# Variable generalized Chaplygin gas in a 5D cosmology

Mustafa Salti*,[1] Oktay Aydogdu†,[1] Hilmi Yanar‡,[1] and Kenan Sogut§[1]

[1] *Department of Physics, Faculty of Arts and Sciences,*
*Mersin University, Mersin-33343, Turkey*

## Abstract

We construct the variable generalized Chaplygin gas (VGCG) defining a unified dark matter-energy scenario and investigate its essential cosmological properties in a universe governed by the Kaluza-Klein (KK) theory. A possible theoretical basis for the VGCG in the KK cosmology is argued. Also, we check the validity of thermodynamical laws and reimpelement dynamics of tachyons in the KK universe.

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* msalti@mersin.edu.tr

† oktaydogdu@mersin.edu.tr (Corresponding author)

‡ hilmiyanar@mersin.edu.tr

§ kenansogut@mersin.edu.tr

## I. INTRODUCTION

The type Ia supernovae (SNe-Ia)[1, 2], cosmic microwave background (CMB)[3], large scale structure (LSS)[4], Wilkinson Microwave Anisotropy Probe (WMAP)[5–7], Sloan Digital Sky Survey (SDSS)[8] and the Planck-2015[9] astrophysical data show that the observable universe expands faster than it should and is nearly flat. It is commonly believed that dark matter and dark energy are the source of speedy expansion phase. In literature, many theoretical models such as the cosmological constant[10], scalar fields[11–16], unified dark matter-energy descriptions[17, 18], dark energy densities[19–23], modified gravity[24] and extra dimensions[25–27] have been introduced in order to explain the dark matter-energy effect. Cai et al.[28] prepared a convenient brief about some well-known theoretical dark energy proposals. Although there are a large number of efforts to discuss of the speedy expansion phase, which is still completely unknown.

Assuming the existence of extra dimensions is a very useful idea to understand the speedy expansion of our universe. The KK theory of gravity is one of the most studied extra dimensional theories and it includes a coupling between electromagnetism and gravity[25, 26]. According to the idea of KK gravity, the universe may have five dimensions. Subsequently, the well-known KK theory is divided into the compact and non-compact branches[29, 30]. The fifth dimension is length-like in the compact form of KK gravity while we have a mass-like fifth dimension in the non-compact version of KK theory. As a matter of fact, the non-compact form is an outcome of the Campbell theorem in which we cannot define any matter in a five-dimensional manifold by hand[29–31]. In further investigations, the original KK gravity turned out to be a base of other extra dimensional models in different perspectives[32–34]. A useful review of extra dimensional unified theories can be found in Ref.[35].

On the other hand, we have two more interesting proposals which emerge from the string theory[36] and are known as Chaplygin gas (CG)[17, 37] and Polytropic gas (PG)[38] models. It is very important to mention here that the CG and PG are unifications of the dark matter and the dark energy. The unified CG description has also been developed into its generalized[39–41], modified[42, 43], variable[44–48], variable generalized[49, 50], variable modified[51] and extended[52–57] versions. Recently, Pourhassan investigated the extended CG unified dark energy model in the Horava-Lifshitz theory of gravity[57] and discussed the unified universe history through the phantom extended CG model[56]. Next, Panigrahi and Chatterjee studied cosmology via thermodynamics of the variable modified[50] and variable generalized[51] CG models. Variable CG proposals are written and constrained by focusing on the union supernovae sample and Barion acoustic oscillation (BAO)[49] considering the $B$ parameter to depend on the scale factor of Friedmann-Robertson-Walker (FRW) metric, i.e. $B \to B_0 a^{-n}$ where $B_0$ is an absolute constant. That is the reason why the extended CG proposal[52–57] cannot be reduced to the variable CG models[49–51]. Although different forms of the CG description were taken into account as dark energy model and used to fit the varied symptoms of cosmological probes, it is not enough to be able to match the observational data[51]. In this paper, we construct a five-dimensional form of the VGCG formulation which can be reduced into the variable, modified and original CG proposals under some limiting conditions. Indeed, it will be very interesting to investigate the CG model and its cosmological implications in the compact KK universe.

The structure of this work is as following: in the next section we implement the KK form of the VGCG model exactly. In the third section, we discuss some physical features of the KK type VGCG definition in order to interpret the model cosmologically. The fourth section is devoted to investigate cosmology via thermodynamical laws. In the last section, we give our final remarks. All numerical calculations and analyzes will be performed by using MATHEMATICA sofware[58].

## II. KK TYPE VGCG

One of the most studied proposals for the role of dark energy is a tachyon scalar field which is defined by a Born-Infeld type lagrangian density[59]

$$\pounds = \upsilon(\varphi)\sqrt{1 + g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi} \tag{1}$$

where $\varphi$ is a scalar field having a self-interacting potential $\upsilon(\varphi)$ and $g^{\mu\nu}$ shows the inverse metric tensor. In a spatially flat KK type FRW (KKFRW) universe, the corresponding energy density and pressure of the tachyon field are defined by

$$\rho_\varphi = \frac{\upsilon(\varphi)}{\sqrt{1-\dot{\varphi}^2}}, \tag{2}$$

and

$$p_\varphi = -\upsilon(\varphi)\sqrt{1-\dot{\varphi}^2}, \tag{3}$$

respectively. Formally, the corresponding equation of state (EoS) is written as

$$p_\varphi = -\frac{\upsilon(\varphi)}{\rho_\varphi}. \tag{4}$$

One can easily show that for the potential $\upsilon(\varphi) = B = constant$ the energy density (2) and the pressure (3) are connected by the original Chaplygin gas (OCG) state equation, i.e. $p = -\frac{B}{\rho}$. From this point of view, it is concluded that the OCG definition coincides with the simplest form of the tachyon scalar field model. Moreover, redefining the self-interacting potential as $\upsilon(\varphi) = Ba^{-n}$, where $n$ is another constant and $a$ denotes the time-dependent cosmic scale factor, yields the variable Chaplygin gas (VCG) model. Note that, $n = 0$ reduces this model into the OCG proposal. Here, we consider an extended form of the VCG, which is known as the VGCG, and assume[60] that

$$p = -\frac{Ba^{-n}}{\rho^\alpha}, \tag{5}$$

where $\alpha$ is a positive parameter with $0 < \alpha \leq 1$.

The KKFRW universe is represented by[61] the following line-element

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)dx_5^2\right], \tag{6}$$

where $k$ implies the cosmic curvature parameter for the closed ($k = -1$), flat ($k = 0$) and open ($k = +1$) universes. Due to the recent astronomical and cosmological data observed by SNe-Ia[2], WMAP[5], SDSS[8], Planck-2015[9] and X-ray[62] have strongly suggested a spatially flat spacetime, we assume that $k = 0$ for further investigations. Next, we suppose that the KKFRW universe is filled with the VGCG, which is defined by the energy momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - g_{\mu\nu}p$ with $u_\mu$, describing the five-velocity vector. Remember that the VGCG (or any other forms of the CG) is a combination of the dark matter and the dark energy, i.e. $\rho = \rho_m + \rho_e$ and $p = p_m + p_e$ where the subscripts $m$ and $e$ imply the dark matter and the dark energy, respectively.

The corresponding Friedmann equations, which govern the evolution of cosmic scale parameter, are obtained as

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3}\rho, \tag{7}$$

$$2H^2 + \dot{H} = \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{8\pi G}{3}p, \tag{8}$$

where the over-dot indicates a time-derivative. In the KKFRW framework, a fluid with an energy density $\rho$ and pressure $p$ satisfies the following conservation law

$$\dot{\rho} + 4H(1+\omega)\rho = 0, \tag{9}$$

with the fact that EoS parameter of the KK type VGCG (KKVGCG) is defined as $\omega = \frac{p}{\rho}$. It is easy to see that the above conservation relation can be rewritten in a very elegant form

$$d(\rho a^4) + p d(a^4) = 0. \tag{10}$$

This equation can be integrated easily and it can be shown that the energy density of the KKVGCG evolves as

$$\rho = \frac{1}{a^4} \sqrt[1+\alpha]{4(1+\alpha) \int B a^{4\alpha+3-n} da + c}, \tag{11}$$

where $c$ is an integration constant. Hence, taking explicitly the above integral, it yields

$$\rho = \left[ \frac{4(1+\alpha)}{4(1+\alpha)-n} \frac{B}{a^n} + \frac{c}{a^{4(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}. \tag{12}$$

It can be seen from the KK form of the Friedmann equations that the speedy expansion phase $\ddot{a} > 0$ is equivalent to

$$\left[ 1 - \frac{(1+\alpha)}{4(1+\alpha)-n} \right] a^{4(1+\alpha)-n} > \frac{c}{4B} \tag{13}$$

which requires $n < 3(1+\alpha)$. Besides, it is known that the cosmological and present energy densities are connected to each other by $\rho = 1.31\rho_c$[56]. Thus, the integration constant $c$ can be expressed in terms of current value of the cosmic scale factor $a_0$. In the entire work, we assume that $a_0 = 1$ for convenience. Thence, it follows that

$$c = 1.72 - \frac{4(1+\alpha)B}{4(1+\alpha)-n}. \tag{14}$$

At this step, we introduce a new parameter written as

$$\Omega = \frac{c}{c + \frac{4(1+\alpha)B}{4(1+\alpha)-n}}. \tag{15}$$

This new parameter transforms the equation (12) into a more convenient form like

$$\rho = \sqrt[1+\alpha]{\Omega a^{-4(1+\alpha)} + (1-\Omega)a^{-n}}. \tag{16}$$

Using the cosmic red shift parameter $z$ in relations helps us to fix free parameters of a theoretical model according to the recent astrophysical data. The red shift parameter and the cosmic scale factor are connected to each other by $z = \frac{1}{a} - 1$. Making use of

$$E(z) \equiv H_0^{-1} H(z) \tag{17}$$

where $H_0 = 67.8 \pm 0.9$ km s$^{-1}$ Mpc$^{-1}$[9] is the recent observable value of the Hubble parameter, one gets

$$H(z) = H_0 \left[\Omega(1+z)^{4(1+\alpha)} + (1-\Omega)(1+z)^n\right]^{\frac{1}{2(1+\alpha)}} . \tag{18}$$

So that there are three free parameters in the KKVGCG model: $B$, $\alpha$ and $n$. In FIG. 1, we analyze the cosmic Hubble parameter $H$ against the red shift parameter $z$. We consider five different $n$ cases in order to discuss our result numerically. Here, the $n = 0$ case (red dashed line) represents the OCG solution and the black squares represent the experimental data[63]. We see that the set $(n, \alpha, B) \equiv (0.5, 0.3, 1.4)$ represented by the blue solid line, denotes the best fit and describes a more meaningful model than the other cases including the OCG proposal.

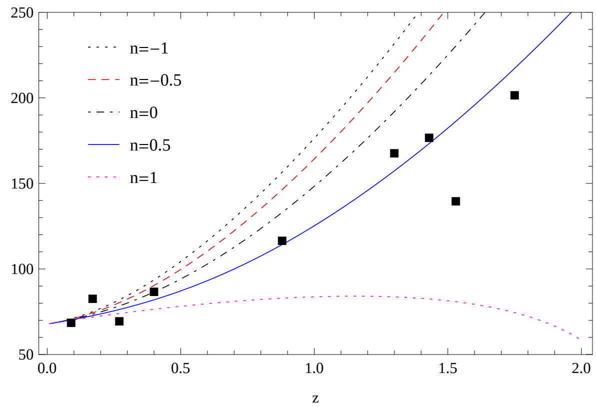


FIG. 1: $H \sim z$ relation with auxiliary parameters $\alpha = 0.3$ and $B = 1.4$.

## III. PHYSICAL PROPERTIES OF THE KKVGCG

### A. EoS parameter

Making use of equations (5) and (16), EoS parameter, describing the KKVGCG, is calculated as

$$\omega \approx -1 + \frac{n}{4(1+\alpha)}, \tag{19}$$

where the parameter $n$ has a prominent influence. Depending on the signature of $n$, the above result implies three different possibilities for the EoS parameter $\omega$. One can see that the Universe tends to be (i) phantom dominated[64], i.e. $\omega < -1$ for $n < 0$, (ii) quintessence dominated[65, 66], i.e. $\omega > -1$ for $n > 0$ or (iii) $\Lambda$CDM (cosmological constant plus cold dark matter) dominated, i.e. $\omega = -1$ for $n = 0$. This conclusion is compatible with the recent astrophysical data[67, 68].

### B. Deceleration parameter

In order to analyze the KKVGCG cosmologically in a different way, one can also calculate the deceleration parameter. Using the relation $q = -\frac{\ddot{a}}{aH^2}$ with (19) gives

$$q = \frac{1}{2} + \frac{3}{2}\frac{p}{\rho} = -1 + \frac{3n}{8(1+\alpha)}. \tag{20}$$

So we must have $n < \frac{8}{3}(1+\alpha)$ which does not violate the previous restriction on $n$, otherwise we get positive $q$ values describing a decelerating universe. See the $q \sim n$ relation for $\alpha = 0.3$ in FIG. 2.

As a matter of fact, we will not mention much in further investigations about the case $n < 0$, because later we will show that checking the thermodynamical stability of the model indicates that the value of $n > 0$.

### C. Statefinder diagnostic

The statefinder parameter cosmology is used to discriminate between different theoretical dark energy candidates. Trajectories in the $(r, s)$-plane corresponding to different cosmological proposals imply qualitatively different behaviors. The statefinders are implemented

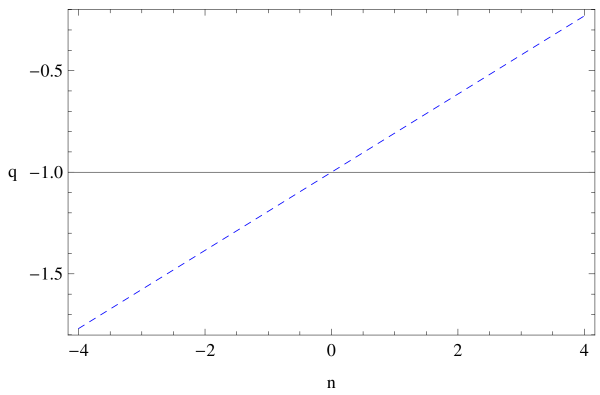


FIG. 2: $q \sim n$ relation with auxiliary parameter $\alpha = 0.3$.

from the cosmic scale factor $a$ as

$$r \equiv \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{21}$$

To begin with, we will write the above relations in a more convenient form as follows

$$r = 1 - 6(1+\omega) + 8(1+\omega)^2, \tag{22}$$

$$s = \frac{4}{9\omega}(1+\omega)(1+4\omega). \tag{23}$$

The $\Lambda$CDM model includes a fixed point $(r, s) \equiv (1, 0)$. The well-known quintessence proposals are given by vertical parts with $r$ decreasing from $r = 1$ to some definite values[69, 70]. If $q = -0.5$, then the current values of statefinders (within the KKVGCG description) are obtained as $r = -\frac{1}{9}$ and $s = \frac{10}{27}$. Making use of equations (22) and (23), we find

$$r_1 = 1 - \frac{9s}{64}\left[20 - 9s - 3\sqrt{(s-4)(9s-4)}\right]. \tag{24}$$

$$r_2 = 1 - \frac{9s}{64}\left[20 - 9s + 3\sqrt{(s-4)(9s-4)}\right]. \tag{25}$$

In FIG. 3, we depict the $(r, s)$-plane for the KKVGCG model and conclude that the $r = r_1$ case includes current values of the statefinders. Thus, we further ignore the case $r = r_2$. It is seen that $r$ first decreases from the $\Lambda$CDM fixed point $r = 1$, $s = 0$ to its minimum value and then increases.

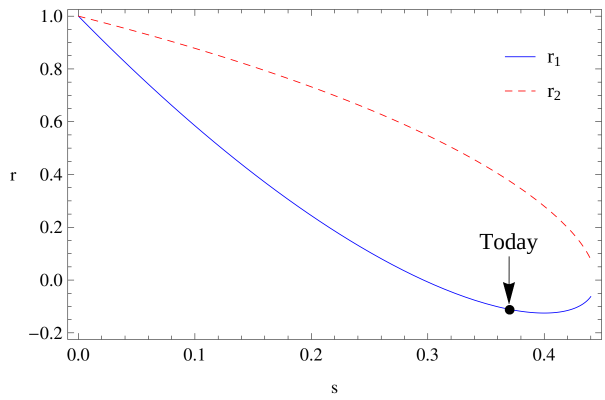


FIG. 3: r-s evolution diagram for the KKVGCG model.

### D. Density perturbations

Density perturbations can be used in order to check the instability of the unified KKVGCG model. Assuming the following definitions[71]

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \qquad \tilde{u}_{\mu} = u_{\mu} + \delta u_{\mu}, \tag{26}$$

$$\tilde{\rho}_{\mu} = \rho_{\mu} + \delta\rho_{\mu}, \qquad \tilde{p}_{\mu} = p_{\mu} + \delta p_{\mu}, \tag{27}$$

where $h_{\mu\nu}$, $\delta u_{\mu}$, $\delta\rho_{\mu}$ and $\delta p_{\mu}$ imply small perturbations around $g_{\mu\nu}$, $u_{\mu}$, $\rho_{\mu}$ and $p_{\mu}$, respectively, the evolution equation for perturbation is written in the Newtonian framework as[71]

$$\ddot{\delta} + 2H\dot{\delta} + \left[\frac{\lambda^2}{a^2}\vartheta_s^2 + 3\frac{\ddot{a}}{a}\right]\delta = 0. \tag{28}$$

Here $\delta = \frac{\delta\rho}{\rho}$, $\lambda$ shows the wavelength of the perturbations and $\vartheta_s$ is the speed of sound which is defined as

$$\vartheta_s^2 = \frac{\partial p}{\partial \rho} = \frac{\dot{p}}{\dot{\rho}} = \frac{p'}{\rho'}, \tag{29}$$

where the prime shows derivative with respect to $z$. Now, making corresponding computation gives

$$\vartheta_s^2 = \frac{4\alpha(1+\alpha) - n\alpha}{4(1+\alpha)}\left[1 - (1+z)^{4(1+\alpha)-n}\left(\frac{4(1+\alpha)-n}{1+\alpha}\frac{0.43}{B} - 1\right)\right], \tag{30}$$

which is implying that $\vartheta_s^2$ remains always positive if we have

$$B \leq 0.43\left[4 - \frac{n}{1+\alpha}\right]. \tag{31}$$

Thus, under this condition, there is no concern about instability of the model or imaginary sound speed.

### E. Scaling solution

One can easily show that the background evolution for the KKVGCG description is equivalent to that for an interaction between the dark energy and the dark matter with the EoS parameter $\omega = -1 + \frac{n}{4(1+\alpha)}$. Taking into account the scaling case for the dark matter-energy density, i.e. $\rho = \rho_m + \rho_e$ with $\rho_e \propto \rho_m a^{4(1+\alpha)-n}$, in the KKFRW framework yields

$$\rho_m + \rho_e = \rho_{cr} \sqrt[1+\alpha]{(1-\Omega_m^0)(1+z)^n + \Omega_m^0 (1+z)^{4(1+\alpha)}}, \tag{32}$$

where $\rho_{cr}$ defines the critical energy density while $\Omega_m^0$ denotes the matter density parameter. Comparing the equation (32) with (16), it is concluded that $\Omega$ can be interpreted as an effective matter density $\Omega_m^0$. If the coupled case is described by

$$\dot{\rho}_m + 4H\rho_m = \Sigma, \tag{33}$$

$$\dot{\rho}_e + \frac{n}{1+\alpha}\rho_e = -\Sigma, \tag{34}$$

then we can characterize the interaction term by [72]

$$\Sigma = 4H \left[\frac{n}{4(1+\alpha)} - 1\right] \frac{\rho_m}{1 + \frac{\rho_m}{\rho_e}}, \tag{35}$$

which describes the interaction between the dark matter and the dark energy. Hence, we can say that background evolution of the KKVGCG is identical to that of dark matter interacting with dark energy. Note that negative values of $\Sigma$ imply energy transition from the dark matter territory to the dark energy one, and positive $\Sigma$ values describe the vice versa situation. This significant event has been observed recently in the Cluster Abell A586[73, 74], however its importance has not been clarified yet.

## IV. THERMODYNAMICS OF THE KKVGCG

We start with the relation of some thermodynamical quantities with the energy density[75] $\rho = \frac{U}{V}$ where $U$ denotes the internal energy and $V = \frac{1}{2}\pi^2 R_h^4$ with the dynamical apparent horizon $R_h = [H^2 + k(1+z)^2]^{-\frac{1}{2}}$ is an extra-dimensional volume of the system. Another significant thermodynamical quantity is the entropy, which is written as $S = \frac{A}{4G}$ where $A = 2\pi^2 R_h^3$ is the surface area of 4-sphere. For the flat KKU, we get $R_h = H^{-1}$ which is known as the Hubble horizon. Therefore, making use of equation (18), we obtain

$$S = \frac{\pi^2}{2GH_0}\left[\Omega(1+z)^{4(1+\alpha)} + (1-\Omega)(1+z)^n\right]^{-\frac{3}{2(1+\alpha)}} = \frac{\pi^{\frac{3}{2}}\sqrt[4]{V}}{2^{\frac{3}{4}}GH_0}. \tag{36}$$

It is clear that the entropy is an incensing function of $V$ (and therefore is an increasing function of time) which shows that the generalized second law of thermodynamics is valid as expected.

On the other hand, focusing on the first law of thermodynamics, tells the temperature is defined as $T = (\rho + p)V/S$. It is interesting to write relations among energy density of the KKVGCG, temperature and entropy. Using our calculations given in the previous sections, it can be obtained that

$$T(\rho) = \sqrt{\frac{3G}{4\pi}}\frac{n\rho^{1/2}}{4(1+\alpha)}, \tag{37}$$

and

$$T(S) = \frac{3nS^5}{(2\pi)^9(1+\alpha)G^7H_0^6}. \tag{38}$$

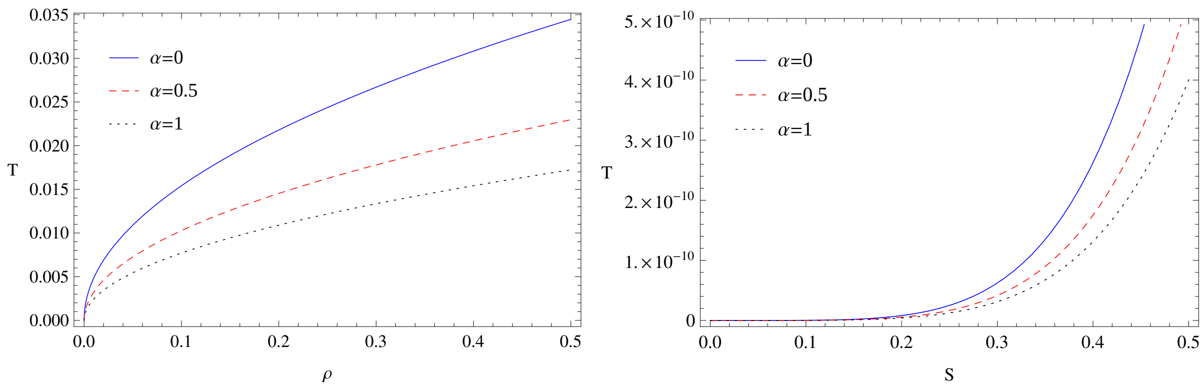


FIG. 4: Graphical analyzes of $T(\rho)$ and $T(S)$ according to three different cases of $\alpha$ with auxiliary parameters $n = 2$, $H_0 = 67.8$ and $8\pi G = 1$.

In FIG. 4, we plot $T \sim \rho$ and $T \sim S$ relations with three different $\alpha$ values. One can see that the temperature of VGCG dominated the KK universe, which increases by increasing energy density just as it is expected. Additionally, it is concluded that the entropy approaches zero as the temperature approaches zero or conversely the temperature approaches zero when the entropy approaches zero. From this point of view, the third law of thermodynamics is satisfied for the KK type VGCG.

At this step, we check the thermodynamical stability condition of the KKVGCG model. For this purpose, we need to check the validity of $T\frac{\partial S}{\partial T} > 0$. Actually, this relation denotes heat capacity which can be rewritten in terms of volume[76]

$$C(V) = V\frac{\partial \rho}{\partial V}\left(\frac{\partial T}{\partial V}\right)^{-1}. \tag{39}$$

Thus, for the KK type VGCG model, we find

$$C = \frac{8(1+\alpha)}{nG}\sqrt[4]{\frac{\pi^2}{2}}V^{\frac{3}{4}}, \tag{40}$$

and consequently conclude that we should use positive n values in order to describe a thermodynamically stable model. See FIG. 5. So, we can reach a significant result for the free parameter $n$ and write $0 < n < \frac{8}{3}(1+\alpha)$ for completeness.

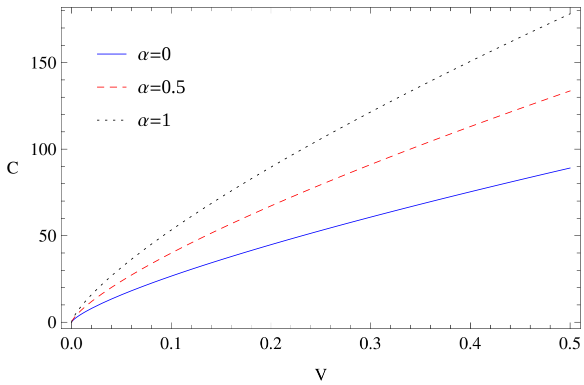


FIG. 5: Heat capacity in terms of V. Here, we take $n = 2$ and $8\pi G = 1$.

## V. 5D TACHYONS

Using equations (2) and (3), we can rewrite the KK form of the Friedmann equations (7) and (8) as

$$6H^2 = \frac{\upsilon(\varphi)}{\sqrt{1-\dot{\varphi}^2}}, \tag{41}$$

and

$$6H^2 + 3\dot{H} = \upsilon(\varphi)\sqrt{1-\dot{\varphi}^2}, \tag{42}$$

with $8\pi G = 1$.

In addition to this, the conservation relation (9) (or (10) of course) is, in turn, identical to the equation of motion for the tachyon field $\varphi$

$$\frac{d\upsilon}{d\varphi} + \upsilon\ddot{\varphi} + 4H\upsilon\dot{\varphi} = 0. \tag{43}$$

Substituting $\upsilon$ from the KK form of Friedmann equations together with the relations for the energy density and pressure into the conservation equation we find the following interesting result, expressing the change of $\upsilon$ in terms of the cosmic Hubble parameter and its time derivative

$$\dot{\varphi}^2 = -\frac{\dot{H}}{2H^2}, \tag{44}$$

and using the Friedmann equation (41) with (44) gives

$$\left(\frac{dH}{d\varphi}\right)^2 - 4H^4 + \frac{1}{9}\upsilon^2 = 0. \tag{45}$$

Now, considering the relation $\upsilon = Ba^{-n}$ assumed in the second section, we obtain that

$$\left(\frac{dH}{da}\right)^3 - \frac{4H^3}{a^3} - \frac{B}{9}\frac{a^{-2n-3}}{H} = 0, \tag{46}$$

and solving this equation gives $H(a)$. After some algebra, we get

$$a(t) = \left(-\frac{n}{2\beta}\right)^{\frac{2}{n}} t^{-\frac{2}{n}} \tag{47}$$

where

$$\beta = \sqrt[4]{\frac{8B}{9(32-n^3)}}. \tag{48}$$

Substituting this solution into the definition $\upsilon(a) = Ba^{-n}$, we find that the tachyon field has the following time-dependency:

$$\varphi(t) = \left[\frac{n(4-n)}{8\beta}\right]^{\frac{2}{4-n}} t^{\frac{2}{4-n}}. \tag{49}$$

Inverting $\varphi(t)$ to get $t(\varphi)$ we finally obtain $\upsilon(\varphi)$. So, in terms of $\varphi(t)$, the potential becomes

$$\upsilon(\varphi) = \left[\frac{32B}{3n^2(4-n)^2}\right]^2 \frac{2\varphi^{4-n}}{32-n^3}. \tag{50}$$

Note that we previously concluded that $n \neq 0$ and the special case $n = 4$ must be interpreted separately, because it implies divergent tachyons.

In FIG. 6, we plot $\varphi(t)$ and $\upsilon(\varphi)$ for three different values of the free parameter $n$. We see that the scalar field function $\varphi$ increases in time and also conclude that the tachyonic self-interaction potential $\upsilon$ increases by increasing scalar field $\varphi$.

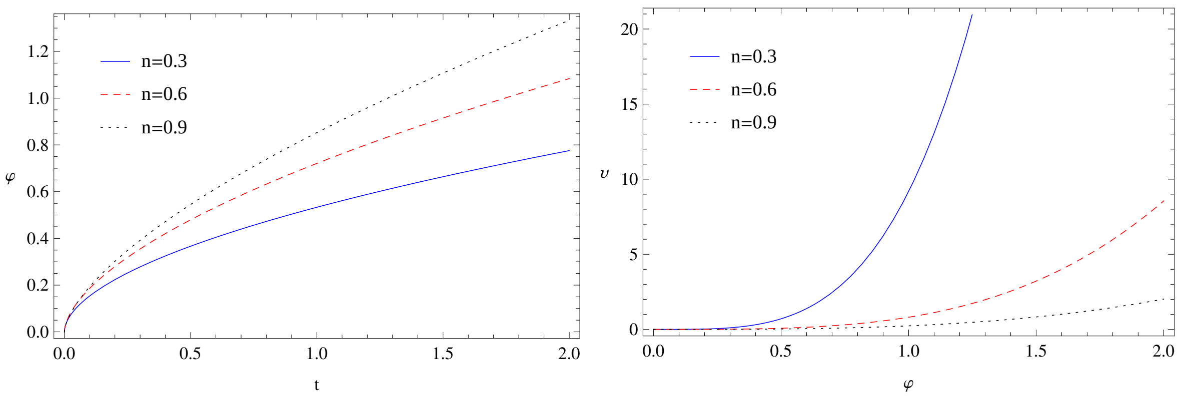


FIG. 6: Graphical analyzes of $\varphi(t)$ and $v(\varphi)$ according to three different cases of $n$ with $B = 0.1$.

## VI. FINAL REMARKS

In this paper, we investigate the KKVGCG unified dark matter-energy model and its cosmological features in the compact KK universe. First, we calculate the corresponding energy density in terms of the cosmic scale factor and then obtain an analytical expression for the Hubble parameter in terms of the red shift parameter. Next, considering these results we discuss some physical properties of the proposal in order to find some constraints for the free parameters used to describe the model. We also focus on thermodynamical quantities of the proposal such as entropy, temperature and heat capacity. We confirm that the thermodynamical laws are satisfied and the model can be stabilized at all times by using positive values of the free parameter $n$. The stability condition of the unified dark matter-energy description is also checked by making use of the speed of sound and it is concluded that there is no concern about instability of the model or imaginary sound speed, if we assume suitable values of $B$.

Furthermore, due to the KKVGCG unified dark matter-energy description is identical to that of a tachyonic scalar field $\varphi$ having self-interacting potential $v(\varphi)$, we also investigate the KKVGCG type reconstruction of tachyon field in order to determine self-interacting potential.

We can summarize our main conclusions in the following way: (i) the KK type VGCG model is consistent with the recent experimental data, (ii) the free parameter $\Omega$ can be interpreted as an effective matter density, (iii) the background evolution of the KK type VGCG model matches with an interaction between the dark energy and the dark matter,

(iv) KK form of the VGCG description is equivalent to a model including a tachyon field and its self-interacting potential.

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