Refutation of Pythagorean triples

Abstract: We evaluate the definition of Pythagorean triples as not tautologous. This invalidates its subsequent use with Ramsey colorings and the computerized proof using the inference of induction as based thereon. These conjectures form a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, \( F \) as contradiction, \( N \) as truthity (non-contingency), and \( C \) as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)


Abstract. The boolean Pythagorean Triples problem has been a longstanding open problem in Ramsey Theory: Can the set \( N = \{1, 2, \ldots \} \) of natural numbers be divided into two parts, such that no part contains a triple \((a, b, c)\) with \(a^2 + b^2 = c^2\)? A prize for the solution was offered by Ronald Graham over two decades ago. We solve this problem, proving in fact the impossibility, by using the Cube-and-Conquer paradigm, a hybrid SAT method for hard problems, employing both look-ahead and CDCL solvers. An important role is played by dedicated look-ahead heuristics, which indeed allowed to solve the problem on a cluster with 800 cores in about 2 days. Due to the general interest in this mathematical problem, our result requires a formal proof. Exploiting recent progress in unsatisfiability proofs of SAT solvers, we produced and verified a proof in the DRAT format, which is almost 200 terabytes in size. From this we extracted and made available a compressed certificate of 68 gigabytes, that allows anyone to reconstruct the DRAT proof for checking.

From: cs.utexas.edu/users/marijn/ptn/

First a definition: A Pythagorean Triple are three natural numbers

\[
1 \leq a \leq b \leq c, \text{ such that } \quad (1.1)
\]

\[
\neg (\neg (\neg (r < q) < p) < (%s > # s)) = (s = s) ; \quad (1.2)
\]

\[
a^2 + b^2 = c^2 \text{ holds. } \quad (2.0)
\]
Remark 2.1: We manipulate Eq. 2.1 in the following steps.

\[ a^2 + b^2 = c^2 \]
\[ c^2 = a^2 + b^2 \]
\[ 0 = a^2 + b^2 - c^2 \]
\[ -1 = a^2 - 1 + b^2 - c^2 \]
\[ -1 = ((a-1)*(a+1)) + ((b-c)*(b+c)) \]
\[ 0 = (1+((a-1)*(a+1)) + ((b-c)*(b+c))) \] (2.1)

\[(s@s)=((%s>#s)+(((p-(%s>#s))&(p+(%s>#s)))+((q-r)&(q+r)))) \] ;
\[ \text{CCCC CCCC CCCC CCCC} \] (2.2)

Remark 3.1: The initial conjecture is that the definition of Eq. 1.2 implies the formula of 2.2.

\[ \sim \sim (\sim(r<q)<p)(%s>#s) \]
\[ ((s@s)=((%s>#s)+(((p-(%s>#s))&(p+(%s>#s)))+((q-r)&(q+r)))) \] ;
\[ \text{CCCC CCCC CCCC CCCC} \] (3.1)

Remark 3.2: Eq. 3.2 as rendered is not tautologous. This refutes the definition of the conjecture, implying the secondary conjecture using colors 4.0 below is moot.

Now the question: Can we colour the natural numbers 1, 2, 3, ... with two colours, say blue and red, such that there is no monochromatic Pythagorean triple? In other words, is it possible to give every natural number one of the colours blue or red, such that for every Pythagorean triple \( a, b, c \) at least one of \( a, b, c \) is blue, and at least one of \( a, b, c \) is red? We prove: The answer is No. (4.0)