

# Factorization of the numbers of the form $n + n^2$

Pedro Hugo García Peláez

All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission of the publisher, except in the case of brief quotations embodied in critical reviews and certain other noncommercial uses permitted by copyright law. For permission requests, write to the publisher at the e-mail below.

[hugo117711@gmail.com](mailto:hugo117711@gmail.com)

© Pedro Hugo García Peláez, 2019

The sum of the natural numbers plus their respective square can be expressed in the form:

$$n^2 + n$$

For example, for the number 14, its sum plus its square is equal to:

$$14 + 14^2 = 210$$

This number can be factored in a special way:

Its factors can be grouped into two blocks:

One of the groups of factors is 14 which in this case can be factored as 2 and 7

The other group is formed  $(14 + 1) = 15$  so the two remaining factors are 3 and 5

It is noteworthy that the factorization of these numbers maintains a symmetry between the two groups of factors since one is  $(n)$  and the other  $(n + 1)$  and the two always appears.

It should be noted that factor 2 will always appear, since it is a consequence of making the sum of a natural and its square and dividing it by the number of terms.

That is, if we sum all numbers between 14 and  $14^2$  all the terms and divide it by  $(14^2 - 14 + 1)$  we get that the sum is  $(14 + 14^2) / 2$

In short, the numbers of the form  $n^2 + n$  can always be divided by two, so factor 2 always appears.

So we will always divide the number  $n^2 + n$  by 2

At this point there are two cases if  $(n)$  is even or if  $(n)$  is odd.

If  $n$  is even:

- The first group of factors is  $(n) / 2$   
The second group of factors is  $(n + 1)$

If  $n$  is odd:

The first group of factors is  $(n)$   
The second group of factors is  $(n + 1) / 2$

It can also be seen as the resolution of the 2nd degree equation  $n^2 + n - a = 0$

Where  $a = (n^2 + n)$

Where the positive root of the equation divided by two is a group of factors and the negative root is the other group of factors, if  $(n)$  it is even.

And where the positive root of the equation is a group of factors and the negative root divided by 2 is the other group of factors, if  $(n)$  it is odd.