Einstein-Whitehead model of GW150914 (?)

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Abstract

A fully relativistic encounter of two 35 solar masses point-like objects, somehow mimicking the binary merger GW150914 described by LIGO Scientific and VIRGO collaborations (Ann. Phys. (Berlin) 529 No 1-2)

Using Maple18 language

One body Whitehead model of Schwarzschild`s solution

```
> restart:
> with(tensor):
```

```
> g[4,4]:=(-1+2*mp/r)*c^2;
```

$$g_{4,\,4} := \left(-1+\frac{2\,mp}{r}\right)c^2 \tag{1}$$

```
> g[4,1]:=(2*mp*x/r^2)*c;
```

$$g_{4,\,1} := \frac{2\,mp\,x\,c}{r^2} \tag{2}$$

```
> g[4,2]:=(2*mp*y/r^2)*c;
```

$$g_{4,\,2} := \frac{2\,mp\,y\,c}{r^2} \tag{3}$$

```
> g[4,3]:=(2*mp*z/r^2)*c;
```

$$g_{4,\,3} := \frac{2\,mp\,z\,c}{r^2} \tag{4}$$

```
> g[1,1]:=1+2*mp*x^2/r^3;
```

$$g_{1,\,1} := 1+\frac{2\,mp\,x^2}{r^3} \tag{5}$$

```
> g[2,2]:=1+2*mp*y^2/r^3;
```

$$g_{2,\,2} := 1+\frac{2\,mp\,y^2}{r^3} \tag{6}$$

```
> g[3,3]:=1+2*mp*z^2/r^3;
```

$$g_{3,\,3} := 1+\frac{2\,mp\,z^2}{r^3} \tag{7}$$

```
> g[1,2]:=2*mp*x*y/r^3;
```

$$g_{1,\,2} := \frac{2\,mp\,x\,y}{r^3} \tag{8}$$

```
> g[2,3]:=2*mp*y*z/r^3;
```

$$g_{2,\,3} := \frac{2\,mp\,y\,z}{r^3} \tag{9}$$

```
> g[3,1]:=2*mp*x*z/r^3;
```

$$g_{3,\,1} := \frac{2\,mp\,x\,z}{r^3} \tag{10}$$

```
> r:=sqrt(x^2+y^2+z^2);
```

$$r := \sqrt{x^2+y^2+z^2} \qquad \textbf{(11)}$$

```
> coord := [x,y,z,t]:
> g_compts := array(symmetric,sparse,1..4,1..4):
> g_compts[4,4]:=g[4,4]:
> g_compts[4,1]:=g[4,1]:
> g_compts[4,2]:=g[4,2]:
> g_compts[4,3]:=g[4,3]:
> g_compts[1,1]:=g[1,1]:
> g_compts[2,2]:=g[2,2]:
> g_compts[3,3]:=g[3,3]:
> g_compts[1,2]:=g[1,2]:
> g_compts[2,3]:=g[2,3]:
> g_compts[3,1]:=g[3,1]:
> g := create([-1,-1],eval(g_compts)):
> ginv:=invert(g,'detg'):
> D1g:=d1metric(g,coord): D2g:=d2metric(D1g,coord):
> Cf1 :=Christoffel1(D1g):
```

Dropping the ignore # label makes the compilation longer.

```
> #RMN:=Riemann(ginv, D2g, Cf1):
> #RICCI:=simplify(Ricci(ginv,RMN));
```

```
> `tensor/Christoffel2/simp`:=proc(x) simplify(x,trig) end proc:
> Cf2:=Christoffel2 (ginv,Cf1):
> Cf2c:=get_compts(Cf2):
> Cf2p:=act(subs,mp=m,Cf2):
> Cf2pc:=get_compts(Cf2p):
```

Two bodies plane motion---

```
> z:=0; vz:=0;
```

$$z := 0$$
$$vz := 0 \qquad \textbf{(12)}$$

```
> b:=simplify(Cf2c[4,4,4]+2*Cf2c[4,4,1]*vx+Cf2c[4,1,1]*vx^2
           +2*Cf2c[4,4,2]*vy+Cf2c[4,2,2]*vy^2+2*Cf2c[4,1,2]*vx*
  vy);
```

$$b := \frac{1}{\left(x^2+y^2\right)^{5/2} c}\Big(2\, mp\, \big(mp\, x^2\, c^2 + mp\, y^2\, c^2 + 2\, c\, mp\, vx\, x\, \sqrt{x^2+y^2} + 2\, c\, mp\, vy\, \sqrt{x^2+y^2}\; y + c\, vx\, x^3 + c\, vx\, x\, y^2 + c\, vy\, x^2\, y + c\, vy\, y^3 + mp\, vx^2\, x^2 + 2\, mp\, vx\, vy\, x\, y + mp\, vy^2\, y^2 + vx^2\, x^2\, \sqrt{x^2+y^2} - vx^2\, \sqrt{x^2+y^2}\; y^2 + 4\, vx\, vy\, x\, \sqrt{x^2+y^2}\; y - vy^2\, x^2\, \sqrt{x^2+y^2} + vy^2\, \sqrt{x^2+y^2}\; y^2\big)\Big) \qquad \textbf{(13)}$$

```
> bp:=simplify(Cf2pc[4,4,4]+2*Cf2pc[4,4,1]*vxp+Cf2pc[4,1,1]*vxp^2
             +2*Cf2c[4,4,2]*vyp+Cf2pc[4,2,2]*vyp^2+2*Cf2pc[4,1,2]*
  vxp*vyp);
```

(14)

$$bp := \frac{1}{\left(x^2+y^2\right)^{5/2} c} \left(2 \left(m^2 x^2 c^2 + m^2 y^2 c^2 + 2 c m^2 vxp\, x \sqrt{x^2+y^2} + c m\, vxp\, x^3 + c m\, vxp\, x y^2 + 2 c\, mp^2\, vyp \sqrt{x^2+y^2}\, y + c\, mp\, vyp\, x^2 y + c\, mp\, vyp\, y^3 + m^2 vxp^2 x^2 + 2 m^2 vxp\, vyp\, x y + m^2 vyp^2 y^2 + m\, vxp^2 x^2 \sqrt{x^2+y^2} - m\, vxp^2 \sqrt{x^2+y^2}\, y^2 + 4 m\, vxp\, vyp\, x \sqrt{x^2+y^2}\, y - m\, vyp^2 x^2 \sqrt{x^2+y^2} + m\, vyp^2 \sqrt{x^2+y^2}\, y^2\right)\right) \quad (14)$$

```
> odevx:=simplify(ax+Cf2c[1,4,4]+2*Cf2c[1,1,4]*vx+Cf2c[1,1,1]*
  vx^2
         +2*Cf2c[1,2,2]*vy^2+2*Cf2c[1,1,2]*vx*vy=b*vx):
```

```
> odevy:=simplify(ay+Cf2c[2,4,4]+2*Cf2c[2,1,4]*vx+Cf2c[2,1,1]*
  vx^2
         +2*Cf2c[2,2,2]*vy^2+2*Cf2c[2,1,2]*vx*vy=b*vy):
```

```
> odevxp:=axp+Cf2pc[1,4,4]+2*Cf2pc[1,1,4]*vxp+Cf2pc[1,1,1]*vxp^2
           +2*Cf2pc[1,2,2]*vyp^2+2*Cf2pc[1,1,2]*vxp*vyp=bp*vxp:
```

```
> odevyp:=simplify(ayp+Cf2pc[2,4,4]+2*Cf2pc[2,1,4]*vxp+Cf2pc[2,1,
  1]*vxp^2
           +2*Cf2pc[2,2,2]*vyp^2+2*Cf2pc[2,1,2]*vxp*vyp=bp*vyp):
```

DATA ---

```
> c:=299792458.0; # m/s
```

$$c := 2.997924580\ 10^{8} \quad (15)$$

```
> G:=6.6738480*10^(-11);      #  *m^3/(kg*s^2);
```

$$G := 6.673848000\ 10^{-11} \quad (16)$$

```
> DTS:=149597870700;  #  m
```

$$DTS := 149597870700 \quad (17)$$

```
> MS:=2.0*10^30;       # *kg
```

$$MS := 2.000000000\ 10^{30} \quad (18)$$

```
> R:=DTS; Rp:=DTS;
```

$$R := 149597870700$$
$$Rp := 149597870700 \quad (19)$$

```
> M:=MS;  Mp:=MS;  # kg
```

$$M := 2.000000000\ 10^{30}$$
$$Mp := 2.000000000\ 10^{30} \quad (20)$$

```
> GM:=G*M;     #   *kg*s^2;
```

$$GM := 1.334769600\ 10^{20} \quad (21)$$

```
> GMp:=G*Mp;   #   *kg*s^2;
```

$$GMp := 1.334769600\ 10^{20} \quad (22)$$

KONTUZ Gravity waves--

```
> M:=35*M;  GM:=35*GM;
```

$$M := 7.000000000\ 10^{31}$$
$$GM := 4.671693600\ 10^{21} \quad (23)$$

KONTUZ---

```
> #M:=1; GM:=G;
```

```
> Mp:=35*Mp; GMp :=35*GMp;
```

$$Mp := 7.000000000\ 10^{31}$$
$$GMp := 4.671693600\ 10^{21} \quad (24)$$

```
> R:=175*10^3; # R:=256*4*R;                    # m
```

$$R := 175000 \quad (25)$$

```
> Rp:=175*10^3; # :=256*4*Rp;                   # m
```

$$Rp := 175000 \quad (26)$$

```
> m:=GM/c^2; mp:=GMp/c^2;
```

$$m := 51979.60146$$
$$mp := 51979.60146 \quad (27)$$

KONTUZ---

```
> #m:=0;
```

```
> Omega:=sqrt(GMp/(4*R^3));
```

Initial value. Guess from Newton's theory

$$\Omega := 466.8207016 \quad (28)$$

```
> Omega := Omega;
```

$$\Omega := 466.8207016 \quad (29)$$

```
> Per:=evalf(2*Pi/Omega);   #  s
```

$$Per := 0.01345952587 \quad (30)$$

```
> Per/(60*24*365);   # years
```

$$2.560792593\ 10^{-8} \quad (31)$$

```
> x0:=R; xp0:=-x0; y0:=0; yp0:=0;
```

$$x0 := 175000$$
$$xp0 := -175000$$
$$y0 := 0$$
$$yp0 := 0 \quad (32)$$

```
> vx0:=0; vxp0:=-vx0; vy0:=x0*Omega; vyp0:=-vy0;
```

$$vx0 := 0$$
$$vxp0 := 0$$
$$vy0 := 8.169362278\ 10^{7}$$
$$vyp0 := -8.169362278\ 10^{7} \quad (33)$$

```
> odex := diff(x(t), t) = vx(t); odexp := diff(xp(t), t) = vxp(t)
  ;
```

$$odex := \frac{d}{dt}\, x(t) = vx(t)$$
$$odexp := \frac{d}{dt}\, xp(t) = vxp(t) \quad (34)$$

```
> odey := diff(y(t), t) = vy(t); odeyp := diff(yp(t), t) = vyp(t)
```

```
;
```

$$odey := \frac{d}{dt} y(t) = vy(t)$$

$$odeyp := \frac{d}{dt} yp(t) = vyp(t) \quad \textbf{(35)}$$

Whitehead two bodies gtavitational interaction (Ll. Bel)------

```
> odevx:=subs(x=(x(t)-xp(t)),y=(y(t)-yp(t)),vx=vx(t),vy=vy(t),ax=
  diff(vx(t),t),odevx):
> odevy:=subs(x=(x(t)-xp(t)),y=(y(t)-yp(t)),vx=vx(t), vy=vy(t),
  ay=diff(vy(t),t),odevy):
> odevxp:=subs(x=(xp(t)-x(t)),y=(yp(t)-y(t)),vxp=vxp(t),vyp=vyp
  (t),axp=diff(vxp(t),t),odevxp):
> odevyp:=subs(x=(xp(t)-x(t)),y=(yp(t)-y(t)),vxp=vxp(t), vyp=vyp
  (t),ayp=diff(vyp(t),t),odevyp):
```

```
> sys:=odex,odexp,odey,odeyp,odevx,odevxp,odevy,odevyp:
> fncs:={x(t),xp(t),vx(t),vxp(t),y(t),yp(t),vy(t),vyp(t)};
```

$$fncs := \{vx(t), vxp(t), vy(t), vyp(t), x(t), xp(t), y(t), yp(t)\} \quad \textbf{(36)}$$

```
> ics:=x(0)=x0, xp(0)=xp0, y(0)=0, yp(0)=0, vx(0)=vx0, vxp(0)=
  vxp0, vy(0)=vy0,
  vyp(0)=-vy0;
```

$$ics := x(0) = 175000, xp(0) = -175000, y(0) = 0, yp(0) = 0, vx(0) = 0, vxp(0) = 0, vy(0) = 8.169362278\ 10^{7}, vyp(0) = -8.169362278\ 10^{7} \quad \textbf{(37)}$$

```
> dsn:=dsolve({sys,ics},fncs,numeric,method=gear,abserr=1.*10^
  (-10),relerr=1.*10^(-10),output=listprocedure);
```

dsn := [*t* = **proc**(*t*) ... **end proc**, *vx*(*t*) = **proc**(*t*) ... **end proc**, *vxp*(*t*) = **proc**(*t*) ... **end proc**, *vy*(*t*) = **proc**(*t*) ... **end proc**, *vyp*(*t*) = **proc**(*t*) ... **end proc**, *x*(*t*) = **proc**(*t*) ... **end proc**, *xp*(*t*) = **proc**(*t*) ... **end proc**, *y*(*t*) = **proc**(*t*) ... **end proc**, *yp*(*t*) = **proc**(*t*) ... **end proc**] **(38)**

```
> x,xp,vx,vxp,y,yp,vy,vyp:=op(subs(dsn,[x(t),xp(t),vx(t),vxp(t),y
  (t),yp(t),vy(t),vyp(t)]));
```

x, xp, vx, vxp, y, yp, vy, vyp := **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc**, **proc**(*t*) ... **end proc** **(39)**

```
> Omega := 466.8207016;
```

$$\Omega := 466.8207016 \quad \textbf{(40)}$$

```
> Per:=evalf(2*Pi/Omega);
```

$$Per := 0.01345952587 \quad \textbf{(41)}$$

```
> Ti:=-Per;
```

$$Ti := -0.01345952587 \quad \textbf{(42)}$$

```
> Tf:=9*Per; r(Tf);
```

$$Tf := 0.1211357328$$

(43)

$$2.49643052300143\ 10^{6} \tag{43}$$

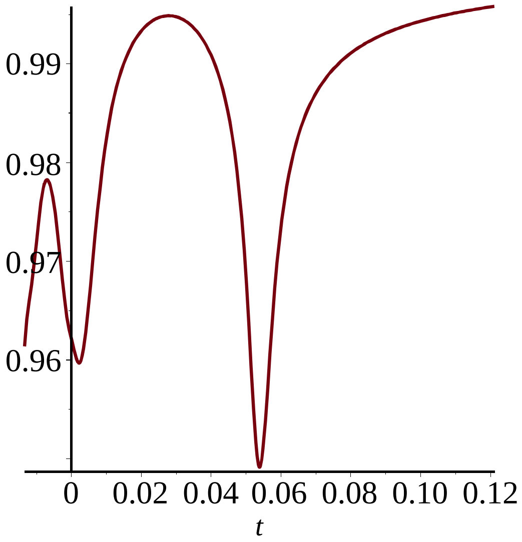
```
> r:=t->sqrt((x(t)-xp(t))^2+(y(t)-yp(t))^2);
```

$$r := t \rightarrow \sqrt{(x(t) - xp(t))^2 + (y(t) - yp(t))^2} \tag{44}$$

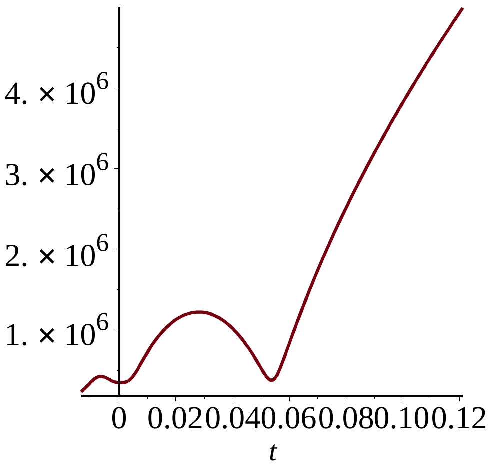
```
> TestV:=t->sqrt(1-(vx(t)^2+vy(t)^2)/c^2);
```

$$TestV := t \rightarrow \sqrt{1 - \frac{vx(t)^2 + vy(t)^2}{c^2}} \tag{45}$$

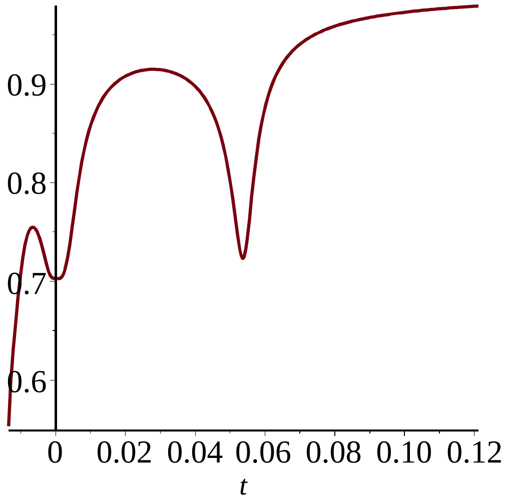
```
> plot(TestV(t),t=Ti..Tf);
```



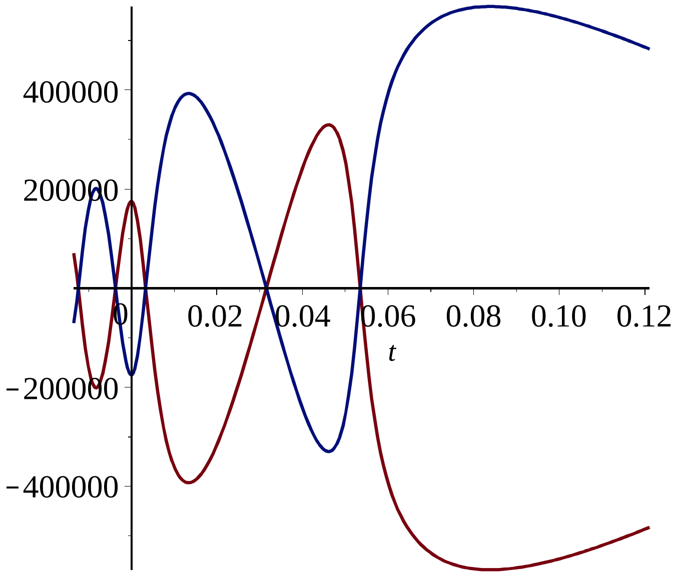

```
> plot(r(t),t=Ti..Tf);
```



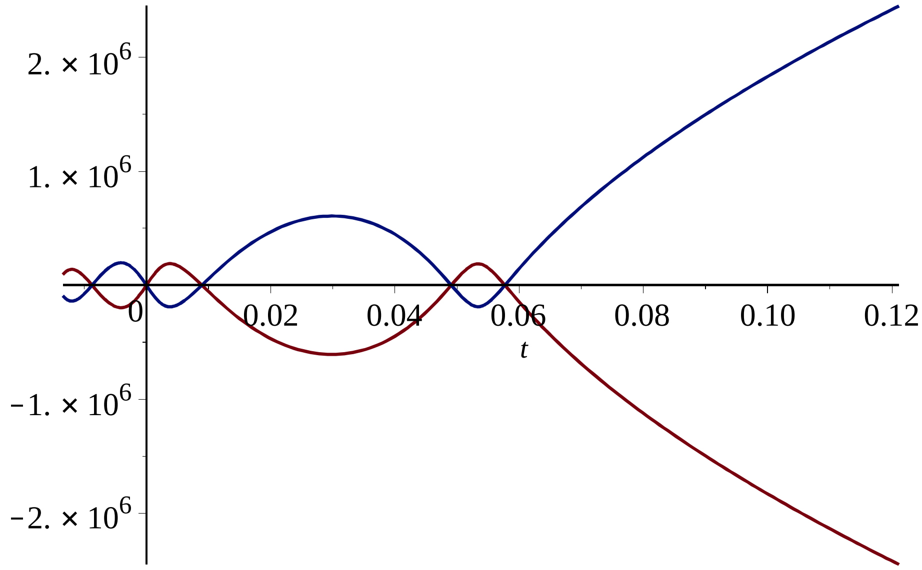

```
> plot(1-2*m/r(t),t=Ti..Tf);
```



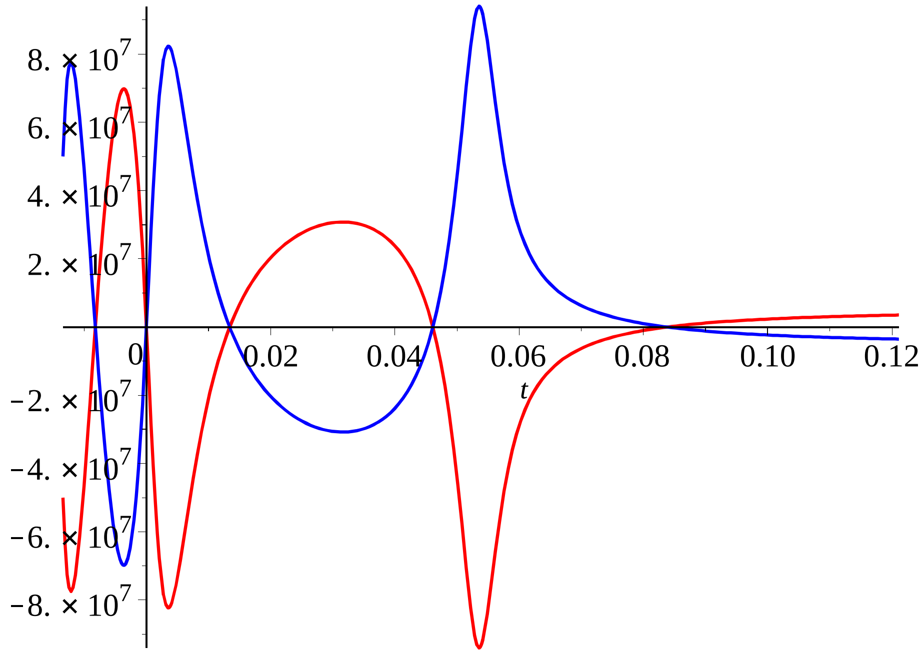

```
> plot([x(t),xp(t)],t=Ti..Tf);
```



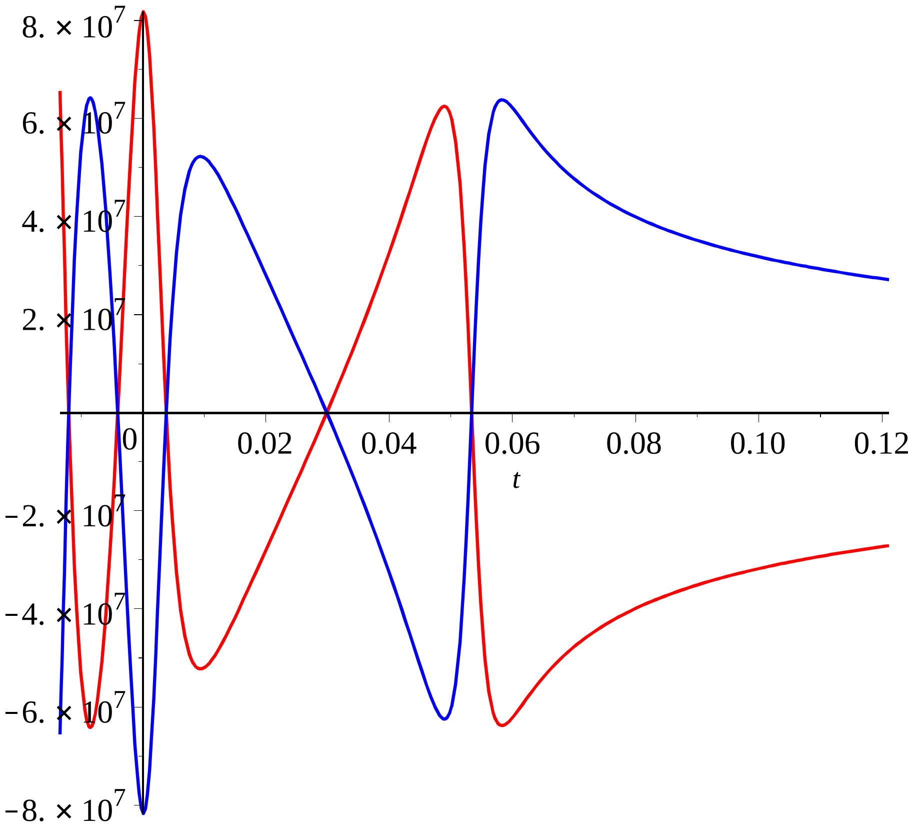

```
> plot([y(t),yp(t)],t=Ti..Tf);
```




```
> plot([vx(t),vxp(t)],t=Ti..Tf,color=[red,blue]);
```



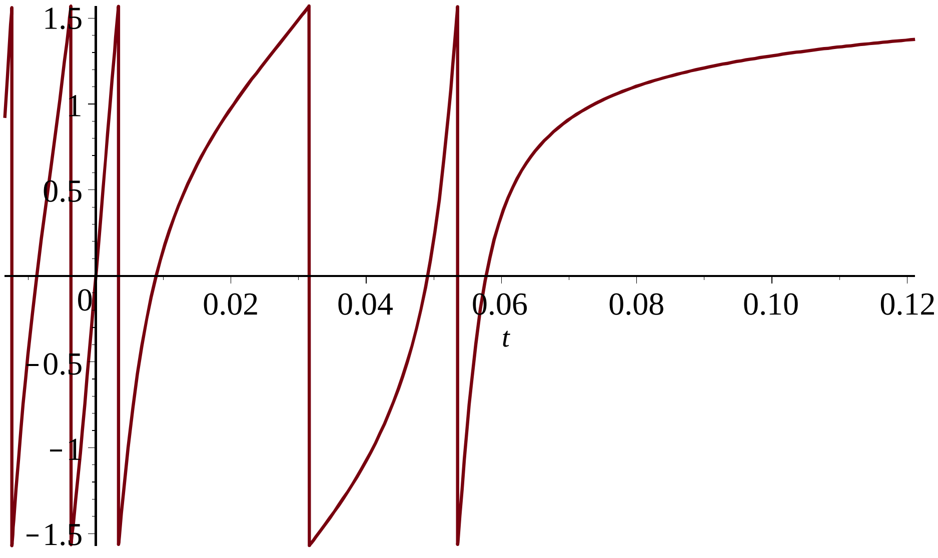

```
> plot([vy(t),vyp(t)],t=Ti..Tf,color=[red,blue]);
```




```
> theta:=t->arctan(y(t)/x(t));
```

$$\theta := t \rightarrow \arctan\left(\frac{y(t)}{x(t)}\right) \quad \textbf{(46)}$$

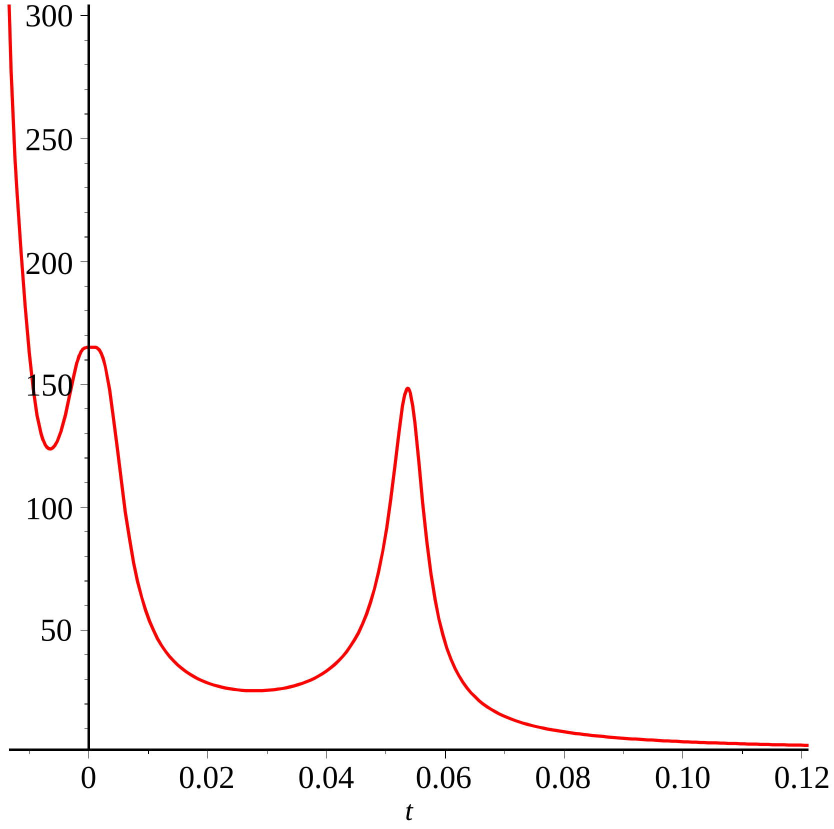
```
> plot([theta(t)],t=Ti..Tf);
```



```
> Omega:=t->sqrt(GM/(4*r(t)^3));
```

$$\Omega := t \to \sqrt{\frac{1}{4}\,\frac{GM}{r(t)^3}} \tag{47}$$

```
> plot([Omega(t)],t=Ti..Tf, color=red);
```



```
> H:=t->(M^2*c^3*(2*c*r(t)+2*vx(t)*x(t)+2*vy(t)*y(t)*mp-M^2*c^4*r
  (t)^2)/(r(t)^2*sqrt(-M^2*c^2*(2*c^2*mp*r(t)^2-c^2*r(t)^3+4*c*
  mp*r(t)*vx(t)*x(t)+4*c*mp*r(t)*vy(t)*y(t)+2*mp*vx(t)^2*x(t)
  ^2+8*mp*vx(t)*vy(t)*x(t)*y(t)+2*mp*vy(t)^2*y(t)^2+r(t)^3*vx(t)
  ^2+r(t)^3*vy(t)^2)/r(t)^3)));
```

$$H := t \to \left(M^2 c^3 \left(2\, c\, r(t) + 2\, vx(t)\, x(t) + 2\, vy(t)\, y(t)\, mp - M^2 c^4 r(t)^2\right)\right) \Big/ \left(r(t)^2 \left(-\frac{1}{r(t)^3}\left(M^2 c^2 \left(2\, c^2\, mp\, r(t)^2 - c^2 r(t)^3 + 4\, c\, mp\, r(t)\, vx(t)\, x(t) + 4\, c\, mp\, r(t)\, vy(t)\, y(t) + 2\, mp\, vx(t)^2 x(t)^2 + 8\, mp\, vx(t)\, vy(t)\, x(t)\, y(t) + 2\, mp\, vy(t)^2 y(t)^2 + r(t)^3 vx(t)^2 + r(t)^3 vy(t)^2\right)\right)\right)^{1/2}\right) \tag{48}$$

Kontuz: +Mp^2, ---

```
> Hp:=t->(Mp^2*c^3*(2*c*r(t)+2*vxp(t)*xp(t)+2*vyp(t)*yp(t)*m-
  Mp^2*c^4*r(t)^2)/(r(t)^2*sqrt(-Mp^2*c^2*(2*c^2*m*r(t)^2-c^2*r
  (t)^3+4*c*m*r(t)*vxp(t)*xp(t)+4*c*m*r(t)*vyp(t)*yp(t)+2*m*vxp
  (t)^2*xp(t)^2+8*m*vxp(t)*vyp(t)*xp(t)*yp(t)+2*m*vyp(t)^2*yp(t)
  ^2+r(t)^3*vxp(t)^2+r(t)^3*vyp(t)^2)/r(t)^3)));
```

$$Hp := t \rightarrow \left(Mp^2\, c^3 \left(2\, c\, r(t) + 2\, vxp(t)\, xp(t) + 2\, vyp(t)\, yp(t)\, m - Mp^2\, c^4\, r(t)^2\right)\right) \Bigg/ \left(r(t)^2 \left(-\frac{1}{r(t)^3} \left(Mp^2\, c^2 \left(2\, c^2\, m\, r(t)^2 - c^2\, r(t)^3 + 4\, c\, m\, r(t)\, vxp(t)\, xp(t) + 4\, c\, m\, r(t)\, vyp(t)\, yp(t) + 2\, m\, vxp(t)^2\, xp(t)^2 + 8\, m\, vxp(t)\, vyp(t)\, xp(t)\, yp(t) + 2\, m\, vyp(t)^2\, yp(t)^2 + r(t)^3\, vxp(t)^2 + r(t)^3\, vyp(t)^2 \right)\right)\right)^{1/2}\right) \quad \textbf{(49)}$$

H=Hp Hamiltonians---

```
> plot([H(t)], t=Ti..Tf);
```