159121610374650778372645618251160887312852 1} \tag{328}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{329}$$

$$\zeta(233) = \zeta(3)\frac{1207805610688348662801082275898479454178944070129817647153441739164876 8}{(2^{233}-1)} \tag{330}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{331}$$

$$\zeta(235) = \zeta(3)\frac{4831222442753394651204329103593917816715776280519270588613766956659507 2}{(2^{235}-1)} \tag{332}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{333}$$

$$\zeta(237) = \zeta(3)\frac{2^{234}}{3155084044247114874255888394183783063977649815849319568074296788022535 3} \tag{334}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{335}$$

$$\zeta(239) = \zeta(3)\frac{77299559084054314419269265657502685067452420488308329417820271306552115 2}{(2^{239}-1)} \tag{336}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{337}$$

$$\zeta(241) = \zeta(3)\frac{309198236336217257677077062630010740269809681953233317671281085226208460 8}{(2^{241}-1)} \tag{338}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{339}$$

$$\zeta(243) = \zeta(3) \frac{2^{240}}{201925378831815351952376857227762116094569588214356452356754994433442260 1} \tag{340}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{341}$$

$$\zeta(245) = \zeta(3) \frac{4947171781379476122833233002080171844316954911251733082740497363619335372 8}{(2^{245} - 1)} \tag{342}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{343}$$

$$\zeta(247) = \zeta(3) \frac{19788687125517904491332932008320687377267819645006932330961989454477341491 2}{(2^{247} - 1)} \tag{344}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{345}$$

$$\zeta(249) = \zeta(3) \frac{2^{246}}{129232242452361825249521188625767754300524536457188129508323196437403046473} \tag{346}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{347}$$

$$\zeta(251) = \zeta(3) \frac{316618994008286471861326912133130998036285114320110917295391831271637463859 2}{(2^{251} - 1)} \tag{348}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{349}$$

$$\zeta(253) = \zeta(3) \frac{12664759760331458874453076485325239921451404572804436691815673250865498554368}{(2^{253} - 1)} \tag{350}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{351}$$

$$\zeta(255) = \zeta(3) \frac{2^{252}}{827086351695115681596935607204913627523357033326004028853268457199379497428 1} \tag{352}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{353}$$

$$\zeta(257) = \zeta(3) \frac{202636156165303341991249223765203838743222473164870987069050772013847976869888}{(2^{257} - 1)} \tag{354}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{355}$$

$$\zeta(259) = \zeta(3) \frac{810544624661213367964996895060815354972889892659483948276203088055391907479552}{(2^{259} - 1)} \tag{356}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{357}$$

$$\zeta(261) = \zeta(3) \frac{2^{258}}{529335265084874036222038788611144721614948501328642578466091812607602878353993} \tag{358}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{359}$$

$$\zeta(263) = \zeta(3) \frac{12968713994579413887439950320973045679566238282551743172419249408886270519672832}{(2^{263} - 1)} \tag{360}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{361}$$

$$\zeta(265) = \zeta(3) \frac{51874855978317655549759801283892182718264953130206972689676997635545082078691328}{(2^{265} - 1)} \tag{362}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{363}$$

$$\zeta(267) = \zeta(3) \frac{2^{264}}{33877456965431938318210482471113262183356704085033125021829876006886584214655561} \tag{364}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{365}$$

$$\zeta(269) = \zeta(3)\frac{8299976956530824887961568205422749234922392500833115630348319621687213132590 61248}{(2^{269}-1)} \tag{366}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{367}$$

$$\zeta(271) = \zeta(3)\frac{33199907826123299551846272821690996939689570003332462521393278486748852530362 44992}{(2^{271}-1)} \tag{368}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{369}$$

$$\zeta(273) = \zeta(3)\frac{2^{270}}{2168157245787644052365470878151248779734829061442120001397112064440741389737955913} \tag{370}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{371}$$

$$\zeta(275) = \zeta(3)\frac{5311985252179727928295403651470559510350331200533194003422924557879816404857991 9872}{(2^{275}-1)} \tag{372}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{373}$$

$$\zeta(277) = \zeta(3)\frac{2124794100871891171318161460588223804140132480213277601369169823151926561943196 79488}{(2^{277}-1)} \tag{374}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{375}$$

$$\zeta(279) = \zeta(3)\frac{2^{276}}{1387620637304092193513901362016799219030290599322956800894151721242074489432291 78441} \tag{376}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{377}$$

$$\zeta(281) = \zeta(3)\frac{3399670561395025874109058336941158086624211968341244162190671717043082499109114871808}{(2^{281}-1)} \tag{378}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{379}$$

$$\zeta(283) = \zeta(3)\frac{1359868224558010349643623334776463234649684787336497664876268686817232999643645948723 2}{(2^{283}-1)} \tag{380}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{381}$$

$\zeta(285)=$

$$\zeta(3)\frac{2^{282}}{888077207874619003848896871690751500179385983566692352572257101594927673236666742023 3} \tag{382}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{383}$$

$\zeta(287)=$

$$\zeta(3)\frac{217578915929281655942979733564234117543949565973839626380202989890757279942983351795712}{(2^{287}-1)} \tag{384}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{385}$$

$\zeta(289)=$

$$\zeta(3)\frac{870315663717126623771918934256936470175798263895358505520811959563029119771933407182848}{(2^{289}-1)} \tag{386}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{387}$$

$\zeta(291)=$

$$\zeta(3)\frac{2^{288}}{568369413039756162463293997882080960114807029482683105646244545020753710871466714894921} \tag{388}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{389}$$

$\zeta(293)=$

$$\zeta(3)\frac{1392505061947402598035070294811098352281277222232573608833299135300846591635093451492556 8}{(2^{293}-1)} \tag{390}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{391}$$

$\zeta(295)=$

$$\zeta(3)\frac{5570020247789610392140281179244393409125108888930294435333196541203386366540373805970227 2}{(2^{295}-1)} \tag{392}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{393}$$

$\zeta(297)=$

$$\zeta(3)\frac{2^{294}}{3637564243454439439765081586445318144734764988689171876135965088132823749577386975327495 3} \tag{394}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{395}$$

$\zeta(299)=$

$$\zeta(3)\frac{891203239646337662742444988679102945460017422228847109653311446592541818646459808955236352}{(2^{299}-1)} \tag{396}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{397}$$

$\zeta(301)=$

$$\zeta(3)\frac{3564812958585350650969779954716411781840069688915388438613245786370167274585839235820945408}{(2^{301}-1)} \tag{398}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{399}$$

$\zeta(303)=$

$$\zeta(3)\frac{2^{300}}{232804111581084124144965221532500361263024959276107000072701765640500719972952766420959700 1} \tag{400}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{401}$$

$\zeta(305)=$

$$\zeta(3)\frac{570370073373656104155164792754625885094411150226462150178119325819226763933734277731351265 28}{(2^{305}-1)} \tag{402}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{403}$$

$\zeta(307)=$

$$\zeta(3)\frac{228148029349462441662065917101850354037764460090584860071247730327690705573493711092540506112}{(2^{307}-1)} \tag{404}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{405}$$

$\zeta(309)=$

$$\zeta(3)\frac{2^{306}}{148994631411893839452777741780800231208335973936708480046529130009920460782689770509414208073} \tag{406}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{407}$$

$\zeta(311)=$

$$\zeta(3)\frac{3650368469591399066593054673629605664604231361449357761139963685243051289175899377480648097792}{(2^{311}-1)} \tag{408}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{409}$$

$\zeta(313)=$

$$\zeta(3)\frac{14601473878365596266372218694518422658416925445797431044559854740972205156703597509922592391168}{(2^{313}-1)} \tag{410}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{411}$$

$\zeta(315)=$

$$\zeta(3)\frac{2^{312}}{9535656410361205724977775473971214797333502331949342722977864320634909490092145312602509316681} \tag{412}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{413}$$

$\zeta(317)=\zeta(3)\times$

$$\frac{233623582053849540261955499112294762534670807132758896712957675855555282507257560158761478258688}{(2^{317}-1)} \tag{414}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{415}$$

$\zeta(319)=\zeta(3)\times$

$$\frac{934494328215398161047821996449179050138683228531035586851830703422221130029030240635045913034752}{(2^{319}-1)} \tag{416}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{417}$$

$\zeta(321)=\zeta(3)\times$

$$\frac{2^{318}}{610282010263117166398577630334157747029344149244757934270583316520634207365897300006560596267593} \tag{418}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{419}$$

$\zeta(323)=\zeta(3)\times$

$$\frac{14951909251446370576765151943186864802218931656496569389629291254755538080464483850160734608556032}{(2^{323}-1)} \tag{420}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}} \tag{421}$$

$\zeta(325)$=$\zeta(3)\times$

$$\frac{59807637005785482307060607772747459208875726625986277558517165019022152321857935400642938434224128}{(2^{325}-1)} \tag{422}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{423}$$

$\zeta(327)$=$\zeta(3)\times$

$$\frac{2^{324}}{39058048656839498649508968341386095809878025551664507793317332257320589271417427200419878161125961} \tag{424}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{425}$$

$\zeta(329)$=$\zeta(3)\times$

$$\frac{956922192092567716912969724363959347342011626015780440936274640304354437149726966410287014947586048}{(2^{329}-1)} \tag{426}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{427}$$

$\zeta(331)$=$\zeta(3)\times$

$$\frac{3827688768370270867651878897455837389368046504063121763745098561217417748598907865641148059790344192}{(2^{331}-1)} \tag{428}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{429}$$

$\zeta(333)$=$\zeta(3)\times$

$$\frac{2^{330}}{2499715114037727913568573973848710131832193635306528498772309264468517713370715340826872202312061513} \tag{430}$$

$$\times\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{431}$$

$\zeta(335),\zeta(337)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(3), \zeta(5)......\zeta(331), \zeta(333)$ are irrational numbers.

4 Postscript

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{\pi}} \tag{432}$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \frac{1}{2^{\pi}} \sum_{n=1}^{\infty} \frac{1}{n^{\pi}} \tag{433}$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \frac{1}{2^{\pi}} \zeta(\pi) \tag{434}$$

$$(1 - \frac{1}{2^{\pi}}) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \tag{435}$$

$$(\frac{2^{\pi}-1}{2^{\pi}}) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \tag{436}$$

$$\zeta(\pi) = \frac{2^{\pi}}{2^{\pi}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \tag{437}$$

=1.17624173838258275887215045193805209116973899002165583496050834623040872376815861833572083732
5571831138945660081 45...

Do the same

$$\zeta(e) = \frac{2^{e}}{2^{e}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{e}} \tag{438}$$

=1.26900960433571711576556986660086110885640446257719048833581592870816524294827307650849451745
076054575828347684218...

I believe that $\zeta(\pi), \zeta(e)$ are irrational numbers.
That is, I believe that the irrational number ζ are irrational numbers.

I also believe that all even value, as well as odd values of ζ, are irrational numbers.

The figures in this paper have been fully verified by WolframAlpha.

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