# Twin Prime Conjecture

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#### Abstract

I proved the Twin Prime Conjecture.

The probability that (6n - 1) is a prime and (6n+1) is also a prime approximately is 4/3 times the square of the probability that a prime will appear in.

I investigated up to  $1 \times 10^{12}$ .

All Twin Primes are executed in hexagonal circulation. It does not change in a huge number (forever huge number).

When the number grows to the limit, the primes occur very rarely, but since Twin Primes are 4/3 times the square of the distribution of primes, the frequency of occurrence of Twin Primes is very equal to 0.

However, it is not 0. Because, primes continue to be generated. Therefore, Twin Primes continue to be generated.

If the Twin Primes is finite, the primes is finite. This is because 4/3 times the square of the probability of primes is the probability of Twin Primes. This is contradiction. Because there are an infinite of primes.

That is, Twin Primes exist forever.

#### key words

Hexagonal circulation, Twin Primes, 4/3 times the square of the probability of primes

### Introduction

In this paper, it is written in advance that 2 and 3 are omitted from primes.

The prime number is represented as (6n - 1) or (6n+1). And, n is positive integer.

All Twin Primes are combination of (6n - 1) and (6n+1).

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That is, all Twin Primes are a combination of 5th-angle and 1th-angle.

1th-angle is (6n+1). 5th-angle is (6n - 1). (6n - 2), (6n), (6n + 2) in are even numbers. (6n - 1), (6n + 1), (6n + 3) are odd numbers. Primes are (6n - 1) or (6n+1). The following is a prime number. There are no primes that are not (6n - 1) or (6n+1). 5 — 6n -1 (Twin prime) 7 ------ 6n+1 11 — 6n -1 (Twin prime) 13 ------ 6n+1 17 — 6n -1 (Twin prime) 19 — 6n+1 23 — 6n -1  $\begin{array}{c} 29 \\ 31 \\ \hline \\ 6n \\ -1 \\ (Twin prime) \\ 31 \\ \hline \\ 6n \\ +1 \end{array}$ . . . . . . . . . . .....

[n is positive integer]

#### Part 1

There are 164 primes from 5 to 1000. Probability is  $\frac{164}{996}$ . In this, there are 34 Twin Primes. Probability is  $\frac{34}{996} = 0.034136546...$ and  $[\frac{164}{996}]^2 \times \frac{5}{4} = 0.0338905824...$  $[\frac{164}{996}]^2 \times \frac{4}{3} = 0.0361499546...$ 

There are 299 primes from 5 to 2000. Probability is  $\frac{299}{1996}$ . In this, there are 60 Twin Primes. Probability is  $\frac{60}{1996} = 0.030060120...$ and  $[\frac{299}{1996}]^2 \times \frac{4}{3} = 0.0299198932...$ 

There are 426 primes from 5 to 3000. Probability is  $\frac{426}{2996}$ . In this, there are 81 Twin Primes. Probability is  $\frac{81}{2996} = 0.027036048...$ and  $\left[\frac{426}{2996}\right]^2 \times \frac{4}{3} = 0.026957171...$  There are 665 primes from 5 to 5000. Probability is  $\frac{665}{9996}$ . In this, there are 125 Twin Primes. Probability is  $\frac{125}{4996} = 0.025020016...$ and  $\left[\frac{665}{4996}\right]^2 \times \frac{4}{3} = 0.023623115...$ 

There are 1227 primes from 5 to 10000. Probability is  $\frac{1227}{29996}$ . In this, there are 204 Twin Primes. Probability is  $\frac{204}{9996} = 0.02040816326...$ and  $[\frac{1227}{9996}]^2 \times \frac{4}{3} = 0.0200897886...$ 

There are 2258 primes from 5 to 20000. Probability is  $\frac{2258}{29996}$ . In this, there are 340 Twin Primes. Probability is  $\frac{340}{19996} = 0.01700340068...$ and  $\left[\frac{2258}{19996}\right]^2 \times \frac{4}{3} = 0.017002013...$ 

There are 3243 primes from 5 to 30000. Probability is  $\frac{3243}{29996}$ . In this, there are 465 Twin Primes. Probability is  $\frac{465}{29996} = 0.01550206694...$ and  $[\frac{3243}{29996}]^2 \times \frac{4}{3} = 0.015584969...$ 

There are 6053 primes from 5 to 60000. Probability is  $\frac{6053}{59996}$ . In this, there are 809 Twin Primes. Probability is  $\frac{809}{59996} = 0.01348423228...$ and  $\left[\frac{6053}{59996}\right]^2 \times \frac{4}{3} = 0.013571738...$ 

There are 6931 primes from 5 to 70000. Probability is  $\frac{6931}{69996}$ . In this, there are 904 Twin Primes. Probability is  $\frac{904}{69996} = 0.012915023716...$ and  $\left[\frac{6931}{69996}\right]^2 \times \frac{4}{3} = 0.0130732657...$ 

There are 6933 primes from 5 to 90000. Probability is  $\frac{6933}{89996}$ . In this, there are 903 Twin Primes. Probability is  $\frac{903}{69996} = 0.012900737185...$ and  $\left[\frac{6933}{69996}\right]^2 \times \frac{4}{3} = 0.01308081164...$ 

There are 9590 primes from 5 to 100000. Probability is  $\frac{9590}{99996}$ . In this, there are 1222 Twin Primes. Probability is  $\frac{1222}{99996}=0.0122204888...$ and  $\left[\frac{9590}{99996}\right]^2 \times \frac{4}{3}=0.0122633943...$  There are 17982 primes from 5 to 200000. Probability is  $\frac{17982}{199996}$ . In this, there are 2158 Twin Primes. Probability is  $\frac{2158}{199996} = 0.0107902...$ and  $[\frac{17982}{199996}]^2 \times \frac{4}{3} = 0.01077884...$ 

There are 25995 primes from 5 to 300000. Probability is  $\frac{25995}{299996}$ . In this, there are 2992 Twin Primes. Probability is  $\frac{2993}{299996} = 0.00997679969...$  and  $[\frac{25995}{299996}]^2 \times \frac{4}{3} = 0.01001123...$ 

There are 33858 primes from 5 to 400000. Probability is  $\frac{33858}{399996}$ . In this, there are 3802 Twin Primes. Probability is  $\frac{3803}{399996}$ =0.009505095... and  $[\frac{33858}{399996}]^2 \times \frac{4}{3}$ =0.00955322...

There are 41536 primes from 5 to 500000. Probability is  $\frac{41536}{499996}$ . In this, there are 4564 Twin Primes. Probability is  $\frac{4564}{499996} = 0.009128073...$  and  $\left[\frac{41536}{499996}\right]^2 \times \frac{4}{3} = 0.009201423...$ 

There are 49096 primes from 5 to 600000. Probability is  $\frac{49096}{599996}$ . In this, there are 4564 Twin Primes. Probability is  $\frac{5330}{599996} = 0.00888339255595...$ and  $\left[\frac{49096}{599996}\right]^2 \times \frac{4}{3} = 0.0089275902...$ 

There are 56540 primes from 5 to 700000. Probability is  $\frac{56540}{699996}$ . In this, there are 6060 Twin Primes. Probability is  $\frac{6060}{699996} = 0.008657192...$  and  $\left[\frac{56540}{699996}\right]^2 \times \frac{4}{3} = 0.00869879...$ 

There are 63948 primes from 5 to 800000. Probability is  $\frac{63948}{799996}$ . In this, there are 6765 Twin Primes. Probability is  $\frac{6765}{799996} = 0.00845629228...$ and  $\left[\frac{63948}{799996}\right]^2 \times \frac{4}{3} = 0.0085195574...$ 

There are 71272 primes from 5 to 900000. Probability is  $\frac{71272}{899996}$ . In this, there are 7471 Twin Primes. Probability is  $\frac{7471}{899996}$ =0.0083011480051... and  $[\frac{71272}{899996}]^2 \times \frac{4}{3} = 0.00836171709...$  There are 78496 primes from 5 to  $1000000=1\times10^{6}$ . Probability is  $\frac{78496}{999996}$ . In this, there are 8168 Twin Primes. Probability is  $\frac{8168}{999996}=0.008168032672...$  and  $[\frac{78496}{999996}]^2 \times \frac{4}{3}=0.0082155617...$ 

There are 148931 primes from 5 to  $2000000=2\times10^6$ . Probability is  $\frac{148931}{1999996}$ . In this, there are 14870 Twin Primes. Probability is  $\frac{14870}{1999996}=0.0074350148...$ and  $[\frac{148931}{1999996}]^2 \times \frac{4}{3}=0.00739351...$ 

There are 216814 primes from 5 to  $3000000=3\times10^{6}$ . Probability is  $\frac{216814}{2999996}$ . In this, there are 20931 Twin Primes. Probability is  $\frac{20931}{2999996}=0.0069770093...$  and  $\left[\frac{216814}{2999996}\right]^{2}\times\frac{4}{3}=0.006964212...$ 

There are 283144 primes from 5 to  $4000000=4\times10^6$ . Probability is  $\frac{216814}{3999996}$ . In this, there are 26859 Twin Primes. Probability is  $\frac{26859}{3999996}=0.0067147567...$  and  $\left[\frac{283144}{3999996}\right]^2 \times \frac{4}{3}=0.006680890...$ 

There are 348511 primes from 5 to  $5000000=5\times10^{6}$ . Probability is  $\frac{348511}{4999996}$ . In this, there are 32462 Twin Primes. Probability is  $\frac{32462}{4999996}=0.00649240519...$  and  $[\frac{348511}{4999996}]^{2} \times \frac{4}{3}=0.006477872...$ 

There are 412847 primes from 5 to  $6000000 = 6 \times 10^6$ . Probability is  $\frac{412847}{5999996}$ . In this, there are 37915 Twin Primes. Probability is  $\frac{37915}{5999996} = 0.00631917087...$  and  $\left[\frac{412847}{5999996}\right]^2 \times \frac{4}{3} = 0.0063126989...$ 

There are 476646 primes from 5 to  $7000000=7\times10^{6}$ . Probability is  $\frac{476646}{6999996}$ . In this, there are 43258 Twin Primes. Probability is  $\frac{43258}{6999996}=0.006179717816...$  and  $\left[\frac{476646}{6999996}\right]^{2} \times \frac{4}{3}=0.0061820862...$ 

There are 539775 primes from 5 to  $8000000=8\times10^{6}$ . Probability is  $\frac{539775}{7999996}$ . In this, there are 48617 Twin Primes. Probability is  $\frac{48617}{7999996}=0.006077128038...$  and  $\left[\frac{539775}{7999996}\right]^2 \times \frac{4}{3} = 0.0060699446...$ 

There are 602487 primes from 5 to  $9000000=9 \times 10^6$ . Probability is  $\frac{602487}{8999996}$ . In this, there are 53866 Twin Primes. Probability is  $\frac{53866}{8999996}=0.00598511377...$  and  $[\frac{602487}{8999996}]^2 \times \frac{4}{3}=0.005975158...$ 

There are 664577 primes from 5 to  $10000000 = 1 \times 10^7$ . Probability is  $\frac{664577}{9999996}$ . In this, there are 58979 Twin Primes. Probability is  $\frac{58979}{9999996} = 0.0058979023...$  and  $[\frac{664577}{19999996}]^2 \times \frac{4}{3} = 0.005888839...$ 

There are 1270605 primes from 5 to  $2000000 = 2 \times 10^7$ . Probability is  $\frac{1270605}{19999996}$ . In this, there are 107406 Twin Primes. Probability is  $\frac{107406}{1999996} = 0.005370301...$  and  $[\frac{1270605}{19999996}]^2 \times \frac{4}{3} = 0.005381459...$ 

There are 2433652 primes from 5 to  $40000000=4\times10^7$ . Probability is  $\frac{2433652}{39999996}$ . In this, there are 196752 Twin Primes. Probability is  $\frac{196752}{39999996}=0.00491880049...$ and  $\left[\frac{2433652}{3999996}\right]^2 \times \frac{4}{3}=0.0049355527...$ 

There are 3562112 primes from 5 to  $60000000=6\times10^7$ . Probability is  $\frac{3562112}{59999996}$ . In this, there are 280557 Twin Primes. Probability is  $\frac{280557}{59999996}=0.00478200038...$  and  $[\frac{3562112}{59999996}]^2 \times \frac{4}{3}=0.00469949762...$ 

There are 4669380 primes from 5 to  $8000000 = 8 \times 10^7$ . Probability is  $\frac{4669380}{79999996}$ . In this, there are 361449 Twin Primes. Probability is  $\frac{361449}{79999996} = 0.00451811272...$  and  $\left[\frac{4669380}{79999996}\right]^2 \times \frac{4}{3} = 0.00454231495...$ 

There are 5761453 primes from 5 to  $10000000 = 1 \times 10^8$ . Probability is  $\frac{5761453}{99999996}$ . In this, there are 440311 Twin Primes. Probability is  $\frac{440311}{9999996} = 0.004403110176...$  and  $[\frac{5761453}{9999996}]^2 \times \frac{4}{3} = 0.0044259124...$ 

There are 11078935 primes from 5 to  $2000000 = 2 \times 10^8$ . Probability is  $\frac{11078935}{199999996}$ . In this, there are 813370 Twin Primes. Probability is  $\frac{813370}{199999996} = 0.004066850081...$ and  $\left[\frac{11078935}{199999996}\right]^2 \times \frac{4}{3} = 0.0040914268...$ 

There are 16252323 primes from 5 to  $3000000=3 \times 10^8$ . Probability is  $\frac{16252323}{299999996}$ . In this, there are 1166479 Twin Primes. Probability is  $\frac{1166479}{299999996}=0.00388826338...$  and  $[\frac{16252323}{299999996}]^2 \times \frac{4}{3}=0.00391315570...$ 

There are 50847530 primes from 5 to  $10000000 = 1 \times 10^9$ . Probability is  $\frac{50847530}{999999996}$ . In this, there are 3424505 Twin Primes. Probability is  $\frac{3424505}{999999996} = 0.00342450501...$  and  $[\frac{50847530}{999999996}]^2 \times \frac{4}{3} = 0.00344729510371...$ 

There are 455052507 primes from 5 to  $1000000000 = 1 \times 10^{10}$ . Probability is  $\frac{455052507}{999999996}$ . In this, there are 27412678 Twin Primes. Probability is  $\frac{27412678}{999999996} = 0.0027412678...$  and  $\left[\frac{455052507}{999999996}\right]^2 \times \frac{4}{3} = 0.0027609704572...$ 

There are 4118054809 primes from 5 to 10000000000=1 × 10<sup>11</sup>. Probability is  $\frac{4118054811}{99999999996}$ . In this, there are 224376047 Twin Primes. Probability is  $\frac{224376047}{9999999996}$ =0.002243760... and  $[\frac{4118054811}{99999999996}]^2 \times \frac{4}{3} = 0.0022611167237...$ 

There are 37607912014 primes from 5 to  $1 \times 10^{12}$ . Probability is  $\frac{37607912014}{9999999996}$ . In this, there are 1870585218 Twin Primes. Probability is  $\frac{1870585218}{99999999996} = 0.001870585218007...$  and  $[\frac{37607912014}{99999999996}]^2 \times \frac{4}{3} = 0.00188580672808544...$ 

#### Part 2

There are 455052507-50847530=404204977 primes from  $1 \times 10^9$  to  $1 \times 10^{10} = 9 \times 10^9$ .

Probability is  $\frac{404204977}{900000000} = 0.04491166411...$ In this, there are 27412678-3424505=23988173 Twin Primes. Probability is  $\frac{23988173}{900000000} = 0.00266535255...$ 

 $[\frac{404204977}{900000000}]^2 \times \frac{4}{3} = 0.00268941009764...$ 

There are 4118054809-455052507=3663002302 primes from  $1 \times 10^{10}$  to  $1 \times 10^{11}=9 \times 10^{10}$ . Probability is  $\frac{3663002302}{9000000000}=0.0407000255777...$ 

In this, there are 224376047-27412678=196963369 Twin Primes. Probability is  $\frac{196963369}{9000000000}$ =0.0021884818777...

 $[\frac{3663002302}{9000000000}]^2 \times \frac{4}{3} = 0.00220865610937....$ 

There are 37607912016-4118054809=33489857207 primes from  $1 \times 10^{11}$  to  $1 \times 10^{12} = 9 \times 10^{11}$ . Probability is  $\frac{33489857207}{90000000000} = 0.0372109524522...$ 

In this, there are 1870585219-224376047=1646209172 Twin Primes. Probability is  $\frac{1646209172}{9000000000}$ =0.0182912130222...

 $[\tfrac{33489857207}{90000000000}]^2\times \tfrac{4}{3}{=}0.0018462066432020...$ 

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There are 17729166164-3760791201=13968374963 primes from 1 \times 10^{12} to 5 \times 10^{12} = 4 \times 10^{12}.
Probability is \frac{13968374963}{40000000000} = 0.0349209374075
In this, there are 8312493001-1870585219=6441907782 Twin Primes. Probability is \frac{6441907782}{40000000000} = 0.0016104769455
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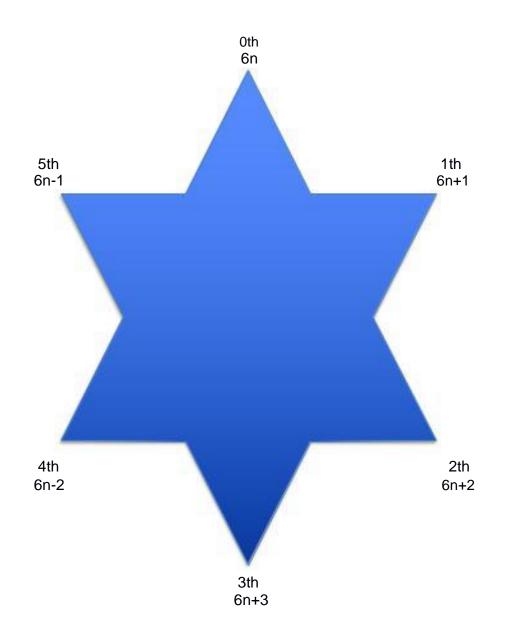
 $\left[\frac{13968374963}{40000000000}\right]^2 \times \frac{4}{3} = 0.001625962492558043761408333...$ 

At first, the correction value was set to 5/4. And the correction value is 4/3.

(It was done by hand calculation up to 200,000, but at this time it was [6/5] at first, gradually moved to [5/4], and then moved to [4/3].

At that time, I didn't know that WolframAlpha and Wolfram Cloud could calculate primes and Twin Primes. )

Calculation depends on WolframAlpha and Wolfram Cloud.



## Discussion

There are four possible primes combination: (6n - 1)(6n - 1), (6n - 1)(6n + 1), (6n + 1)(6n - 1), (6n + 1)(6n + 1), (6n + 1)(6n + 1), (6n + 1)(6n + 1), Each with the same probability. At this time, Twin Prime is only (6n - 1)(6n + 1). The probability of (6n - 1)(6n + 1) is [1/4]. That is, when Primes comes out, the probability that it is Twin Primes is 1/[1-(1/4)=3/4]. This is the reason for the constant [4/3].

(number)	(Sexy prime)	(Twin Prime)	(Cousin Primes)
	,		
400000			
	9184		
600000	10688		5334
700000		6061	6085
800000			6798
900000	15000	7472	7471
1000000			
2000000			14742
3000000			20826
4000000	53224		
5000000	64481		
6000000	75417		
7000000			
8000000	96705		
9000000	107042	53866	53468
10000000	117207		
90000000	801602		401025
$100000000 = 1 \times 10^8 \dots$	879908		

I wrote below the distribution of Sexy Primes, Twin primes and Cousin primes.

First, say 6n - 1 = 6n + 5

 $(6n - 1) \times 5 = 6(5n - 1) + 1 = 1$ th-angle.  $(6n + 1) \times 5 = 6(5n) + 5 = 5$ th-angle. and  $(6n-1) \times 7 = 6(7n-2) + 5 = 5$ th-angle.  $(6n+1) \times 7 = 6(7n+1) + 1 = 1$ th-angle. and  $(6n-1) \times 11 = 6(11n-2) + 1 = 1$ th-angle.  $(6n+1) \times 11 = 6(11n+1) + 5 = 5$ th-angle. and  $(6n-1) \times 13 = 6(13n-3) + 5 = 5$ th-angle.  $(6n+1) \times 13 = 6(13n+2) + 1 = 1$ th-angle. and  $(6n-1) \times 17 = 6(17n-3) + 1 = 1$ th-angle.  $(6n + 1) \times 17 = 6(17n+2) + 1 = 5$ th-angle. and  $(6n-1) \times 19 = 6(19n - 4) + 5 = 5$ th-angle.  $(6n + 1) \times 19 = 6(19n+3) + 1 = 1$ th-angle. and  $(6n-1) \times (6n-1) = 6(6n^2 - 2n) + 1 = 1$ th-angle.  $(6n - 1) \times (6n + 1) = 6(6n^2 - 1) + 5 = 6(6n^2) - 1 = 5$ th-angle. and  $(6n+1) \times (6n-1) = 6(6n^2-1) + 5 = 6(6n^2) - 1 = 5$ th-angle.  $(6n+1) \times (6n+1) = 6(6n^2+2n)+1 = 1$ th-angle.

In this way, prime multiples of (6n - 1) or (6n+1) of primes fill 5th-angle, 1th-angle, and the location of primes becomes little by little narrower.

However, every time the hexagon is rotated once, the number of locations where the prime number exists increases by two.

The probability of a twin prime [(6n - 1)(6n + 1) combinations] is obtained by multiplying 6/5 times the square of the probability of a prime will occur.

The probability that a twin prime will occur 6/5 times the square of the probability that a prime will occur in a huge number, where the probability that a prime will occur is low from the equation (1).

While a prime number is generated, Twin Primes be generated.

And, as can be seen from the equation below, even if the number becomes large, the degree of occurrence of primes only decreases little by little.

$$\pi(x) \sim \frac{x}{\log x} \quad (x \to \infty) \tag{1}$$

 $log(10^{20}) = 20 log(10) \approx 46.0517018$   $log(10^{200}) = 200 log(10) \approx 460.517018$   $log(10^{2000}) = 2000 log(10) \approx 4605.17018$   $log(10^{20000}) = 20000 log(10) \approx 46051.7018$  $log(10^{200000}) = 200000 log(10) \approx 460517.018$  (Expected to be larger than  $\log(10^{200000})$ )

As x in  $\log(x)$  grows to the limit, the denominator of the equation also grows extremely large. Even if primes are generated, the frequency of occurrence is extremely low. The generation of Twin Primes is approximately the square of the generation frequency of primes, and the generation frequency is extremely low.

However, as long as primes are generated. Twin Primes are generated with a very low frequency.

When the number grows to the limit, the denominator of the expression becomes very large, and primes occur very rarely, but since twins are the square of the distribution of primes, the frequency of occurrence of twins is very equal to 0.

However, it is not 0. Therefore, Twin Primes continue to be generated.

However, when the number grows to the limit, the probability the twin prime appearing is almost 0 because it is of 4/3 times the square of the probability of the appearance of the primes.

It is a subtle place to say that almost 0 appears.

Use a contradiction method. If the Twin Primes is finite, the primes is finite. This is because 4/3 times the square of the probability of primes is the probability of Twin Primes.

This is contradiction. Because there are an infinite of primes.

That is, Twin Primes exist forever.

Proof end.

## Appendix

Twin primes are expressed  $6m \pm 1.$  (m is positive integer) As you can see from the bottom, primes that satisfy this, that is, certain primes, end with 3 or 7.

For example, 2, 3, 5, 7, 17, 23, 103, 107, 137, 283, 313, 347, 373, 397, 443, 467, 577, 593, 653, 773, 787, 907, 1033, 1117, 1423, 1433, 1613, 1823, 2027, 2063, 2137, 2153, 2203, 2287, 2293, 2333, 2347, 2677 etc.

If write 3 or 7 separately, except 2 and 5, it looks like below.

3, 23, 103, 283, 313, 373, 443, 593, 653, 1033, 1423, 1433, 1613, 1823, 2063, 2153, 2203, 2293, 2333....

7, 17, 107, 137, 247, 397, 467, 577, 787, 907, 1117, 2027, 2137, 2287, 2347, 2677....

Primes are forever. Therefore, Twin Primes are forever.

 $12 = 6 \times 2$  $18 = 6 \times 3$  $30 = 6 \times 5$  $42 = 6 \times 7$  $102 = 6 \times 17$  $138 = 6 \times 23$  $618 = 6 \times 103$  $642 = 6 \times 107$  $822 = 6 \times 137$  $1698 = 6 \times 283$  $1878 = 6 \times 313$  $2082 = 6 \times 347$  $2238 = 6 \times 373$  $2382 = 6 \times 397$  $2658 = 6 \times 443$  $2802 = 6 \times 467$  $3462 = 6 \times 577$  $3558 = 6 \times 593$  $3918 = 6 \times 653$  $4638 = 6 \times 773$  $4722 = 6 \times 787$  $5442 = 6 \times 907$  $6198 = 6 \times 1033$  $6702 = 6 \times 1117$  $8538 = 6 \times 1423$  $8598 = 6 \times 1433$  $9678 = 6 \times 1613$  $10938 = 6 \times 1823$  $12162 = 6 \times 2027$  $12378 = 6 \times 2063$  $12822 = 6 \times 2137$  $12918 = 6 \times 2153$  $13218 = 6 \times 2203$  $13722 = 6 \times 2287$  $13758 = 6 \times 2293$  $13998 = 6 \times 2333$  $14082 = 6 \times 2347$  $16062 = 6 \times 2677$  $17418 = 6 \times 2903$ 

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## Postscript

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Thanks to fried-turnip, it was decided whether 4/3 would be a constant.