

# The characteristics of primes

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## Abstract

The prime numbers has very irregular pattern. The problem of finding pattern in the prime numbers is the long-open problem in mathematics. In this paper, we try to solve the problem axiomatically. We propose some natural properties of prime numbers.

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## 1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our goal is to axiomatize the structure of primes.

## 2 The pattern of numbers

These below are some patterns of number.

Let  $t_n$  denote the  $n$ th triangular number. Then

$$t_n = \binom{n+1}{2} \quad n \geq 1,$$

where  $\binom{n+1}{2}$  is the binomial coefficients [Bur02, p. 15].

Let  $F_n$  be the  $n$ th Fibonacci number. Then

$$F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},$$

where  $n$  is a positive integer [Wei03, p. 1042].

Let  $B_n$  be the  $n$ th Bernoulli number. Then

$$B_n = (-1)^{n+1} n \zeta(1 - n),$$

where  $\zeta(1 - n)$  is the Riemann zeta-function [Wei03, p. 189].

If  $p(n)$  denotes the total number of partitions of  $n$ , then

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}},$$

where  $n$  is a positive integer [Har99, p. 116].

**Postulate 2.1** (Peano Postulates). Given the number 0, the set  $\mathbf{N}$ , and the function  $\sigma$ . Then:

1.  $0 \in \mathbf{N}$ .
2.  $\sigma : \mathbf{N} \rightarrow \mathbf{N}$  is a function from  $\mathbf{N}$  to  $\mathbf{N}$ .
3.  $0 \notin \text{range}(\sigma)$ .
4. The function  $\sigma$  is one-to-one.
5. If  $I \subset \mathbf{N}$  such that  $0 \in I$  and  $\sigma(n) \in I$  whenever  $n \in I$ , then  $I = \mathbf{N}$ .

We define  $1 = \sigma(0)$ ,  $2 = \sigma(1)$ ,  $3 = \sigma(2)$ , etc. [Eis96, p. A64].

### 3 The axiomatic approach

In this section, we try to solve the problem of prime number axiomatically. We propose the basic characteristics of prime numbers. The prime numbers stands on several basic assumptions. Given a prime number  $p$ . Then

**Postulate 3.1.**  $2 \leq p$ .

Postulate 3.1 says that 2 is the least prime number. 2 is the only one even prime number. 2 is also assumed as a generator of primes.

**Postulate 3.2.**  $4 \nmid p$ .

Postulate 3.2 holds generally for any prime number. Postulate 3.2 expresses the indivisibility of primes over the number 4. The number 4 is the least composite number which cannot divide any prime number.

**Definition 3.3.** For an integer  $n > 1$ , where  $\tau(n)$  denote the number of positive divisors of  $n$ . The function  $\chi(n)$  is defined by

$$\chi(n) = \begin{cases} 0 & \text{if } \tau(n) = 2 \\ 1 & \text{if } \tau(n) > 2. \end{cases}$$

**Postulate 3.4.**  $(-1)^{\chi(p)} = 1$ .

Postulate 3.4 shows the connection between  $-1$  and the prime number  $p$ . For a composite number  $k$ , then  $(-1)^{\chi(k)} = -1$ . Hence,  $(-1)^{\chi(n)} = \pm 1$  for  $n \geq 2$ .

$\sigma(n)$  denotes the sum of positive divisors of  $n$ . Then

**Postulate 3.5.**  $3 \leq \sigma(p)$ .

Postulate 3.5 expresses that 3 is the lower bound of  $\sigma(p)$ . Postulate 3.1 can deduce Postulate 3.5. Given  $2 \leq p$ . Then  $2 + 1 \leq p + 1$  implies  $3 \leq \sigma(p)$ . Postulate 3.5 is the one of basic patterns of the function  $\sigma(n)$ .

**Definition 3.6.** Given an integer  $n > 1$ , let  $\Delta(n)$  denote the number of positive divisors of  $n$  besides 1 and  $n$ .

**Postulate 3.7.**  $\Delta(p) = 0$ .

Postulate 3.7 shows that the zeros of such a function are any prime number. By our observation, we get the estimation. Let  $p_n$  be the  $n$ th prime, where  $n$  is a positive integer. Then

$$\frac{p_{n+1}}{p_n} \leq 1.7.$$

We have tested for several positive integers of  $n$ . The result can be seen in the table below.

$n$	$p_{n+1}/p_n$
1	1.500
2	1.667
3	1.400
4	1.571
5	1.182
6	1.308
7	1.118
8	1.211
9	1.261
10	1.069

Table 1: The ratio of  $p_{n+1}/p_n$

## References

- [Bur02] D. M. Burton. *Elementary Number Theory*. McGraw-Hill Companies, Inc., New York, 5th edition, 2002.
- [Eis96] M. Eisenberg. *The Mathematical Method*. Prentice-Hall, Inc., Upper Saddle River, NJ, 1996.
- [Har99] G. H. Hardy. *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*. Chelsea, New York, 3rd edition, 1999.
- [Rie59] B. Riemann. Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse. *Monatsber. Akad. Berlin*, pages 671–680, 1859.
- [Wei03] E. W. Weisstein. *CRC Concise Encyclopedia of Mathematics*. Chapman & Hall/CRC, Boca Raton, FL, 2nd edition, 2003.