Quantization/normalization of the mutual energy principle

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Abstract

This author has introduced the mutual energy principle, the mutual energy principle successfully solved the problem of conflict between the Maxwell equations and the law of the energy conservation. The mutual energy flow theorem is derived from the mutual energy principle. The mutual energy flow is consist of the retarded wave and the advanced wave. The mutual energy flow theorem tell us the total energy of the energy flow goes through any surfaces between the emitter to the absorber are all same. This property is required by the photon and any quantum. Hence, this author has linked the mutual energy principle to the photon. However there is still a problem, the field of an emitter or the field of an absorber is decrease according to the distance. If the current of a source or sink for a photon is constant. The energy of the photon which equals the inner product of the current and the field will depended on the distance between the the source and the sink of the photon. If the distance increase the amount of photon energy will decrease to infinite small. This is not correct. The energy of a photon should be a constant $E = hv$. The energy of the photon cannot decrease. In order overcome this difficulty, this author make a quantization for the mutual energy principle. It is assume that the retarded wave sent from the emitter has collapse back in all direction. But the mutual energy flow build a channel between the source and sink. Since the energy can only go through this channel, the total energy of a photon must go through this channel. Hence, the total energy of the mutual energy flow has to be normalized to the energy of one photon. The wave energy will increased in the direction of the channel. The amplitude of the wave does not decrease on the direction along the channel. The advanced wave also does not decrease on the direction of the channel. The electromagnetic wave in the space between an emitter (source) and an absorber (sink) look like a wave inside a wave guide. The wave in a guide the amplitude does not decrease alone the wave guide if the loss of energy can be omitted. This wave guide can be called the nature wave guide. In the wave guide the advanced wave leads the the retarded wave, hence the retarded wave can only goes at the direction where has strong advanced wave. This normalization process successfully quantized the the mutual energy flow.

Keywords: Poynting; Maxwell; Schrödinger; Self-energy; Mutual energy; Mutual energy flow; Reciprocity theorem; Radiation; Newton's third law; Action; Reaction; Advanced wave; Photon; Electron; Wave and particle duality; Huygens Principle; Quantization; Normalization.

1 Introduction

1.1 Action at a distance and the absorber theory

The theory about advanced wave became most interesting work for this author. This author noticed the absorber theory of Wheeler and Feynman[1, 2]. The absorber theory is based on the action-at-a-distance [9, 11, 6]. In the absorber theory, any current source sends half retarded wave and half advanced wave. For a source we only notice the source sends the retarded wave, we did not notice it also sends the advanced wave. Some one will argue that if in the same time the source sends the retarded wave, it also sends the advanced wave, the source loss the energy from the retarded wave and acquire the energy from the advanced wave, and hence, it doesn't send any energy out. However we all know that the source can send the energy out. This means the absorber theory also has some thing which is not self-consistence. This is also the reason that the absorber theory has not been widely accepted. But any way, the absorber theory accepts the advanced wave as a real wave instead of some virtual wave. This author is inspired by this a lot. The transactional interpretation of John Cramer has introduced the advance wave to the whole quantum mechanics [3, 4]. Stephenson offered a good tutorial about the advanced wave [10].

1.2 The mutual energy theorems

W.J. Welch introduced a reciprocity theorem in arbitrary time-domain [12] in 1960 (this will be referred as Welch's reciprocity theorem in this article). In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula[8], (this will be referred as Rumsey's reciprocity theorem). In early of 1987 Shuang-ren Zhao (this author) has introduced the concept of mutual energy and the mutual energy theorem [7] (this will be referred as Zhao's mutual energy theorem). In the end of 1987 Adrianus T. de Hoop introduced the time domain cross-correlation reciprocity theorem[5], (this will be referred as Hoop's reciprocity theorem). Welch's reciprocity theorem is a special case of the Hoop's reciprocity theorem.

Assume there are two current sources $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is the current of a transmitting antenna. $\boldsymbol{J}_2$ is the current of a receiving antenna. The field of $\boldsymbol{J}_1$ is described as $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$. The field of the current $\boldsymbol{J}_2$ is $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$. Assume $\boldsymbol{J}_2$ has a some distance with $\boldsymbol{J}_1$. Hoop's reciprocity theorem can be written as,

$$-\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{J}_1(t)\cdot\boldsymbol{E}_2(t+\tau)dV=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t+\tau)dV \qquad (1)$$

if $\tau = 0$, we have,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV = \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \tag{2}$$

This is Welch's reciprocity theorem. The Fourier transform of Hoop's reciprocity theorem can be written as,

$$-\iiint_{V_1} \boldsymbol{J}_1(\omega) \cdot \boldsymbol{E}_2(\omega)^* dV = \iiint_{V_2} \boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2(\omega)^* dV \tag{3}$$

Where "*" is the complex conjugate operator. In this article if the variable t is applied in a formula, it is in time-domain. If ω is applied, it is in Fourier frequency domain. This is the Rumsey's reciprocity theorem and is also Zhao's mutual energy theorem. Hence this 4 theorems can be seen as one theorem in different domain: time-domain or Fourier domain.

Shuang-ren Zhao noticed that this theorem is an energy theorem, hence the two fields in the formula must all physic waves. The other author referred the theorem as reciprocity theorem, as a reciprocity theorem, it can be a mathematical theorem. One of the two fields can be virtual instead of real. If it is virtual even it is an advance wave, that can be easily accepted. If the two fields are all real as the mutual energy theorem required, we must first accept the advanced wave. The advanced wave are not obey the traditional causality consideration.

1.3 The mutual energy flow theorems

This author introduced the mutual energy flow theorem 30 years later than the mutual energy theorem. The mutual energy flow theorem is following,

$$-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$$

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{4}$$

Here $-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$ is the energy offered by the current $\boldsymbol{J}_1(t)$ of emitter. $\int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt$ is the energy received by the absorber. $\int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$ is the energy flow goes through any surface Γ. Γ is at the place between the source and the sink. The mutual energy flow theorem is stronger than the mutual energy theorem. It tells us how the energy flow goes from the the source to the sink.

1.4 The mutual energy principle and self-energy principle

This author also introduced the mutual energy principle and the self-energy principle. Which can overcome the difficulty that the Maxwell equations, which conflict with the energy conservation law. The mutual energy flow theorem for two charges in an empty space are following,

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$
$$=\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV$$
$$+\int_V(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dV \tag{5}$$

where $\partial=\frac{\partial}{\partial t}$. From the mutual energy principle the mutual energy flow theorem and mutual energy theorem Eq.(54) can be derived.

It should be noticed that, mutual energy theorem is actually energy conservation law for a space has only two charges, one is emitter and another is absorber. This is artificial space in which there is only two charges and nothing else. If the mutual energy principle is the axiom of the electromagnetic field theory, the mutual energy theorem is derived as an energy conservation law. Or we can say the energy conservation law can be derived through the mutual energy principle. The mutual energy theorem can also be derived through Maxwell equations, but it is only can be derived as a energy theorem: or mutual energy theorem, not a energy conservation law. Since there is also the self-energy, self-energy flow in the space. This author introduced the self-energy principle, self-energy principle tell us that the self-energy flow does not transfer any energy. The self-energy flow is canceled by the time-reversal self-energy flow. After introducing the self-energy principle, we can also prove that the mutual energy theorem is an energy theorem (energy law).

1.5 The problem of the mutual energy flow theorem

The mutual energy flow theorem and the mutual energy theorem is energy conservation law. The total energy of the mutual energy flow has very good property because it does not change for any surfaces between the source and the sink. It has the shape very thin in two ends (source and sink) and very thick in the middle between the two ends. Hence, it looks particle in the place of source and sink. These properties are all properties a photon or a particle needs. However there is still a problem that is the energy of the photon will decrease with the distance between the two ends. The farther the distance between the source and the sink, the smaller the energy of the photon.

This article try to solve this problem. The quantization or normalization method for the mutual energy principle is introduced.

2 Quantization/normalization of the mutual energy principle

The mutual energy flow theorem can be written as,

$$-\int_{t=-\infty}^{\infty}\iiint_{V}\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt$$

$$=\int_{t=-\infty}^{\infty}\oint\!\!\!\oint_{\Gamma}(E_1(t)\times H_2(t)+E_2(t)\times H_1(t))\cdot\hat{n}d\Gamma dt$$

$$=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dVdt \tag{6}$$

$\boldsymbol{J}_1(t)$ and $\boldsymbol{J}_2(t)$ are the current intensity of the emitter and absorber, assume that this is a constant. $\boldsymbol{E}_1(t)$ is the electric field of the retarded wave and $\boldsymbol{E}_2(t)$ the advanced wave which are decrease with the distance between the emitter and the absorber: $r=||\boldsymbol{x}_2-\boldsymbol{x}_1||$. $\boldsymbol{x}_1$ is position of the emitter and $\boldsymbol{x}_2$ is the position of the absorber. Hence, we have,

$$\boldsymbol{J}_1(t),\boldsymbol{J}_2(t)\sim const \tag{7}$$

$$\boldsymbol{E}_1(t)\sim\frac{1}{r}\boldsymbol{J}_1(t) \tag{8}$$

$$\boldsymbol{E}_2(t)\sim\frac{1}{r}\boldsymbol{J}_2(t) \tag{9}$$

$$\lim_{r\to\infty}\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dVdt\sim\lim_{r\to\infty}\frac{1}{r}=0 \tag{10}$$

$$\lim_{r\to\infty}\int_{t=-\infty}^{\infty}\iiint_{V}\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt\sim\lim_{r\to\infty}\frac{1}{r}=0 \tag{11}$$

We know a photon has constant energy, $hv=const$. where h is Planck constant. v is the frequency. In order to make,

$$\lim_{r\to\infty}\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dVdt=const \tag{12}$$

where *const* take a constant value which is the energy of a photon $const=hv$. where h is Planck constant, v is the frequency of the light. We need either

$$\boldsymbol{J}_1(t)\sim r,\quad \boldsymbol{J}_2(t)\sim r \tag{13}$$

$$\boldsymbol{E}_1(t) \sim \frac{1}{r}\boldsymbol{J}_1(t) = const \tag{14}$$

$$\boldsymbol{E}_2(t) \sim \frac{1}{r}\boldsymbol{J}_2(t) = const \tag{15}$$

This means when the distance between the two charge increased, the current of the charge intensity needs to increase. Or

$$\boldsymbol{J}_1(t), \boldsymbol{J}_2(t) \sim const \tag{16}$$

$$\boldsymbol{E}_1(t) \sim \boldsymbol{J}_1(t) \tag{17}$$

$$\boldsymbol{E}_2(t) \sim \boldsymbol{J}_2(t) \tag{18}$$

This tell us that the electromagnetic fields do not decrease with the distance r.

This author thought the above both are possible. First option let us see Eq.(16-18). This author assumes that if the waves (retarded wave and the advanced wave) all collapse back in all directions, but the mutual energy flow still keep a energy channel from the source to the sink. Hence, the waves has collapse back in all other direction, the waves amplitude will increase at the directions of the the energy channel. This make the field amplitude not decrease at the direction of along the energy channel. The energy channel is at the direction of the mutual energy flow.

The self-energy flow collapse back has no any price to the photon system. Hence, if the energy of a photon becomes smaller, it can send more times until the energy of a whole photon is sent out. This also can be simulated by increase the current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$, hence, we can let

$$\boldsymbol{J}_1 \sim r, \qquad \boldsymbol{J}_2 \sim r \tag{19}$$

The wave will looks like it is propagate inside a wave guide. The shape of the wave guide is very thin in the two ends of wave guide. It is very wide in the middle between the two ends. Hence, we can have,

$$||\boldsymbol{E}_1(\boldsymbol{x}_1)|| = ||\boldsymbol{E}_1(\boldsymbol{x}_2)|| \qquad ||\boldsymbol{E}_2(\boldsymbol{x}_1)|| = ||\boldsymbol{E}_2(\boldsymbol{x}_2)|| \tag{20}$$

It is clear that there is,

$$||\boldsymbol{E}_1(\boldsymbol{x}_3)|| < ||\boldsymbol{E}_1(\boldsymbol{x}_2)|| \qquad ||\boldsymbol{E}_2(\boldsymbol{x}_3)|| < ||\boldsymbol{E}_2(\boldsymbol{x}_2)|| \tag{21}$$

Here $\boldsymbol{x}_3$ is a point between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. $||\cdot||$ is the norm of the vector. But the energy go through each stream line should be still equal.

This author assume that the wave sends according to Maxwell equations which is a wave decrease with the distance r, since only in one direction which is direction of the absorber there is the advanced wave, which leads the retarded wave go along that direction. All the waves of self energy flow collapse back to the source and re-send out. The waves which collapse back are time-reversal waves. When the wave is re-sent out, only in the direction of the absorber can produce a mutual energy flow. hence the value of the energy flow is increased in the direction of absorber. The retarded wave in the mutual energy flow goes alone the direction of absorber. The advanced wave goes to the direction of the emitter. Hence, the waves does not decrease with the distance r.

This can be seen as a normalization process, the whole mutual energy of the mutual energy flow,

$$\int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt = const = hv \tag{22}$$

The mutual energy flow calculated through the retarded wave and the advanced wave in the space produced a photon which is decrease with the distance between the source and think. After the normalization the photon energy must equal to hv. Where h is the Planck constant. v is the frequency of the photon.

Since after this normalization a photon with correct energy is produced we also call this is a quantization process.

It seams the energy has collapse from all space to the absorber, but actually the self-energy flow collapse back through the time-reversal wave. After the self-energy is collapsed back the energy is re-sent out many many times, only at the direction of the absorber produced the mutual energy flow is produced. The mutual energy flow need to be increased to a whole photon.

2.1 Microscopic Maxwell equations, Macroscopic Maxwell equations

This author assume the solutions of the microscopic Maxwell equation, which are the retarded wave and the advanced wave is the wave decrease with the distance r. The electromagnetic field for a photon that is quantized electromagnetic field, which is not decrease with the distance r. The macroscopic wave which is the wave average the photon energy from an absorber to a $4\pi r^2$ area, and hence, also reduce with the distance r. Hence, the macroscopic wave and microscopic wave are all retarded wave and advanced wave satisfy the Maxwell equations, the amplitude of the wave reduce with the distance r. How ever, the amplitude of the quantized wave is not reduce with the distance r.

The result of Maxwell equations

$$E_{mic} = ||\boldsymbol{E}_{mic}|| \sim \frac{1}{r} \tag{23}$$

$$E_{mac} = ||\boldsymbol{E}_{mac}|| \sim \frac{1}{r} \tag{24}$$

$$E_{quantized} = ||\boldsymbol{E}_{quantized}|| \sim cost \tag{25}$$

$\boldsymbol{E}_{mic}$ is the microscopic wave. E_{mac} is the macroscopic wave. $E_{quantized}$ is quantized wave.

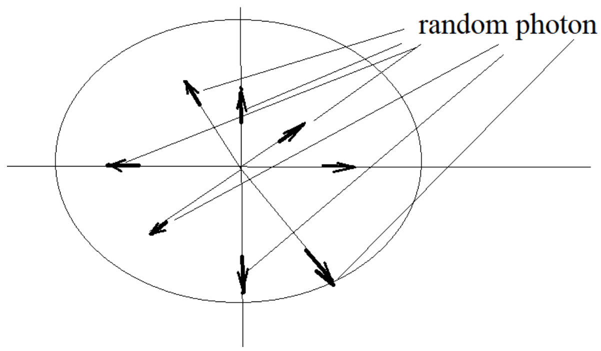

Figure 1: A photon does not reduce the amplitude of the electromagnetic field, but if average each photon to the whole area $4\pi r^2$, the energy will decrease with the fact $\frac{1}{r^2}$, and the field will decrease with the distance fact $\frac{1}{r}$..

is because a statistics average. $\boldsymbol{E}_{mac}$ is the field of the Maxwell equations. The field sends in one direction (one of a absorber) is $E_{quantized} = ||\boldsymbol{E}_{quantized}|| \sim const$. However, the probability this absorber in that direction receive a photon is decrease with the area of the sphere with the radio equal to r. If we consider the probability

$$\boldsymbol{E}_{mac} = \sqrt{P(a)}\boldsymbol{E}_{quantized} \sim \frac{1}{r} \tag{26}$$

where $P(a)$ is the probability to whole sphere.

Hence, in the electromagnetic field theory, we often have the field which decreases with the distance r that is because the statistic average effect. Hence, the Maxwell equations still can be established on the base of statistic average. The field we obtained from Maxwell equations is,

$$\boldsymbol{E}_{mac} = \frac{1}{T}\int_{t=0}^{T} \boldsymbol{E}(t)dt \sim \frac{1}{r} \tag{27}$$

The field of the Maxwell equation is still correct. The only thing need to know is that it is average result to time. Figure 1 shows the photon is send at different directions. The photon energy is not decrease with the distance. But the field of Maxwell which is a average on the whole sphere with area $4\pi r^2$. This leads the averaged field decrease with the distance of distance r.

This author still assume that in the beginning the emitter and the absorber send the broadcast wave which is the wave decrease with distance r, and satisfies the Maxwell equations. Assume that the emitter sends the retarded wave, assume that the absorber sends advanced wave. This wave all belong to broadcast wave which is decrease the amplitude with distance r. If the two waves do not synchronized, the wave is returned to its source or think throguh the time-reversal waves corresponding to the two broadcast waves. If the two waves are synchronized, the advanced wave become the lead wave to the retarded wave, which increase the amplitude of both the retarded wave and the advanced wave in the direction of energy channel. The two waves do not decrease with the distance between the two charges (emitter and the absorber). However, if we average this kind wave in all direction, we can obtained the original wave of the Maxwell equations offered for single charge, which decrease with the distance r.

Hence, we can say that the advanced wave produced a wave guide to the retarded wave. The wave of this synchronized wave are wave in a guide. Wave-guide wave is focalized broadcast wave. This focalization process can be referred as quantization of the the mutual energy principle. After this quantization, the energy of the mutual energy flow does not decrease with the distance $r = ||\boldsymbol{x}_2 - \boldsymbol{x}_1||$. (1) The photon energy does not decrease with distance between emitter and absorber and (2) the photon energy are equal at any surfaces between the emitter and absorber are two most important properties of the photon.

2.2 The original mutual energy principle

The original mutual energy principle related to the Maxwell equations should be seen also the result of the statistic average.

From that we obtained the result the lager the distance between the emitter to the absorber, the smaller the energy of the photon. This result is not correct, but it is good enough to a macroscopic result. In the macroscopic situation how big the energy of a photon is no sense. If the photon is small, with more photon will have the same effect. The number of photon is also no since to the macroscopic situation. Hence in the macroscopic wireless wave situation, there are infinite photons, Maxwell equations and original mutual energy principle all still can be applied.

The mutual energy principle batter than the Maxwell equations at the point it overcome the problem of the conflict the Maxwell equations with the energy conservation law.

2.3 The mutual energy flow theorem

The mutual energy flow theorem still can be established after the quantization of the mutual energy principle. The reason is that the mutual energy principle is also established on a wave guide according to the original theorem, after the quantization that is just like the field restricted on the wave guide. A original derivation of the mutual energy principle and mutual energy flow theorem are still effective.

3 Huygens principle

3.1 Huygens sources

Considering,

$$\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) \cdot \hat{n} = \boldsymbol{E}_1(t) \cdot \boldsymbol{H}_2(t) \times \hat{n} = \boldsymbol{E}_1(t) \cdot J_{h2}(t) \tag{28}$$

$$\boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) \cdot \hat{n} = \hat{n} \cdot \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) = \hat{n} \times \boldsymbol{E}_2(t) \cdot \boldsymbol{H}_1(t) = K_{h2} \cdot \boldsymbol{H}_1(t) \tag{29}$$

we can defined the Huygens source,

$$\sigma_2 = \begin{cases} J_{h2}(t) = \boldsymbol{H}_2(t) \times \hat{n} \\ K_{h2}(t) = \hat{n} \times \boldsymbol{E}_2(t) \end{cases} \tag{30}$$

The inner product becomes,

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \cdot J_{h2}(t) + \boldsymbol{H}_1(t) \cdot \boldsymbol{J}_{h2}(t)) d\Gamma \tag{31}$$

$$\hat{n} \cdot \boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) = \hat{n} \times \boldsymbol{E}_1(t) \cdot \boldsymbol{H}_2(t) = K_{h1}(t) \cdot \boldsymbol{H}_2(t) \tag{32}$$

$$\boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) \cdot \hat{n} = \boldsymbol{E}_2(t) \cdot \boldsymbol{H}_1(t) \times \hat{n} = \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1} \tag{33}$$

The Huygens source is,

$$\sigma_1 = \begin{cases} J_{h1}(t) = \boldsymbol{H}_1(t) \times \hat{n} \\ K_{h1}(t) = \hat{n} \times \boldsymbol{E}_1(t) \end{cases} \tag{34}$$

The inner product becomes,

$$\begin{aligned} (\xi_2, \xi_1) &= \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma \\ &= \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{H}_2(t) \cdot K_{h1}(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1}(t)) d\Gamma \end{aligned} \tag{35}$$

In the above, we have choose ξ_1 as the retarded wave, ξ_2 as the advanced wave.

$$(\xi_2, \xi_1) = (\xi_2, \sigma_1) = (\sigma_2, \xi_1) \tag{36}$$

where,

$$(\xi_2, \sigma_1) \equiv \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{H}_2(t) \cdot K_{h1}(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1}(t)) d\Gamma \tag{37}$$

$$(\xi_1, \sigma_2) \equiv \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \cdot J_{h2}(t) + \boldsymbol{H}_1(t) \cdot \boldsymbol{J}_{h2}(t)) d\Gamma \tag{38}$$

From the above we can see the field ξ_1 and ξ_2 can be replaced as the corresponding Huygens sources σ_1 and σ_2.

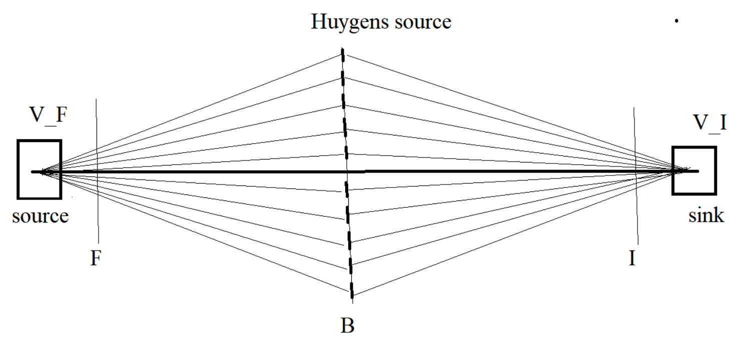

Figure 2: The source is inside the volume V_F, the sink is inside of the volume V_I. F is the surface of the volume V_F, I is the surface of the volume V_I. B is the surface between the source and the sink. .

3.2 Generalized Huygens principle

In the quantum mechanics there is the bra $\langle\xi|$ and the ket $|\xi\rangle$, which can be used as the definition for the inner product, i.e.,

$$\langle\xi_1|\xi_2\rangle \equiv \langle\xi_1||\xi_2\rangle \equiv (\xi_1,\xi_2)$$

$$\equiv \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t)\times\boldsymbol{H}_2(t)+\boldsymbol{E}_2(t)\times\boldsymbol{H}_1(t))\cdot\hat{n}d\Gamma \tag{39}$$

where

$$\xi_1 \equiv [\boldsymbol{E}_1,\boldsymbol{H}_1] \tag{40}$$

$$\xi_2 \equiv [\boldsymbol{E}_2,\boldsymbol{H}_2] \tag{41}$$

The mutual energy flow theorem can be written as,

$$\langle\xi_F|\xi_I^F\rangle = \langle\xi_F^B|\xi_I^B\rangle = \langle\xi_F^I|\xi_I\rangle \tag{42}$$

ξ_F^B is the field at B produced by the source at F. The subscript F is used to express the final point or the place of the sink. The superscript B is the place of the field. Here B is the surface between the I and F. I is the initial source place. The mutual energy flow theorem and the mutual energy theorem together can be rewritten as,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_I} \boldsymbol{J}_I(t)\cdot\boldsymbol{E}_F^I(t)dV$$

$$= \langle\xi_F|\xi_I^F\rangle = \langle\xi_F^B|\xi_I^B\rangle = \langle\xi_F^I||\xi_I\rangle$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V_F} \boldsymbol{E}_I^F(t) \cdot \boldsymbol{J}_F(t) dV \tag{43}$$

The details, can be found in Figure 2. In the above ξ_I^F is field at F, which is produced by the source I. ξ_F is the field at F produced by the source at F. $\langle \xi_F | \xi_I^F \rangle$ is the inner product at the surface F. $\langle \xi_F^B | \xi_I^B \rangle$ is a inner product at the surface B. $\langle \xi_F^I | \xi_I \rangle$ is a inner product at the surface I. We use F to express the surface of the volume V_F. I is the surface of the volume V_I. B is any surface between I and F. We assume V_I is the source, V_F is the sink.

The above mutual energy flow theorem is also suitable to quantum mechanics, can be written as

$$\langle \Psi_b | \Psi_a^b \rangle = \langle \Psi_b^c | \Psi_a^c \rangle = \langle \Psi_b^a | \Psi_a \rangle \tag{44}$$

$\langle \Psi_b^c | \Psi_a^c \rangle$ is the inner produce on the surface c.$\langle \Psi_b | \Psi_a^b \rangle$ is the inner product on the surface b $\langle \Psi_b^a | \Psi_a \rangle$ is the inner product on the surface a.

In the following discussion we only use electromagnetic field as example, the result is also suitable to the quantum mechanics.

We have known that in quantum mechanics there is,

$$\sum_i |q_i\rangle\langle q_i| = 1 \tag{45}$$

In our situation, our inner product is at the surface B. That is a integral on the surface B and, hence, we can rewritten the above formula as,

$$\sum_i |\xi_{Bi}\rangle\langle \xi_{Bi}| = 1 \tag{46}$$

Substitute the above formula to the mutual energy flow theorem Eq.(43), the mutual energy flow theorem can be written as,

$$\langle \xi_F | \xi_I^F \rangle = \sum_i \langle \xi_F^B | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^B \rangle = \langle \xi_F^I | \xi_I \rangle \tag{47}$$

Considering the mutual energy flow theorem about the surface B and F we have,

$$\langle \xi_F^B | \xi_{Bi} \rangle = \langle \xi_F | \xi_{Bi}^F \rangle = \langle \xi_F | G_B^F | \xi_{Bi} \rangle \tag{48}$$

In the above $\langle \xi_F^B | \xi_{Bi} \rangle$is a inner product at the surface B. $\langle \xi_F | \xi_{Bi}^F \rangle$ is a inner product at F. The first equal sign is because the mutual energy flow theorem, the inner product can be moved from the place of B to the place F. In the second equal sign, we have considered the definition of G_B^F, hence, $|\xi_{Bi}^F\rangle = G_B^F |\xi_{Bi}\rangle$. Where G_B^F is the gain from $|\xi_{Bi}\rangle$to $|\xi_{Bi}^F\rangle$. G_B^F is a operator, or matrix. Hence, we have,

$$\langle \xi_F | \xi_I^F \rangle = \sum_i \langle \xi_F | \xi_{Bi}^F \rangle \langle \xi_{Bi} | \xi_I^B \rangle$$

$$= \sum_i \langle \xi_F | G_B^F | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^B \rangle$$

$$= \sum_i \langle \xi_F | G_{Bi}^F | \xi_{Bi} \rangle \langle \xi_{Bi} | G_I^{Bi} | \xi_I \rangle$$

$$= \langle \xi_F^I | \xi_I \rangle \tag{49}$$

The above formula actually has problem, G_B^F is still the gain operator, but it actually is the gain operator from $|\xi_{Bi}\rangle$ to $|\xi_I^F\rangle$. And hence can be written as G_{Bi}^F. G_{Bi}^F actually is the matrix element. In the above, insert the unit operator $\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1$,

$$AB = \sum_j A|\xi_j\rangle\langle\xi_j|B$$

$$= \sum_j A_j^k |\xi_j\rangle\langle\xi_j| B_i^j$$

$$= \sum_j A_j^k B_i^j \tag{50}$$

The two matrix AB after insert the unit operator become the matrix element expression. Hence, Eq(49) can be rewritten as,

$$\langle \xi_F | \xi_I^F \rangle = \sum_i \langle \xi_F | G_{Bi}^F | \xi_{Bi} \rangle \langle \xi_{Bi} | G_I^{Bi} | \xi_I \rangle = \langle \xi_F^I || \xi_I \rangle \tag{51}$$

This is the Huygens principle for the retarded wave. In the surface B, $|\xi_{Bi}\rangle$ is the unit Huygens source, $\langle \xi_{Bi} | \xi_I^{Bi} \rangle = \langle \xi_{Bi} | G_I^{Bi} | \xi_I \rangle$ is the value of the Huygens source, $|\xi_{Bi}\rangle\langle\xi_{Bi}||\xi_I^{Bi}\rangle$is the Huygens source. $\langle \xi_F | G_B^F | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^{Bi} \rangle$ is the contribution of the Huygens source to the surface F. $\sum_i \langle \xi_F | G_B^F | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^B \rangle$ is all contributions of the Huygens sources to the surface F.

From the above Eq.(49), we still have also,

$$\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1 \tag{52}$$

It should be noticed that, in Eq.(47), the unit operator Eq.(46) is inserted to the inner product $\langle \xi_F^B | \xi_I^B \rangle$. In Eq.(49) to obtained the unit operator Eq.(52) we have to apply the mutual energy flow theorem $\langle \xi_F^B | \xi_{Bi} \rangle = \langle \xi_F | \xi_{Bi}^F \rangle$.

In other side,

$$\langle \xi_F^I | = \langle \xi_F | T_F^I | = \langle \xi_F | T_F^B T_B^I | \tag{53}$$

Where T_F^I is the gain to the left vector $\langle \xi_F |$, Hence,

$$\langle \xi_F^I | \xi_I \rangle = \langle \xi_F | T_F^B T_B^I | \xi_I \rangle \tag{54}$$

Insert $\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1$ to the above we obtain,

$$\langle \xi_F^I | \xi_I \rangle = \sum_i \langle \xi_F | T_F^B | \xi_{Bi} \rangle \langle \xi_{Bi} | T_B^I | \xi_I \rangle$$

$$= \sum_i \langle \xi_F | T_F^{Bi} | \xi_{Bi} \rangle \langle \xi_{Bi} | T_{Bi}^{I} | \xi_I \rangle = \langle \xi_F | \xi_I^F \rangle \tag{55}$$

This is the Huygens principle for the advanced wave.

3.3 Application of the Huygens principle to more surfaces

The received energy at the final point F is

$$\langle \xi_F || \xi_I^F \rangle \tag{56}$$

Substitute the following 3 formulas

$$\sum_i |\xi_{Ai}\rangle\langle\xi_{Ai}| = 1 \tag{57}$$

$$\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1 \tag{58}$$

$$\sum_i |\xi_{Ci}\rangle\langle\xi_{Ci}| = 1 \tag{59}$$

to the above formula we have,

$$\begin{aligned}
\langle \xi_F | \xi_I^F \rangle &= \langle \xi_F | G_I^F \xi_I \rangle \\
&= \langle \xi_F G_C^F G_B^C G_A^B G_I^A \xi_I \rangle \\
&= \sum_{kji} \langle \xi_F | G_C^F | \xi_{Ck} \rangle \langle \xi_{Ck} | G_B^C | \xi_{Bj} \rangle \langle \xi_{Bj} | G_A^B | \xi_{Ai} \rangle \langle \xi_{Ai} | G_I^A | \xi_I \rangle \\
&= \sum_{kji} \langle \xi_F | G_{Ck}^F | \xi_{Ck} \rangle \langle \xi_{Ck} | G_{Bj}^{Ck} | \xi_{Bj} \rangle \langle \xi_{Bj} | G_{Ai}^{Bj} | \xi_{Ai} \rangle \langle \xi_{Ai} | G_I^{Ai} | \xi_I \rangle
\end{aligned} \tag{60}$$

In the above, we have considered that,

$$\xi_I^F = G_I^F \xi_I \tag{61}$$

where G_I^F is the gain from ξ_I to ξ_I^F, it is clear that we have,

$$G_I^F = G_C^F G_B^C G_A^B G_I^A \tag{62}$$

There is also the similarly formula for the advanced wave.

3.4 The amplitude of the field is not change alone the streamline

According to the assumption of this article, the amplitude of the field alone the streamline will not change, hence we have,

$$G_I^F = \exp(-iHT) \tag{63}$$

$$||G_I^F|| = 1 \tag{64}$$

$$\begin{aligned} G_I^F &= G_C^F G_B^C G_A^B G_I^A \\ &= \exp(-i\sum_{i=1}^{N} H\delta T) \end{aligned} \tag{65}$$

Here H is the Hamilton. This formula is the theory base of the path integral. In the path integral the wave does not decrease with distance. When this author work out the "updated version of the path integral", the streamline integral introduced by this author. In that time this author also noticed that Feynman path integral does not decrease with distance. In the beginning this author thought that it is a mistake of Feynman. Now we know that Feynman is correct at this point. In the future this author will correct the streamline integral using the quantization of the mutual energy principle. After that the wave will not decrease with distance along the streamline. This offers a theory basis to the path integral. This concept can be further wider to that the wave can be calculated at any path, on which the amplitude of the wave does not change. Any paths differ than energy streamline will have very smaller effect and will cancel each other, hence Feynman define the path integral on all paths still can obtained correct result.

However, this author has only proved that the electromagnetic field does not decrease along the streamline. Feynman widen it to all paths, the electromagnetic field does not decrease along any paths that has not been proved as correct.

4 Numerical calculation

The retarded wave for the photon inside the nature wave guide can be calculated according to the boundary condition there is only one source and one sink in the space. The retarded wave started from the source end at the sink. The space between the source and the sink is the nature wave guide. The wave inside the wave guide should satisfy the Maxwell equations. The advanced wave can be calculated exact same way. Hence, the retarded wave and the advanced wave are same. Perhaps the finite element method can be applied for this kind of calculation.

The author introduce an approximate method, we need to obtained the stream line of the photon, which can be obtained by the traditional mutual energy flow theorem. That means we first calculate the retarded wave sending from the emitter to free space, the amplitude of the wave is calculated according to $\sim \frac{1}{r}$. Then calculated the advanced wave sending from absorber, the amplitude of the wave is calculate according to $\sim \frac{1}{r}$. The mutual energy flow is calculated through the retarded wave and the advanced wave in free space. This way we calculate the stream line from the source to the sink. It should be largely correct (perhaps it is exactly correct, the author has not proved it). This way even the energy of photon is not correct but we obtained the streamline.

When we obtained the streamline, we have known that this kind of calculation the mutual energy flow will decrease with the distance. The mutual energy flow is equal to,

$$\int\limits_{t=-\infty}^{\infty} \iiint\limits_{V} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{66}$$

If $\boldsymbol{J}_2(t) \sim const$, $\boldsymbol{E}_1(t) \sim \frac{1}{r}$, $\int_{t=-\infty}^{\infty} \iiint_V \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \sim \frac{1}{r}$. According to the mutual energy flow theorem the mutual energy flow $\int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt \sim \frac{1}{r}$.

In order to make the mutual energy flow do not decrease according to $\frac{1}{r}$ We increase the $\boldsymbol{J}_2(t), \boldsymbol{J}_2(t)$ according to

$$\boldsymbol{J}_1(t) := \boldsymbol{J}_1(t)\ r, \qquad \boldsymbol{J}_2(t) := \boldsymbol{J}_2(t)\ r \tag{67}$$

This means we use $\boldsymbol{J}_1(t)\ r$ to replace $\boldsymbol{J}_1(t)$. Use $\boldsymbol{J}_2(t)\ r$ to replace $\boldsymbol{J}_2(t)$, This way,

$$\int\limits_{t=-\infty}^{\infty} \oiint\limits_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma dt = const \tag{68}$$

The mutual energy flow will keep do not change when the distance r increase. In this way, the $\boldsymbol{E}_1(t)$, $\boldsymbol{H}_1(t)$ and $\boldsymbol{E}_2(t)$, $\boldsymbol{H}_2(t)$ can be calculated using the currents of the point charges $\boldsymbol{J}_1(t)$ and $\boldsymbol{J}_2(t)$. The stream line and the energy flow intensity can be calculated according to Eq.(68).

5 The collapse of the wave

In quantum mechanics, there is the concept of wave function collapse. I generally agree that wave is collapsed. However the wave function collapse cannot described by any mathematical formula. There are also many different wave collapse. Fist there are collapse in the measure time that means wave is collapsed when the particle is measured, this belong to Copenhagen interpretation. John Cramer introduced the concept of the continuous collapse, which means the wave continuously collapses to the light ray. According the John Cramer's transactional interpretation, the retarded wave and the advanced wave both collapse to the light ray.

This author suggest that the both wave collapse instead of the light ray, but the mutual energy flow. The mutual energy flow is more concrete concept for light ray. the light ray looks a line but the shape of the mutual energy flow is thin in the two ends and thick in the middle between the two ends.

Since the the collapse cannot be described. This author introduced the time-reversal waves. We assume that the retarded wave and the advanced wave collapse back with the time-reversal waves. Here this author means the self-energy flow all collapse back. Hence retarded wave and the advanced wave will be resend out from the emitter and the absorber. The mutual energy flow will

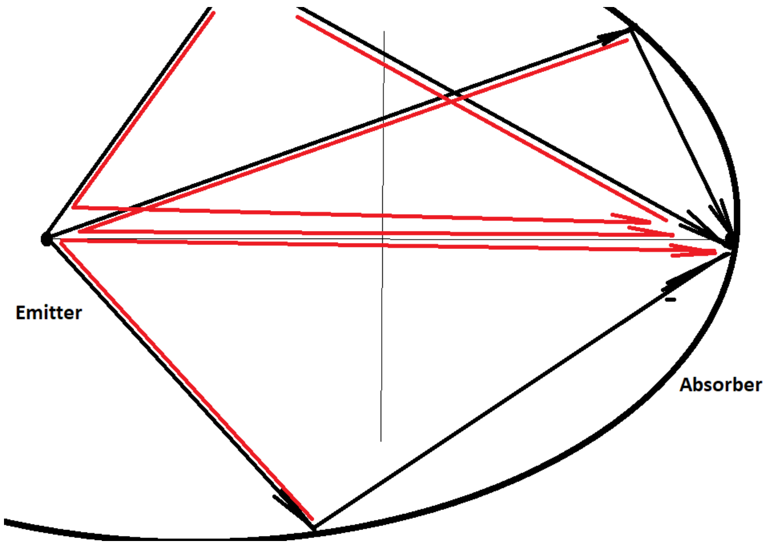

Figure 3: Waves collapse are through two process (1) collapse back(time-reversal self-energy flow) (2) The mutual energy flow accumulatively increases. .

increased more, the self-energy flow collapse back until the mutual energy flow increase to the level of a whole photon.

Hence the wave collapse in consist of two processes one is the the two self-energy flows for the retarded wave and the advanced wave collapse back to the emitter and absorber, the second is the mutual energy flow accumulatively increase. This two process together build the wave collapse process. The all this two processes can be mathematically described. Hence we can offer a wave collapse process described mathematically.

This author also introduced a possibility that the mutual energy flow collapse back with anti-mutual energy flow. Especially in case an absorber obtained a photon energy that is not enough as a whole photon, it will be returned through time-reversal mutual energy flow. In this situation, stimulated radiation, the absorber or emitter in the neighbor will have big chance to send the retarded wave or advanced wave of a photon. All half photon, and the partial photon will collapse back, the energy is resend out. W have applied this method to explain why there is not half photon or partial photon.

This author introduced the concept of time-reversal waves and time-reversal mutual energy flow which is anti-photon all still correct. Figure 3 shows the collapsed waves. The black line shows the wave collapse. The red line further tell us the collapse is through a collapse back (time-reversal wave for the self-energy flow or time-reversal mutual energy flow).

6 Conclusion

We have know the photon is a wave package, that means the shape of photon looks like a pollywog, that means it very thin at the two ends and thick in the middle between the two ends. The two ends are the position of emitter and the absorber. This author looks a theory can produce a wave package looks like pollywog from the concept of the retarded wave and the advanced wave. John Cramer introduced the method of superposition. The retarded wave superposed with the advanced wave. However the superposition cannot produced a wave package which does not send energy out at infinite big sphere. This author looks function which has 0 values every where but not 0 only at the link line from emitter position to the absorber position. This function consist of the retarded wave from the emitter and the advanced wave from absorber. This author found the mutual energy flow satisfies this requirement. The mutual energy flow is starting from the emitter and ends at the absorber. The mutual energy flow theorem guarantees that the energy go through any surface between the emitter to the absorber is all equal. Mutual energy flow actually is also the the product of the superposition of the retarded wave and the advanced wave. However it is the energy flow of superposed fields of the advanced wave and the retarded wave subtract the energy flows of the retarded wave and the advanced wave themselves. The mutual energy is also the surface inner product of the retarded wave and the advanced wave.

However, the free space electromagnetic field decrease with the distance between the source point and the field point. This leads the mutual energy flow also decrease with the distance between the two charges the emitter and the absorber. The photon is not decrease with the distance between the emitter and the absorber.

In order to overcome this difficulty, this author normalized the mutual energy flow. So the mutual energy flow should always the energy of one photons. This normalization process ask the field of the retarded field are equal at the emitter position and the absorber position. It is same to the advanced wave. That means the field of the photon is not decrease with distance. This can be understand by the superposition of the retarded wave and the advanced wave, the two wave is superposed only at the linkage line between the emitter and the absorber, hence, the field are become much strong in that direction. This is also because the retarded wave and the advanced wave are lead wave of each other, the wave has no other direction can go. The energy flow must be one photons energy.

The inner product of the retarded wave and the advanced wave (the mutual energy flow) can guarantee that the energy flow only goes from the source to the sink. Hence, this author choose the mutual energy flow to describe the photon. The mutual energy flow has very good property that is it is equal at any surface between the emitter to the absorber. This property is the most important property the photon needs. However, the mutual energy flow decrease when the distance between the emitter and absorber increase. It is known that the photon energy is a constant, hence, this author normalized the mutual energy flow. A wave which does not decrease in amplitude is applied to calculate the mutual energy flow.

In this article this author introduced the concept of quantization of the mutual energy, the mutual energy flow, mutual energy principle. According this theory, the emitter (source) sends retarded wave and the absorber (sink) sends advanced wave. Both emitter and absorber send the waves randomly. If there is no an advanced wave synchronized to the retarded wave, bother waves are returned through the time-reversal waves. These waves are broadcast waves, the amplitude of the broadcast wave decrease with the distance r. In case there is advanced wave synchronized with the retarded wave, the both waves the retarded wave and the advanced wave increase the amplitude on the direction alone the line linked the emitter and the absorber. All energy will go through the nearby of the line links both charges. The amplitude of the wave will not decrease alone the energy streamline. This process can be seen as a focalization process of the traditional Maxwell theory and the mutual energy principle. This focalization process make the mutual energy flow between the emitter and the absorber as a constant even the distance between the emitter to the absorber increased. This result make the mutual energy flow become a quantum or photon. Through statistical average process to the quantization of the mutual energy principle, a traditional mutual energy principle will obtained. The Maxwell theory can also be obtained through the statistical average process to the quantization of the mutual energy principle.

The traditional mutual energy principle is the foundation of the quantization of the mutual energy principle. The traditional mutual energy principle offers a consist theory without conflict to the law of the energy conservation. After a small step of the quantization, a final new theory for the electromagnetic fields is produced.

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