Machine learning algorithm in a caloric view point of cosmology

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Abstract

In the present work, we mainly discuss the variable polytropic gas (VPG henceforth) proposal, which is describing a self-gravitating gaseous sphere and can be considered as a crude approximation for realistic stellar definitions, from a caloric perspective. In order to reach this aim, we start with reconstructing the VPG model by making use of thermodynamics. And then, the auxiliary parameters written in the proposal are fitted by focusing on updated experimental dataset published in literature. We also discuss the model in view of the statistical perspective and conclude that the caloric VPG model (cVPG henceforth) is in good agreement with the recent astrophysical observations. With the help of the statistical discussions, we see that the cVPG model is suitable for the statistical cosmology and can be used to make useful predictions for the future of the universe via the machine learning (ML henceforth) methods like the linear regression (LR henceforth) algorithm. Moreover, according to the results, we also perform a rough estimation for the lifetime of the universe and conclude that the cosmos will be torn apart after 51Gyr which means our universe has spent 21 percent of its lifetime.

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I. INTRODUCTION

Identifying dark sectors of the universe, at least the part containing dark energy (DE henceforth) and dark matter (DM henceforth), is one of the challenging issues of modern theoretical physics[1]. Among them, the dark energy sector is considered as the generator of the current speedy expansion epoch[2–11]. There are plenty of proposals introduced for getting an expression for the nature of the major part of cosmos: cosmological constant[12], modified gravity theories[13, 14], scalar fields[15–19], braneworld models[20, 21], energy densities[22–31, 41, 42], assuming extra dimensions[43–45] and so on. For a convenient brief about theoretical proposals, one can check Ref.[46] and references therein. Among these models, the Hobbit[29], the CG[30] family (including its generalized[31], variable[32], modified[33], variable modified[34], variable generalized[35] and extended versions[36–40]) and the PG[41, 42] have a very significant feature: these proposals automatically remove the cosmic coincidence puzzle due to expressing the dark cosmos with a single fluid.

Data science is the investigation of producing and making use of sophisticated algorithms and cutting-edge numerical investigation methods to extract information from data[47]. For instance, fitting free parameters defined a theoretical proposal is one of the important issues in contemporary physics. The most often used method in modern cosmology is to analyze the luminosity distance data sets for a specific family of cosmic objects[48–51]. Recently, a new technique including observational Hubble constant values (OHV henceforth), which is also called cosmic chronometer data, has come forward to check some astrophysical tests[52–57]. On the other hand, scientists across different disciplines are focusing on using artificial intelligence (AI henceforth) to make their jobs much easier. For instance, the AI can be used to identify whether a tumor is benign or malignant in medicine[58], to predict future market prices in economy[59] or to forecast weather conditions in meteorology[60], etcetera. The main idea behind such use of the AI is the ML algorithm. As a matter of fact, the ML is a subset of the AI and includes advanced methods that enable computers to figure things out from the data. One of the fundamental algorithms is the LR which provides a base to build on and figure out other ML algorithms. This technique is mostly applied to forecast and find out cause-effect relationship between variables. In the present work, we apply the LR algorithm to the cVPC cosmology in order to discuss its reliability and show that the model is in good agreement with the recent astrophysical data sets and can be used to make meaningful predictions for the future of our cosmos. To reach this aim, we (i) reconstruct the VPC proposal[61] via thermodynamics, (ii) perform a constraining analysis including the most recent OHV in order to find best values of the free parameters written in the cVPG model, (iii) discuss the model from a statistical perspective and (iv) apply the LR algorithm to the cVPG model.

The layout of the paper is as follows. In the next section, we construct the cVPG proposal. Next, in the third section, we perform a data analysis to fit the auxiliary parameters given in the model. In the fourth section, we study the model from a statistical perspective in order to check its statistical reliability and whether the cVPG proposal can take a role in the ML field. In the fifth section, we apply the LR algorithm to the cVPG proposal. In the sixth section, according to the best fitting values of the auxiliary parameters given in the model, we perform a rough estimation for the lifetime of the cosmos. The final section is devoted to closing remarks.

All numerical calculations and analyzes are performed by using the Mathematica[62] and the Pyton[63] softwares.

II. CONSTRUCTING THE THEORETICAL MODEL

We start with assuming the universe is described by the flat Friedmann-Robertson-Walker (FRW henceforth) type line-element. Due to the recent cosmological observations[2–11, 64] have strongly indicated a spatially flat geometry of the universe, we use the line-element $ds^2 = dt^2 - a^2(t)\sum_{i=1}^{3}(dx_i)^2$ where the time-dependent function $a(t)$ is the cosmic scale factor.

On the other hand, the VPG model is given[61] by the equation-of-state (EoS henceforth) $p = \beta a^{-n}\rho^{1+\frac{1}{\xi}}$, where β, n and the polytropic index ξ denote real constants. In the present work, we suppose that the cosmos is filled with a perfect fluid, which is described by the energy-momentum tensor $T_{\mu\nu} = (\rho_t + p_t)u_\mu u_\nu - g_{\mu\nu}p_t$ where u_μ, $\rho_t = \rho + \rho_{bm}$, $p_t = p + p_{bm}$ and $g_{\mu\nu}$ represent the four-velocity vector, total energy density, total pressure and the metric tensor, respectively. Note that the subscript bm stands for the baryonic matter, which is represented by the EoS parameter $\omega_{bm} = 0$.

Consequently, from the continuity equation $T^{\mu\nu}_{;\nu} = 0$, we can write

$$\dot{\rho}_{bm} + 3H\rho_{bm} = 0, \tag{1}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble expansion parameter and the dot indicates a derivative with respect to the cosmic time t. From equation (1), it is found that $\rho_{bm} = \frac{\rho^0_{bm}}{a^3}$ where ρ^0_{bm} is an integration constant and represents present value of the energy density of baryonic matter.

With the help of thermodynamics, we have the following expression

$$p = -\left(\frac{\partial U}{\partial V}\right)_S = \beta a^{-n}\rho^{1+\frac{1}{\xi}} \tag{3}$$

where $U = \rho V$, $V = a^3$ and S represent the energy, volume and the entropy, respectively. Therefore, performing the corresponding integration gives the following relation

$$U^{-\frac{1}{\xi}} = c(S) - \frac{3\beta}{3+n\xi}a^{-\frac{3+n\xi}{\xi}}, \tag{4}$$

where $c(S)$ denotes an integration constant which may depend on the entropy only or be a universal constant. Subsequently, we find

$$\rho = \left[\frac{(n\xi+3)V^{\frac{n}{3}}}{3\beta}\right]^{\xi}\left\{1+\left(\frac{V}{\nu}\right)^{\frac{n}{3}+\frac{1}{\xi}}\right\}^{-\xi}, \tag{5}$$

$$p = (-1)^{1-\xi}(\beta V)^{n\xi/3}\left(\frac{n\xi+3}{3}\right)^{\xi+1}\left\{1+\left(\frac{V}{\nu}\right)^{\frac{n}{3}+\frac{1}{\xi}}\right\}^{-(1+\xi)}, \tag{6}$$

where ν has the dimension of volume and it is given by

$$\nu = \left\{-\frac{(n\xi+3)c}{3\beta}\right\}^{-\frac{3\xi}{n\xi+3}}. \tag{7}$$

For the pressureless case, let V_c represents the critical volume of the system. Thus, we may write $V_c = (-1)^{\frac{3\xi}{n\xi+3}}\nu$. Subsequently, it can be written that

$$\rho = \left[\frac{(n\xi+3)V^{\frac{n}{3}}}{3\beta}\right]^{\xi}\left\{1-\left(\frac{V}{V_c}\right)^{\frac{n}{3}+\frac{1}{\xi}}\right\}^{-\xi}. \tag{8}$$

Next, for the corresponding heat capacity, one should check whether the condition $C_V > 0$ is satisfied in the constant volume case. It is generally written that

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \rho}{\partial T}\right)_V. \tag{9}$$

Moreover, making use of the expression $T = \left(\frac{\partial U}{\partial S}\right)_V$, the temperature of the VPG can be calculated as a function of entropy and volume. Thus, it is found that

$$T = -\xi\left[c - \frac{3\beta}{n\xi+3}V^{-\frac{n\xi+3}{3\xi}}\right]^{-(\xi+1)}\left(\frac{\partial c}{\partial S}\right)_V. \tag{10}$$

It can be seen that the cVPG remains at zero temperature for any value of pressure and volume of the fluid if the integration constant c is a universal constant. This makes the stability of the cVPG proposal questionable. From this point of view, for an extensive discussion, we assume that $\left(\frac{\partial c}{\partial S}\right)_V \neq 0$ which means the temperature of the cVPG varies during its speedy expansion. From fundamental physical assumptions, it is generally known that the function $c(S)$ must yield positive value of temperature. Previously, Santos et al.[71] used the Jacobian identity to get a definition for the $c(S)$ function in a cosmological scenario including the modified CG. Later, Panigrahi and Chatterjee[35] and Askin et al.[72] performed similar investigations for the generalized CG and the original PG models, respectively. So, after performing a dimensional analysis, we see from equation (4) that $[U] = [c]^{-\xi}$. Due to $[U] = [S][T]$, one can write that $[c] = [S]^{-\frac{1}{\xi}}[T]^{-\frac{1}{\xi}}$. It is easy to conclude that we cannot get an analytic expression of the function $c(S)$ from this result. From this point of view, an empirical description of $c(S)$ can be assumed. Hence, the function $c(S)$ should depend on entropy only $c = T_c^{-\frac{1}{\xi}} S^{-\frac{1}{\xi}}$ where T_c is a constant parameter having the dimension of temperature. Thus, one can reach the following expressions for the temperature and entropy of the system

$$T = T_c(-1)^{\xi+1}\left[\frac{V}{V_c}\right]^{-\frac{(n\xi+3)(\xi+1)}{3\xi}}\left(1-\left[\frac{V}{V_c}\right]^{\frac{n\xi+3}{3\xi}}\right)^{-(\xi+1)}, \tag{11}$$

$$S = (-1)^{-\xi}V^{\frac{n\xi+3}{3}}\left[\frac{3\beta}{n\xi+3}\right]^{-\xi}\left[\frac{T^{-\xi}}{T_c}\right]^{\frac{1}{\xi+1}}\left[1-\left(\frac{T}{T_c}\right)^{\frac{1}{\xi+1}}\right]^{\xi}. \tag{12}$$

Here, it is seen that, for the third law of thermodynamics ($T = 0$ and $S = 0$), we should have $V = V_c$ and $T = T_c$. Now, making use of equation (12), one can reach the following expression for the heat capacity

$$C_V = \frac{(-1)^{1-\xi}\xi}{\xi+1}V^{\frac{n\xi+3}{3}}\left[\frac{n\xi+3}{3\beta}\right]^{\xi}\left[\frac{T^{-\xi}}{T_c}\right]^{\frac{1}{\xi+1}}\left[1-\left(\frac{T}{T_c}\right)^{\frac{1}{\xi+1}}\right]^{1+\xi}. \tag{13}$$

We can see that $C_V > 0$ if ξ and n are negative even numbers and $0 < T < T_c$. So, it can be said that T_c represents the maximum value of temperature.

We can now proceed our investigation to get expressions for the energy density, pressure and the EoS parameter of the VPG as a function of temperature. So, we find the following expressions

$$\rho = (-\beta)^{-\xi}V^{\frac{n\xi}{3}}\left[\frac{n\xi+3}{3}\right]^{\xi}\left[1-\left(\frac{T}{T_c}\right)^{\frac{1}{\xi+1}}\right]^{\xi}, \tag{14}$$

$$p = (-1)^{1-\xi} \frac{V^{\frac{n\xi}{3}}}{\beta^{\xi}} \left[\frac{n\xi + 3}{3} \right]^{\xi+1} \left[1 - \left(\frac{T}{T_c} \right)^{\frac{1}{\xi+1}} \right]^{\xi+1}, \tag{15}$$

$$\omega = -\frac{n\xi + 3}{3} \left[1 - \left(\frac{T}{T_c} \right)^{\frac{1}{\xi+1}} \right]. \tag{16}$$

In the early inflation phase when temperature is very high (note that $T = 10^{32}$ K at the Planck phase), i.e. $T \to T_c$, we get $\omega \sim 0$ indicating a dust dominated universe. Besides, in the late-time speedy expansion phase when the temperature is very low, i.e. $T \to 0$, it is found that $\omega \sim -1 - \frac{n\xi}{3}$. If $n\xi > 0$, we get $\omega > -1$ which means the case represents a quintessence type evolution and the big rip fate of the universe is avoided. Next, if $n\xi < 0$, we have $\omega < -1$ describing a phantom type DE. It is known that taking $n = 0$ reduces the VPG model into the original PG model[61]. So, for the late-time speedy expansion phase, we get $\omega = -1$ which means this specific case ($n = 0$) points to the ΛCDM model.

Moreover, in order to analyze the cVPG cosmologically in a different way, we can also focus on the deceleration parameter. Making use of the relation $q = -\frac{\ddot{a}}{aH^2}$ with the result (16) yields

$$q = \frac{1}{2} + \frac{3\omega}{2} = -1 - \frac{n\xi}{2} + \frac{n\xi + 3}{2} \left(\frac{T}{T_c} \right)^{\frac{1}{\xi+1}}. \tag{17}$$

In order to describe an accelerating universe, we need to have negative q values. In the next section, we fit the model parameters according to recent observations. After that, we can analyze the deceleration parameter numerically in order to test the consistency of the cVPG model.

III. DATA ANALYSIS

We may write the Friedman equation (FE henceforth) as $H^2 = \frac{8\pi G}{3}(\rho_{bm} + \rho_{dm} + \rho_{de})$. Here, one can assume the following useful dimensionless fractional densities

$$\Omega_{bm} = \frac{\rho_{bm}}{\rho_c}, \qquad \Omega_{dm} = \frac{\rho_{dm}}{\rho_c}, \qquad \Omega_{de} = \frac{\rho_{de}}{\rho_c}. \tag{18}$$

Here the critical density is given by $\rho_c = \frac{3H_0^2}{8\pi G}$ where H_0 shows the present value of the cosmic Hubble parameter. Planck-results[10, 11] have indicated that $H_0 = 67.8^{+0.9}_{-0.9}$ KmSec^{-1}Mpc^{-1}. From this point of view, the FE can be rewritten in a simple elegant form

$$\sum_{i=1}^{3} \Omega_i = E, \tag{19}$$

with $\Omega_i = (\Omega_{bm}, \Omega_{dm}, \Omega_{de})$ and the dimensionless Hubble parameter $E = \frac{H}{H_0}$. According to the Planck-results[10, 11], we have $\Omega^0_{bm} + \Omega^0_{dm} + \Omega^0_{de} = 0.049 + 0.278 + 0.673$. Also, we can define $\Omega^0_{bm} = \frac{\rho^0_{bm}}{\rho_c}$, $\Omega^0_{vpg} = \frac{\Omega^0_{vpg}}{\rho_c}$, $\Omega^0_{dm} = \frac{\rho^0_{dm}}{\rho_c}$ and $\Omega^0_{de} = \frac{\rho^0_{de}}{\rho_c}$. Subsequently, one can write that

$$\frac{\Omega^0_{bm}}{\Omega^0_{dm} + \Omega^0_{de}} = \frac{\Omega^0_{bm}}{\Omega^0_{vpg}} = \frac{\rho^0_{bm}}{\rho^0_{vpg}}. \tag{20}$$

Moreover, it is generally known that the present energy density is related[66] to the cosmological density by $\rho^0 = 1.31\rho_{cos}$, where $\rho^0 = \rho^0_{bm} + \rho^0_{vpg}$, which means one should assume $\rho^0 = 1.31$ when $a_0 = 1$. With the help of equation (20), one can obtain that $(\rho^0_{bm}, \rho^0_{vpg}) = (0.05, 1.26)$.

Additionally, the integration constant written in the equation (4) can be given in terms of the free parameter of the model by making use of the above results. Hence, we get

$$c = (1.26)^{-\frac{1}{\xi}} + \frac{3\beta}{n\xi + 3}. \tag{21}$$

Focusing on the FE, we can write the cosmic Hubble parameter in the following form

$$H = H_0\sqrt{\frac{1 - \Omega^0_{vpg}}{V} + \frac{\Omega^0_{vpg}}{\rho^0_{vpg}}\left[\frac{(n\xi + 3)V^{\frac{n}{3}}}{3\beta}\right]^{\xi}\left\{1 - \left(\frac{V}{V_c}\right)^{\frac{n}{3}+\frac{1}{\xi}}\right\}^{-\xi}}. \tag{22}$$

Besides, one can also rewrite the above relation as a function of the cosmic red-shift parameter z. Remember that the $z \sim a$ relation is given by $z + 1 = \frac{1}{a}$ with $a_0 = 1$. Hence, it is obtained that

$$H(z) = H_0\sqrt{0.049(1+z)^3 + 0.754\left\{\frac{3\beta(1+z)^n}{n\xi + 3} - \left[(1.26)^{-\frac{1}{\xi}} + \frac{3\beta}{n\xi + 3}\right](1+z)^{-\frac{3}{\xi}}\right\}^{-\xi}}. \tag{23}$$

In order to fit the free parameters of the model, we can use the set of recent OHV[73–86] given in Table I[61] and Planck-results[10, 11]. On this purpose, one can minimize χ^2, which can be defined as $\chi^2 = \sum_i^{27}[H_{obs}(z_i) - H_{theo}(z_i)]^2\sigma_i^{-2}$, in order to reach best-fitting values of the auxiliary parameters. Here, H_{obs}, H_{theo} and σ_i show observational value of the Hubble parameter, theoretical value of the Hubble parameter and the corresponding uncertainty, respectively. Consequently, the best-fitting values of auxiliary parameters are calculated as

$$(\xi, \beta, n) = (-10, -0.8, -2) \tag{24}$$

with

$$\chi^2 = 15.5184. \tag{25}$$

TABLE I: The recent observable $H(z)$ dataset.

z	$H_{obs}(z)$	σ	Ref.	z	$H_{obs}(z)$	σ	Ref.	z	$H_{obs}(z)$	σ	Ref.
0.0708	69.00	$\mp$19.68	[74]	0.4247	87.10	$\mp$11.20	[79]	0.7810	105.0	$\mp$12.00	[76]
0.1200	68.60	$\mp$26.20	[74]	0.4300	86.45	$\mp$3.680	[77]	0.8750	125.0	$\mp$17.50	[76]
0.1700	83.00	$\mp$8.000	[75]	0.4497	92.80	$\mp$12.90	[79]	0.9000	117.0	$\mp$23.00	[75]
0.1990	75.00	$\mp$5.000	[76]	0.4783	80.90	$\mp$9.000	[80]	1.3000	168.0	$\mp$17.00	[84]
0.2400	79.69	$\mp$2.650	[77]	0.4800	97.00	$\mp$62.00	[81]	1.4300	177.0	$\mp$18.00	[84]
0.2800	88.80	$\mp$36.60	[74]	0.5700	92.40	$\mp$4.500	[82]	1.5300	140.0	$\mp$14.00	[75]
0.3500	84.40	$\mp$7.000	[78]	0.5930	104.0	$\mp$13.00	[76]	1.7500	202.0	$\mp$40.00	[85]
0.3802	83.00	$\mp$13.50	[76]	0.6800	92.00	$\mp$8.000	[76]	1.9650	186.5	$\mp$50.40	[76]
0.4000	95.00	$\mp$17.00	[75]	0.7300	97.30	$\mp$7.000	[83]	2.3400	222.0	$\mp$7.000	[86]

In FIG. 1, we depict the evolutionary nature of the cosmic Hubble parameter according to the cVPG model in the 1σ confidence region. Note that, in FIG. 1, the green dots represent the recent observable values.

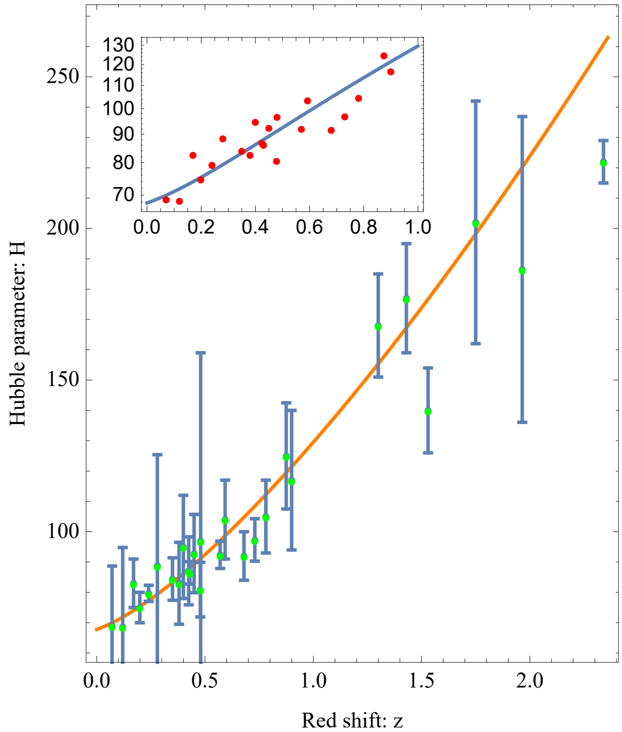

FIG. 1: $H \sim z$ relation for OHV dataset according to the cVPG model in the 1σ confidence region.

On the other hand, with the help of best fitting values of the auxiliary parameters given in the model, we can now plot the heat capacity in order the check the stability condition of the model. It can be seen from FIG. 2 that the cVPG model is always stable due to positive values of the heat capacity.

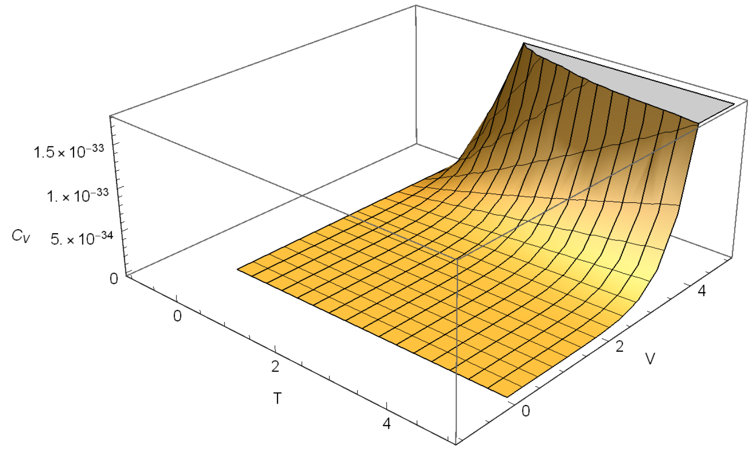

FIG. 2: The evolutionary nature of heat capacity with best fitting values of the free parameters. Here, we assumed that $T_c = 10^{32}K$.

Additionally, we need to test the consistency of the model cosmologically in order to demonstrate its reliability. Focusing on the expression (17) with the help of best fitting values of the model parameters given in equation (24), we plot $q \sim T$ relation in FIG. 3.

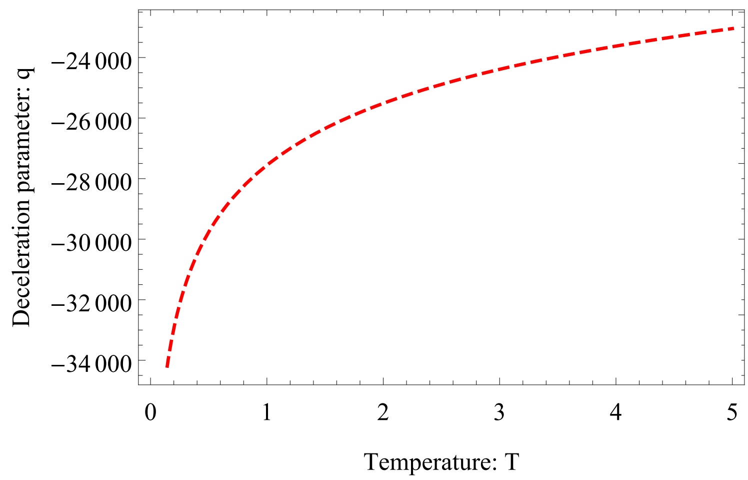

FIG. 3: The graphical analysis of $q \sim T$ relation with the help of best fitting values of the free parameters. Here, we assumed that $T_c = 10^{32}K$.

It is seen from FIG. 3 that the deceleration parameter takes negative values. So, the model is reliable and can yield cosmologically meaningful results.

IV. STATISTICAL ANALYSIS

We can investigate the corresponding correlation between values of H_{obs} and H_{theo} with the help of correlation parameter Υ. This coefficient is taken into account in statistics to determine how strong the corresponding relationship is. Hence, in order to achieve this goal, the following expressions can be used:

$$K_x = \sqrt{\frac{\Sigma_{n=1}^{27}\left(H_{obs}(z_n) - \overline{H}_{obs}\right)^2}{n-1}}, \tag{26}$$

$$K_y = \sqrt{\frac{\Sigma_{n=1}^{27}\left(H_{theo}(z_n) - \overline{H}_{theo}\right)^2}{n-1}}, \tag{27}$$

where $\overline{H}_{obs}$ and $\overline{H}_{theo}$ represent mean values

$$\overline{H}_{obs} = \Sigma_{n=1}^{27}\frac{H_{obs}(z_n)}{27}, \qquad \overline{H}_{theo} = \Sigma_{n=1}^{27}\frac{H_{theo}(z_n)}{27}. \tag{28}$$

With the help of the above expressions, the correlation coefficient is written as

$$\Upsilon = \frac{\Sigma_{n=1}^{27}\left(H_{obs}(z_n) - \overline{H}_{obs}\right)\left(H_{theo}(z_n) - \overline{H}_{theo}\right)}{(n-1)K_x K_y}. \tag{29}$$

Note that the values of Υ should always interpolate in the interval $[-1, +1]$. The case $\Upsilon = +1$ ($\Upsilon = -1$) implies the points are on a perfect straight line with positive (negative) slope. The zero value case indicates that there is no relationship at all. Additionally, the absolute value $|\Upsilon|$ represents strength of the relationship: the larger the absolute value of correlation coefficient, the stronger the linear relationship. For the cVPG model, we calculate that $r = 0.967657$ which means there is a strong positive relationship between H_{obs} and H_{theo}.

In literature, the two most used scaling techniques are the normalization and the standardization. The normalization method typically shows re-scaling the corresponding values of a quantity into a range of $[0, 1]$. On the other hand, the standardization typically means re-scaling the corresponding dataset to have a standard deviation of 1 and a mean of 0. We deal with the word normalization informally in statistics, and therefore the term *normalized data* can have multiple meanings. In most situations, while normalizing a dataset, we generally eliminate the units of measurement. This assumption enables us to compare data from different places more easily. Re-scaling a dataset to have values between 0 and 1 is usually called feature scaling. One possible expression to achieve this goal is

$$\Xi_{Norm} = \frac{x_i - x_{min}}{x_{max} - x_{min}}. \tag{30}$$

As a matter of fact, the terms normalization and standardization are sometimes taken into account interchangeably, but it should be known that they usually refer to different things. The standardization is usually called as z-score, and points of a dataset can be standardized with the following expression

$$\Xi_{Stand} = \frac{x_i - \overline{x}}{\sigma_s}, \tag{31}$$

where σ_s is the standard deviation. Making use of the z-score method is very common in statistical investigations. The method allows to compare different sets of data and to determine probabilities for a dataset by considering standardized tables, which is called z-tables.

In FIGs. 4 and 5, using H_{obs} and H_{theo} values, we compared our theoretical results with the recent observations from the statistical perspective. It is concluded from FIGs. 4 and 5 that H_{obs} and H_{theo} data sets are in a good agreement. Thus, we can use our theoretical results also in the ML techniques like the LR in order to make predictions for the future of our universe.

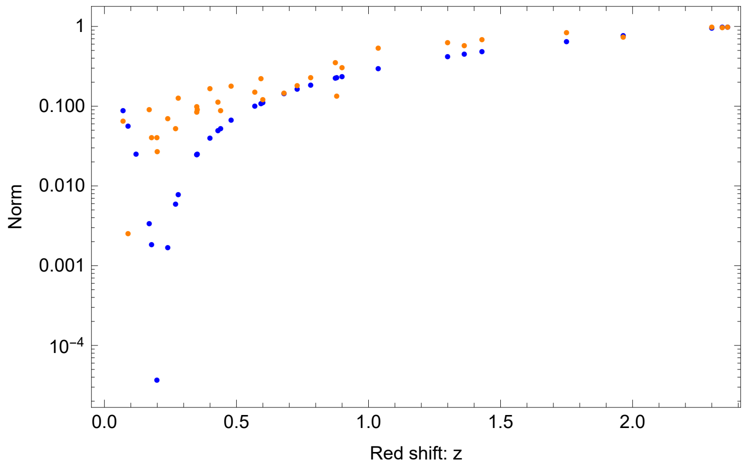

FIG. 4: The graphical analyses of Ξ_{Norm} for H_{obs} and H_{theo} values with the help of best fitting values of the free parameters. Here, the orange dots represent observational values while the blue ones show theoretical values.

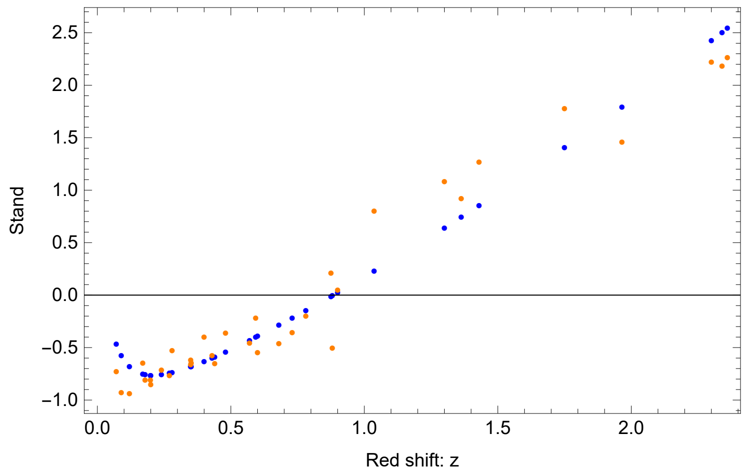

FIG. 5: The graphical analyses of Ξ_{Stand} for H_{obs} and H_{theo} values with the help of best fitting values of the free parameters. Here, the orange dots represent observational values while the blue ones show theoretical values.

V. THE LR ANALYSIS

Now, let's focus on the role of LR in cVPG cosmology. In the LR algorithm, it is known that, for a function $y(x)$, we have

$$y_i = b_0 + b_1 x_i, \tag{32}$$

where

$$b_0 = \overline{y} - b_1 \overline{x}, \tag{33}$$

$$b_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}, \tag{34}$$

$$\overline{x} = \sum_{i=1}^{n} \frac{x_i}{n}, \tag{35}$$

$$\overline{y} = \sum_{i=1}^{n} \frac{y_i}{n}. \tag{36}$$

Here, we focus on the $H(z)$ function (23) to investigate the role of ML in the cVPG cosmology. First of all, we use best fitting values of the free parameters given in the relation (23) and calculate corresponding $H(z)$ values (see the third column in Table II) in order to let the computer learn how the function $H(z)$ depends on the red shift parameter z. Making use of the Python software, we let the computer select random 9 rows given in the third column of Table II and estimate the other 18 rows after learning the evolution of $H(z)$.

TABLE II: Numerical values for the $H(z)$ function and the deviation percentages.

z	$H_{obs}(z)$	$H_{theo}(z)$	$H_{est}(z)$	$\frac{H_{theo}}{H_{obs}}$	$\frac{H_{est}}{H_{obs}}$	D_{to}	D_{eo}
0.0708	69.000[74]	69.950	69.955	1.013768	1.013841	0.013768	0.013841
0.1200	68.600[74]	71.820	71.824	1.046939	1.046997	0.046939	0.046997
0.1700	83.000[75]	67.850	73.965	0.817470	0.891145	0.182530	0.108855
0.1990	75.000[76]	75.310	75.305	1.004133	1.004067	0.004133	0.004067
0.2400	79.690[77]	77.310	77.312	0.970134	0.970159	0.029866	0.029841
0.2800	88.800[74]	77.010	79.384	0.867230	0.893964	0.132770	0.106036
0.3500	84.400[78]	83.250	83.255	0.986374	0.986434	0.013626	0.013566
0.3802	83.000[76]	85.010	85.012	1.024217	1.024241	0.024217	0.024241
0.4000	95.000[75]	86.190	86.191	0.907263	0.907274	0.092737	0.092726
0.4247	87.100[79]	87.690	87.689	1.006774	1.006762	0.006774	0.006762
0.4300	86.450[77]	88.020	88.015	1.018161	1.018103	0.018161	0.018103
0.4497	92.800[79]	91.150	89.237	0.982220	0.961606	0.017780	0.038394
0.4783	80.900[80]	91.040	91.044	1.125340	1.125389	0.125340	0.125389
0.4800	97.000[81]	93.670	91.153	0.965670	0.939722	0.034330	0.060278
0.5700	92.400[82]	101.17	97.083	1.094913	1.050682	0.094913	0.050682
0.5930	104.00[76]	98.650	98.653	0.948558	0.948587	0.051442	0.051414
0.6800	92.000[76]	104.78	104.78	1.138913	1.138913	0.138913	0.138913
0.7300	97.300[83]	114.50	108.42	1.176773	1.114286	0.176773	0.114286
0.7810	105.00[76]	112.23	112.23	1.068857	1.068857	0.068857	0.068857
0.8750	125.00[76]	126.58	119.47	1.012640	0.955760	0.012640	0.044240
0.9000	117.00[75]	121.44	121.44	1.037949	1.037949	0.037949	0.037949
1.3000	168.00[84]	155.36	155.36	0.924762	0.924762	0.075238	0.075238
1.4300	177.00[84]	167.26	167.26	0.944972	0.944972	0.055028	0.055028
1.5300	140.00[75]	181.15	176.68	1.293929	1.262000	0.293929	0.262000
1.7500	202.00[85]	199.48	198.19	0.987525	0.981139	0.012475	0.018861
1.9650	186.50[76]	220.18	220.18	1.180590	1.180590	0.140590	0.180590
2.3400	222.00[86]	260.70	260.70	1.174324	1.174324	0.174324	0.174324

In Table II, we also provide the estimated values of the cosmic Hubble parameter and the deviation percentages D_{to} (between H_{theo} and H_{obs}) and D_{eo} (between H_{est} and H_{obs}). The corresponding mean values of the deviation percentages are

$$\left(\overline{D}_{to}, \overline{D}_{eo}\right) = (0.078372, 0.072647)\,. \tag{37}$$

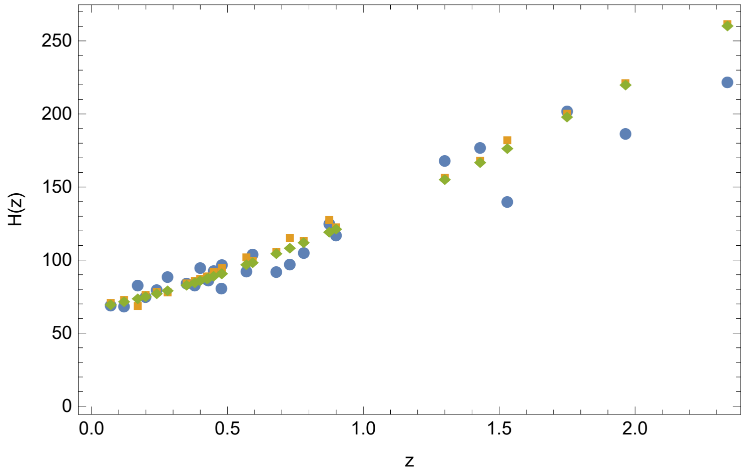

FIG. 6: Plots of $H(z)$ datasets with the help of best fitting values of the free parameters. Here, the blue circles represent observed values, the orange squares are the theoretical values and the green rhombuses shows the estimated values.

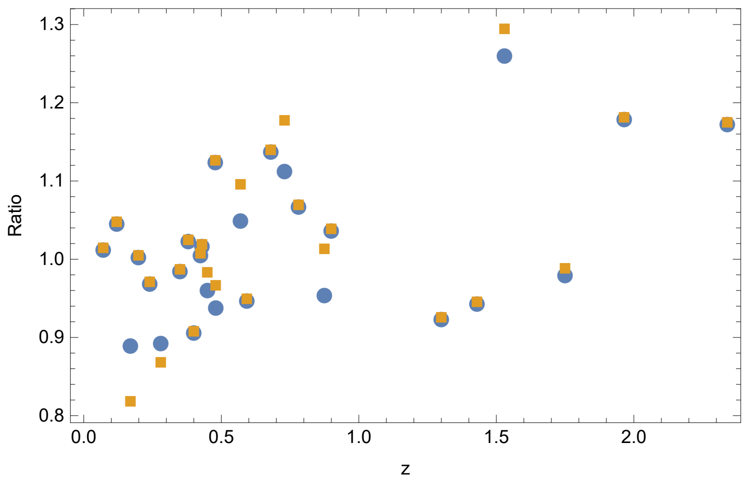

FIG. 7: Comparing $\frac{H_{theo}}{H_{obs}} \sim z$ (the orange squares) and $\frac{H_{est}}{H_{obs}} \sim z$ (the blue circles) relations with the help of best fitting values of the free parameters.

In FIGs. 6 and 7, in order to compare H_{obs}, H_{theo} and H_{est} values, we plot $H(z)$ data sets and the ratios $\frac{H_{theo}}{H_{obs}}$ and $\frac{H_{est}}{H_{obs}}$. One can see from FIG. 7 that the corresponding values of the ratios are concentrated around 1. It is significant to emphasize here that the ML algorithm estimated meaningful values (as a matter of fact better than the theoretical ones) with the help of the cVPG proposal. Thus, one can say that the model is suitable to make interesting predictions for the cosmos.

Moreover, the R-squared statistic helps us to measure the goodness-of-fit of a trend. As a matter of fact, it implies how significantly the slope of a fitted-line differs from zero. The approach is mainly based on partitioning the Total Sum of Squares (TSS henceforth) into the Error Sum of Squares (ESS henceforth) and Regression Sum of Squares (RSS henceforth)[87, 88]. In the R-squared analysis, we have the following expressions[88]

$$TSS = \sum_i (y_i - \overline{y})^2, \tag{38}$$

$$ESS = \sum_i (y_i - \hat{y}_i)^2, \tag{39}$$

$$RSS = \sum_i (\hat{y}_i - \overline{y})^2, \tag{40}$$

$$R^2 = 1 - \frac{ESS}{TSS} = \frac{RSS}{TSS}, \tag{41}$$

where $0 \leq R^2 \leq 1$ and y_i and $\hat{y}_i$ represent the observed and the estimated variables for the independent variable x_i, respectively. In can be said that the case $R^2 = 1$ shows there is no deviation between the actual observed dataset and the estimated values. For the cVPG proposal, we calculated that $R^2 = 0.88$, which clearly indicates that the model provides a very good fit for the recent observable dataset.

VI. ESTIMATING LIFETIME OF THE COSMOS

Based on the various evolutionary characteristics of the cosmic Hubble parameter, the final fate of our cosmos can be divided into the three main categories: the big rip, little rip and the pseudo rip[89]. If $H(t) \to \infty$ when $t \to constant$, the big rip will occur at a certain time. If $H(t) \to \infty$ when $t \to \infty$, the little rip fate will happen. Note that this case has no singularities in the future. The pseudo rip case is identified with $H(t) \to constant$. With the growth of cosmic time $a \to \infty$ (or $1 + z \to 0$) and the help of best fitting values of the free parameters, it is found that

$$H(a) = \frac{\dot{a}}{a} = H_0 \sqrt{\frac{\Omega^0_{vpg}}{\rho^0_{vpg}} \left(\frac{n\xi + 3}{3\beta}\right)^{\xi} a^{n\xi}}. \tag{42}$$

Therefore, the above result indicates the little rip fate of our universe. By solving this equation, one can find the following relation

$$a = \left[1 - \frac{n\xi H_0}{2}\sqrt{\frac{\Omega^0_{vpg}}{\rho^0_{vpg}}\left(\frac{n\xi + 3}{3\beta}\right)^{\xi}}(t - t_0)\right]^{-\frac{2}{n\xi}}, \tag{43}$$

where t_0 represents the present value of time. Substituting the above result in the general expression of the cosmic Hubble parameter, it can be calculated that

$$H = \frac{2}{\frac{2}{H_0\sqrt{\frac{\Omega^0_{vpg}}{\rho^0_{vpg}}\left(\frac{n\xi+3}{3\beta}\right)^{\xi}}} - n\xi(t - t_0)}. \tag{44}$$

So, when $t \to \frac{2}{n\xi H_0}\left[\frac{\Omega^0_{vpg}}{\rho^0_{vpg}}\left(\frac{n\xi+3}{3\beta}\right)^{\xi}\right]^{-\frac{1}{2}} + t_0$, we have $H(t) \to \infty$ which means the cosmos will have a little rip after $t - t_0 = \frac{2}{n\xi H_0}\left[\frac{\Omega^0_{vpg}}{\rho^0_{vpg}}\left(\frac{n\xi+3}{3\beta}\right)^{\xi}\right]^{-\frac{1}{2}}$. Therefore, the lifetime of our cosmos can be determined by free parameters of the model. Now, let us perform a rough estimation. So, it is concluded that the universe will be torn apart after 51Gyr. It is generally accepted that 13.8Gyr is the current age of the universe, so one can say that the cosmos has spent 21 percent of its life.

VII. CLOSING REMARKS

Here, we have investigated the cVPG model of a self-gravitating gaseous sphere from theoretical, numerical and statistical perspectives. We have shown that the theoretical proposal is thermodynamically stable and in good agreement with the recent astrophysical observations when assuming suitable values of the free parameters given in the model. The best-fitted form of the theoretical cVPG model in the 1σ confidence region is

$$p = -0.8a^2\rho^{0.9}. \tag{45}$$

On the other hand, we have focused on the ML mechanism in cVPG cosmology. Why the ML algorithm in astronomy and cosmology? There are three significant reasons. One of them is the automation. Size of a observational dataset indicates that manual intervention is possible only in a highly exceptional circumstance. From this point of view, such analyses need to be automated. The second reason is the acceleration. An ML algorithm can generate shortcuts to expensive simulations. The third one is the superhuman performance. Trained algorithms can sometimes outperform the ability of designed algorithms. The statistical analyses of the model have indicated that the suggested theoretical model can be used also to propose meaningful predictions for the fate of our cosmos and the future experiments via the ML algorithm, which is helping systems to gain the ability learning from augmented data. With the help of this model, we have applied the LR algorithm to the model and let the computer estimate $H(z)$ values. It has been concluded that the ML algorithm produces meaningful $H(z)$ dataset which means it can improve the theoretical predictions. Subsequently, we have performed a rough estimation for the lifetime of our cosmos and seen that the universe has spent 21 percent of its life which means it will be torn apart after 51Gyr.

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