Does the Polytropic gas yield better results in a 5D framework?

H.I. Ay,[1] O. Aydogdu*,[1] and M. Salti†[1]

[1]*Department of Physics, Faculty of Arts and Science,*
Mersin University, Mersin, TR 33343, Turkey

Abstract

In this work, we study the polytropic gas (PG) cosmology in a d-dimensional (dD) form of the flat Friedmann-Robertson-Walker (FRW) framework. In this context, we focus on the evolution of the corresponding energy density as a first step. Next, we use the most recent data from the Type Ia Supernova (SN Ia), observational values of the cosmic Hubble parameter (OHD) and the updated Planck-results to place constraints on the free parameters defined in the model. We show that the 5D form of the scenario is more compatible with the recent observations. Moreover, according to the best values of the auxiliary parameters, we compte age of the cosmos theoretically.

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* oktaydogdu@mersin.edu.tr
† msalti@mersin.edu.tr

I. INTRODUCTION

In the recent decades, the striking observational data has strongly suggested the existence of a cosmological constant or dark energy (DE). Moreover, the cosmic microwave background (CMB) radiation[1–3], SN Ia[4, 5], gravitational clustering (GC)[7] and the Planck[6] observations indicate that the universe has been moved into a speedy expansion phase by an exotic form of negative-pressure contents, called dark matter (DM) and the DE. The best current observational data[6] show up that dark contents (DM plus DE) constitute $27.8 + 67.3 = 95.1$ percent of the total energy in the observable cosmos. The mass energy of the remaining part includes baryons and also non-relativistic DM.

The earliest proposal of the DE is known as the cosmological constant with associated mass energy density

$$\rho_\Lambda \approx 18.78 \times 10^{-30} \Omega_\Lambda h^2 gcm^{-3}, \tag{1}$$

where $h = H_0/100/[kmsec^{-1}Mpc^{-1}]$ shows the then-favored dimensionless Hubble parameter, H_0 implies the present value of Hubble constant and $\Omega_\Lambda \approx 0.67$ indicates the dimensionless energy density parameter. Although, the cosmological constant with cold DM (ΛCDM) model gives a nice interpretation for the speedy enlargement phase, it remains sensible that making use of a time-dependent dark energy density may yield better results (for a good review see Ref.[8] and references therein). Due to the earliest DE models are suffering from famous cosmological constant issues such as the cosmological coincidence and the fine-tuning puzzles, many physicists have tried to propose different ideas in order to handle the mysterious puzzle. Actually, any inflationary proposal may be used to reach this aim if various values for its auxiliary parameters are assumed. One of the theoretical dark universe proposals is based on assuming a scalar field description which is coupled minimally with gravity[9–12]. Besides, a non-minimal coupling case in scalar field proposals leads to other theoretical candidates of the dark content in scalar-tensor theories (see Ref.[13] and references therein). Chaplygin gas (CG)[14, 15] or the PG[16, 17] definitions emerge from the string theory[18] and invoke matter with interesting properties. Still, the others[19–22] explain the cosmic accelerated enlargement behavior with the help of the quantum vacuum polarization, topological defects or the particle creation. Subsequently, it was resulted that extra-dimensional braneworld assumptions, which interpret the speedy expansion phase by formulating the general relativity in a 5D framework, could help us to explain the current observational cosmic phenomenon[23–28].

It is very important to emphasize here that the CG and the PG models indicate a unified dark matter-energy scenario. The CG model was also developed into its generalized[29–31], modified[32, 33], variable[34–38], variable generalized[39, 40], variable modified[41] and extended[42–47] forms. Although various forms of the CG model have been taken into account in order to fit the different symptoms of astronomical probes, the CG family is not able enough to match the recent astrophysical measurements[41].

Constraining the free parameters given in a theoretical point of view of the dark universe is one of the interesting topics in modern cosmography. Focusing on the luminosity distance data[48, 49], X-ray gas mass fraction of galaxy clusters[50, 51], size of the baryonic acoustic oscillation (BAO) peak[52] and the CMB measurements[53, 54] are the most often considered methods. Recently, a new way including the observational values of the cosmic Hubble parameter, which is connected with the differential ages of the oldest galaxies, was considered to check some theoretical descriptions[55–62]. In this work, we extend the original PG model to its dD form and then fit the model parameters according to the current experimental values of the Hubble parameter.

We organize this paper as follows: in the next section, we introduce the dD form of the PG cosmological scenario. In the third section of this study, we focus on the recent astrophysical results to fit free parameters of the PG type unified dark universe scenario. Subsequently, in the fourth section, we discuss cosmological properties of the model by focusing on evolutionary natures of some well-known cosmological parameters. Note that all numerical analyses will be done by making use of the MATHEMATICA sofware[63].

II. THE dD FORM OF THE PG MODEL

We shall start with focusing on an extra dimensional, isotropic and homogeneous space-time model which is represented by the following type of the FRW metric[64]

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 dx_n^2\right], \tag{2}$$

where

$$dx_n^2 = d\varphi_1^2 + \sin^2\varphi_1 d\varphi_2^2 + ... + \sin^2\varphi_1 \sin^2\varphi_2 ... \sin^2\varphi_{n-1} d\varphi_n^2. \tag{3}$$

Here, $n = d - 2$ and $a(t)$ stands for the number of spacetime dimensions and the cosmic scale factor, respectively.

Equation-of-motion (EoM) is basically defined by the following form of the Einstein field equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{4}$$

where the energy-momentum tensor is represented by the expression

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - pg_{\mu\nu}. \tag{5}$$

Here, ρ and p are the energy density and pressure of the cosmic perfect fluid, respectively, and U_μ denotes the velocity vector. We assume that the cosmic fluid is a mixture of the ordinary baryonic matter (BM) and the PG, thence, ρ and p can be written as $\rho = \rho_m + \rho_{pg}$ and $p = p_{pg}$ with $p_m = 0$. Substituting the metric (2) into the EoM (4) results in two independent field equations[64];

$$\frac{(d-1)(d-2)}{2}\frac{\dot{a}^2}{a^2} = 8\pi G\rho, \tag{6}$$

$$\frac{(d-2)\ddot{a}}{a} + \frac{(d-2)(d-3)}{2}\frac{\dot{a}^2}{a^2} = -8\pi G p, \tag{7}$$

where $(. = \frac{d}{dt})$. Moreover, the above equations yield the relation

$$\dot{\rho} + (d-1)(\rho + p)\frac{\dot{a}}{a} = 0. \tag{8}$$

Considering the theoretical definition of the cosmic Hubble parameter, i.e. $H \equiv \frac{\dot{a}}{a}$ estimating the expansion rate of the cosmos, we can rewrite equations (6), (7) and (8) in the following forms

$$\frac{(d-1)(d-2)}{2}H^2 = 8\pi G\rho, \tag{9}$$

$$(d-2)(\dot{H} + H^2) + \frac{(d-2)(d-3)}{2}H^2 = -8\pi G p, \tag{10}$$

$$\dot{\rho} + (d-1)(\rho + p)H = 0. \tag{11}$$

One can decompose the equation (11) into two conserving relations for the energy densities of the BM and PG as written below

$$\dot{\rho}_m + (d-1)\rho_m H = 0, \tag{12}$$

$$\dot{\rho}_{pg} + (d-1)(\rho_{pg} + p_{pg})H = 0. \tag{13}$$

In this dark substances scenario, we can introduce the following fractional densities

$$\Omega_m = \frac{\rho_m}{\rho_c}, \qquad \Omega_{pg} = \frac{\rho_{pg}}{\rho_c}, \tag{14}$$

where

$$\rho_c = \frac{(d-1)(d-2)H_0^2}{16\pi G} \tag{15}$$

denotes the critical density. Hence, equation (9) can be rewritten in a very nice form $\Omega_m + \Omega_{pg} = E$, where $E = \frac{H}{H_0}$ represents the dimensionless Hubble parameter.

The equation of state (EoS) describing the PG[16, 17] model is written as

$$p_{pg} = \beta\rho_{pg}^{1+\frac{1}{\xi}}, \tag{16}$$

where both β and ξ are real parameters. Note that assuming the case including $\xi = -\frac{1}{2}$ with $\beta = -B$ transforms the above formulation into the form of the original CG proposal[14, 15]. Moreover, taking $\xi = -\frac{1}{\alpha+1}$ with $\beta = -B$ reduces it to the formulation of the generalized CG energy density[29–31].

We can rewrite the conservation laws (12) and (13) as given below

$$\frac{d\rho_m}{da} + \frac{(d-1)}{a}\rho_m = 0, \tag{17}$$

$$\frac{d\rho_{pg}}{da} + \frac{(d-1)}{a}\rho_{pg}(1+\beta\rho_{pg}^{\frac{1}{\xi}}) = 0. \tag{18}$$

Consequently, solving the above differential equations yields

$$\rho_m = \rho_m^0 a^{1-d}, \tag{19}$$

$$\rho_{pg} = \frac{1}{\left[ca^{\frac{d-1}{\xi}} - \beta\right]^{\xi}}, \tag{20}$$

where ρ_m^0 and c are integration constants. It is concluded that, for $\beta < ca^{\frac{d-1}{\xi}}$, the polytropic energy density always have positive values for any odd or even number of ξ. But, in the case of $\beta > ca^{\frac{d-1}{\xi}}$, the energy density is positive only for even values of ξ. On the other hand, for $\beta = ca^{\frac{d-1}{\xi}}$, we get $\rho_{pg} \to \infty$ and the dD polytropic energy density has a finite-time type-III[65] singularity at $a_s = \left[\frac{\beta}{c}\right]^{\frac{\xi}{d-1}}$. Thus, we see that the type-III singularity takes place at past ($a_s < 1$) for $\frac{\beta}{c} < 1$, at the present time ($a_s = 1$) for $\frac{\beta}{c} = 1$, and at future ($a_s > 1$) in the case of $\frac{\beta}{c} > 1$. Moreover, it is generally accepted that the cosmological and current total energy densities are connected to each other[43], i.e. $\rho_{pre} = 1.3\rho_{cos}$. Consequently, the integration constant c can be written in terms of a_0 which denotes the present value of $a(t)$. In the entire study, we take $a_0 = 1$ for simplicity. So, it follows that $c = \sqrt[-\xi]{1.3} + \beta$.

Taking time derivative of the relation (20) yields

$$\dot{\rho}_{pg} = c(1-d)Ha^{\frac{d-1}{\xi}}\rho_{pg}^{1+\frac{1}{\xi}}. \tag{21}$$

Therefore, making use of this result in the conservation equation (13) together with the EoS $p_{pg} = \omega_{pg}\rho_{pg}$ leads to the following expression

$$\omega_{pg} = -1 - \frac{c}{\beta a^{\frac{1-d}{\xi}} - c}. \tag{22}$$

We find that the dD PG proposal mimics the cosmological constant or incompressible fluid ($\omega_{pg} \to -1$) at early time phase ($a \to 0$) and also it can cross the phantom line when $\frac{\beta}{c} > a^{\frac{d-1}{\xi}}$. Additionally, one can see that, for the 4D framework ($d = 4$), our result (22) can be reduced to that one obtained previously by Malekjani[66]. Besides, using the conservation equation (13) together with the fractional densities (14), we can write

$$\rho_{pg} = \rho_{pg}^0 a^{(1-d)(1+\omega_{pg})}, \tag{23}$$

where ρ_{pg}^0 indicates a constant describing the present value of the PG energy density.

III. FITTING THE MODEL PARAMETERS

As we mentioned before, one can fix the free parameters β and ξ according to the recent cosmographic experiments by rewriting the theoretical expressions in terms of the red shift parameter. Note that the red shift parameter z is related to $a(t)$ by $z = \frac{1}{a} - 1$. Inserting equations (19) and (23) in the Friedmann equation (9) and using the dimensionless fractional densities given in equation (14), we find that

$$E(z) = \left[\Omega_m^0(1+z)^{d-1} + \Omega_{pg}^0(1+z)^{(d-1)[1+\omega_{pg}(z)]}\right]^{\frac{1}{2}} \tag{24}$$

where

$$\omega_{pg}(z) = -1 - \frac{c}{\beta(1+z)^{\frac{d-1}{\xi}} - c}. \tag{25}$$

Here, present day ($z = 0$) non-relativistic matter and PG energy densities are denoted by Ω_m^0 and Ω_{pg}^0, respectively, which satisfy the relation $\Omega_m^0 + \Omega_{pg}^0 = 1$.

In further steps of this section, we focus on the data from the SN Ia[67], OHD and the Planck-telescope[6] results in order to fix the free parameters of the PG model.

A. Constraints from SN Ia observations

The SN Ia observations include information about the luminosity distance. The Hubble-free definition of the luminosity parameter is written as

$$d_L = (1+z)\int_0^z \frac{dz'}{E(z')}. \tag{26}$$

Subsequently, for the SN Ia dataset, χ^2 function is given by[68]

$$\chi^2_{SN} = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i)]^2}{\sigma_i^2}, \tag{27}$$

where the theoretical distance modulus is written as

$$\mu_{theo} = 5\log_{10} d_L(z_i) + \mu_0, \tag{28}$$

with

$$\mu_0 = 42.38 - 5\log_{10} h. \tag{29}$$

Here, $\mu_{obs}(z_i)$ and σ_i denote the observed distance modulus and the uncertainty in the distance modulus, respectively. For the minimization of χ^2_{SN} with respect to the set of 580 data points obtained in the SN Ia measurements[67], we have[69]

$$\tilde{\chi}^2_{SN} = P - \frac{Q^2}{R}, \tag{30}$$

where

$$P = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i; \mu_0 = 0)]^2}{\sigma_i^2}, \tag{31}$$

$$Q = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i; \mu_0 = 0)]}{\sigma_i^2}, \tag{32}$$

$$R = \sum_i^{580} \frac{1}{\sigma_i^2}. \tag{33}$$

The best-fitting values of Ω^0_m, Ω^0_{pg}, β, ξ and χ^2_{min} describing the minimum value of χ^2_{SN} are given in TABLE I. It is important to mention here that the best value of Ω^0_{pg} is consistent with the recent observations. According to the Planck-satellite results[6].

TABLE I: χ^2_{min} value and the best-fitting values of the free model parameters obtained by using SN Ia data in the 1σ confidence region.

Dimension	χ^2_{min}	Ω^0_m	Ω^0_{pg}	β	ξ
$d=4$	570.449	0.04	0.96	-0.64	-5.3
$d=5$	580.845	0.06	0.94	-0.50	2.1

B. Constraints from OHD and Planck-results

Considering the most recent OHD[70–83] given in TABLE II (here, DGA and RBAO means the Differential Galactic Age and the Radial BAO, respectively), we further investigate the validity of the constraints on the free parameters given in the dD Polytropic gas proposal. Using equation (24), we can write

$$H(z) = H_0\sqrt{\Omega^0_m(1+z)^{d-1} + \Omega^0_{pg}(1+z)^{(d-1)[1+\omega_{pg}(z)]}}. \tag{34}$$

TABLE II: The recent observable $H(z)$ dataset[70].

z	$H_{obs}(z)$	σ	Method, Ref.	z	$H_{obs}(z)$	σ	Method, Ref.
0.0708	69.00	$\mp$19.68	DGA, [71]	0.4800	97.00	$\mp$62.00	DGA, [78]
0.1200	68.60	$\mp$26.20	DGA, [71]	0.5700	92.40	$\mp$4.500	RBAO, [79]
0.1700	83.00	$\mp$8.000	DGA, [72]	0.5930	104.0	$\mp$13.00	DGA, [73]
0.1990	75.00	$\mp$5.000	DGA, [73]	0.6800	92.00	$\mp$8.000	DGA, [73]
0.2400	79.69	$\mp$2.650	RBAO, [74]	0.7300	97.30	$\mp$7.000	RBAO, [80]
0.2800	88.80	$\mp$36.60	DGA, [71]	0.7810	105.0	$\mp$12.00	DGA, [73]
0.3500	84.40	$\mp$7.000	RBAO, [75]	0.8750	125.0	$\mp$17.50	DGA, [73]
0.3802	83.00	$\mp$13.50	DGA, [73]	0.9000	117.0	$\mp$23.00	DGA, [72]
0.4000	95.00	$\mp$17.00	DGA, [72]	1.3000	168.0	$\mp$17.00	DGA, [81]
0.4247	87.10	$\mp$11.20	DGA, [76]	1.4300	177.0	$\mp$18.00	DGA, [81]
0.4300	86.45	$\mp$3.680	RBAO, [74]	1.5300	140.0	$\mp$14.00	DGA, [72]
0.4497	92.80	$\mp$12.90	DGA, [76]	1.7500	202.0	$\mp$40.00	DGA, [82]
0.4783	80.90	$\mp$9.000	DGA, [77]	1.9650	186.5	$\mp$50.40	DGA, [73]
				2.3400	222.0	$\mp$7.000	RBAO, [83]

FIG. 1 displays the evolutionary nature of the $H(z)$ function in the 1σ confidence region where the dots indicate the recent OHD.

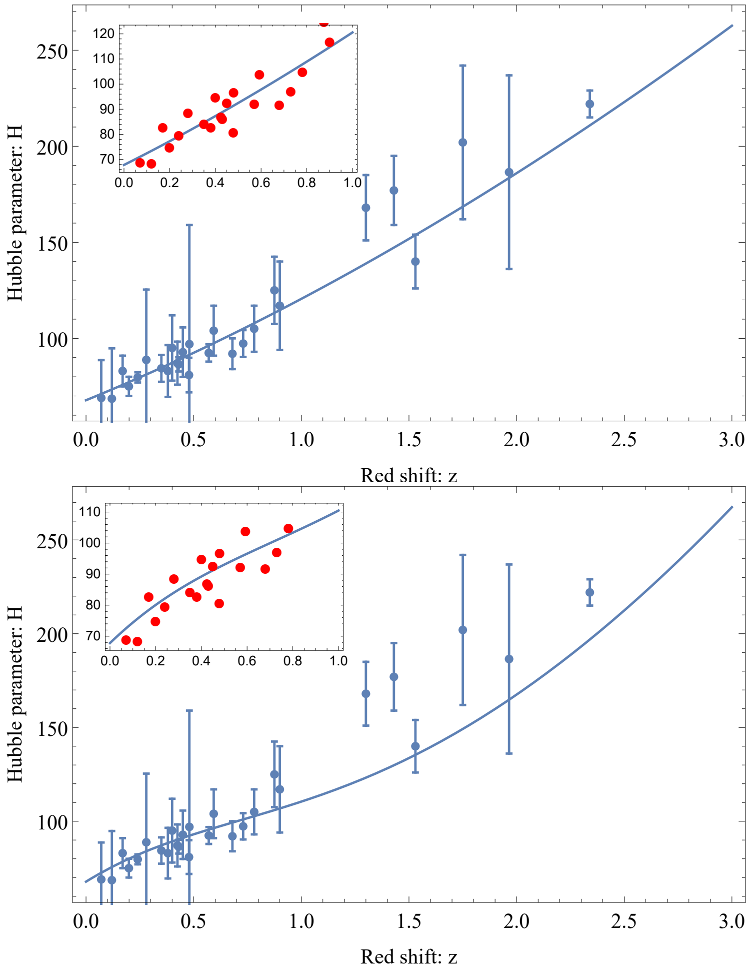

FIG. 1: $H \sim z$ relation with auxiliary parameters given in TABLE II for $d = 4$ (the upper one) and $d = 5$ (the bottom one) cases in the 1σ confidence region.

While performing this analysis, we find the best-fit values of the parameters Ω^0_m, Ω^0_{pg}, β and ξ (see TABLE III).

TABLE III: Best values of the auxiliary parameters obtained with the help of the OHD and the Planck-results.

Dimension	Ω^0_m	Ω^0_{pg}	β	ξ
$d = 4$	0.036	0.964	-0.60	-5.35
$d = 5$	0.055	0.945	-0.45	2.30

C. Age of the universe

Since we can write $H(z) = -\frac{\dot{z}}{1+z}$, one can express the age of the universe as

$$t_0 = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)E(z)}. \tag{35}$$

Remember that the dD PG model can mimic the cosmological constant at early time, i.e. $\lim_{a\to 0} \omega_{pg} = -1$. So, one can consider the ΛCDM parametrization of the dD PG proposal and write

$$t_0 = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)\left[\Omega_m^0 (1+z)^{d-1} + \Omega_{pg}^0\right]^{\frac{1}{2}}}. \tag{36}$$

Now, let's define $\Omega_m^0 (1+z)^{d-1} = \Omega_{pg}^0 \sinh^2\theta$. Consequently, the integral (36) can be transformed into the following form

$$t_0 = \frac{1}{(d-1)H_0\sqrt{\Omega_{pg}^0}} \int_{\theta_0}^\infty \frac{d\theta}{\sinh\theta}, \tag{37}$$

where

$$\theta_0 = \ln\left\{\frac{1}{\Omega_{pg}^0} + \sqrt{\frac{1-\Omega_{pg}^0}{\Omega_{pg}^0}}\right\}. \tag{38}$$

Next, after solving the integral given in equation (37), we reach at the conclusion

$$t_0 = \frac{1}{(d-1)H_0\sqrt{\Omega_{pg}^0}} \ln\left[\frac{1+\sqrt{\Omega_{pg}^0}}{1-\sqrt{\Omega_{pg}^0}}\right]. \tag{39}$$

In the limit $\Omega_{pg}^0 \to 0$, we get $t_0 = \frac{2}{(d-1)H_0}$. Moreover, taking $d = 4$ gives the same result as obtained previously for the Einstein-de Sitter model by Farooq[84]. In FIG. 2, we plot t_0 in terms of $\frac{1}{H_0}$ versus Ω_m^0 and Ω_{pg}^0, respectively, in the 5D spacetime model. We consider only the cosmological constant limit of the model and find that the cosmic age of the universe increases by incorporating the dark constituents. The present total age of the universe for the 5D polytropic scenario is the same (cosmic concordance) as estimated by the flat coasting k-matter (one-component liquid with $w = -\frac{1}{3}$) proposal, i.e. $t_0 = H_0^{-1}$[85].

It can be seen that the units of the Hubble parameter include inverse time. Thus, the inverse of H_0 has come to be known as the Hubble time[86]:

$$t_H = \frac{1}{H_0} = \frac{9.78}{h} Gyr. \tag{40}$$

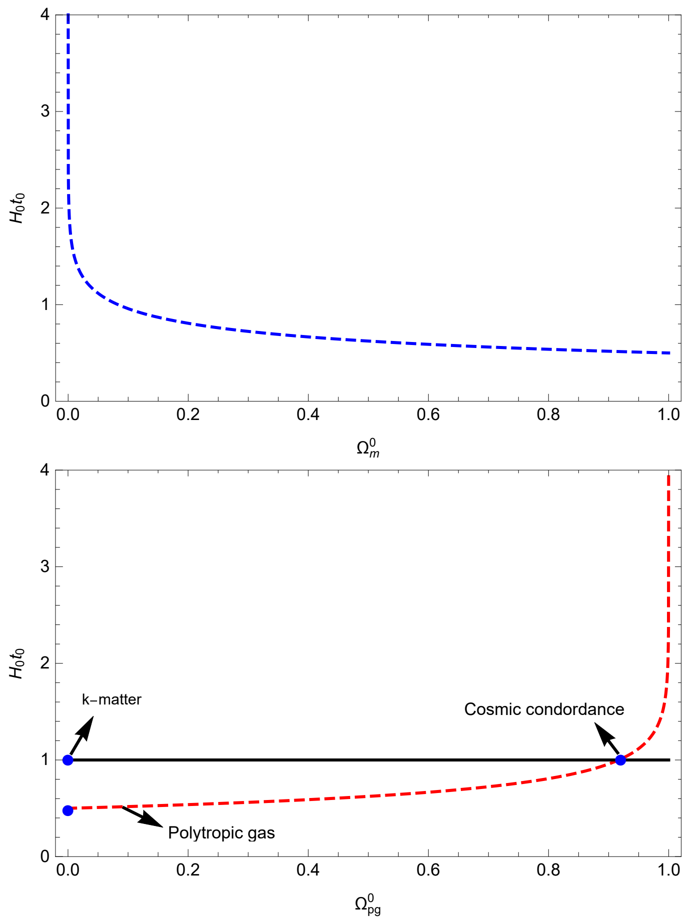

FIG. 2: The cosmic age t_0 in terms of $\frac{1}{H_0}$ versus Ω_m^0 and Ω_{pg}^0 in the 5D framework.

So, it can be written that

$$t_0 = \frac{t_H}{(d-1)} \ln \left[\frac{1 - \sqrt{\Omega_{pg}^0}}{1 + \sqrt{\Omega_{pg}^0}} \right]^{\sqrt{\Omega_{pg}^0}} . \tag{41}$$

Now, using equation (36) with the best-fitting values given in TABLE I and TABLE III and considering the present-day value of the then-favored dimensionless Hubble parameter $h = 0.678 \pm 0.009$[6], we can calculate the age of the universe in different dimensional frameworks. In TABLE IV, we present the theoretical values we have.

TABLE IV: Age of the cosmos according to the best values of the free parameters.

	t_0^{4D} (Gyr)	t_0^{5D} (Gyr)
SN Ia	22.510$\mp$0.3	15.585$\mp$0.205
OHD+Planck-2015	22.665$\mp$0.295	13.14$\mp$0.18

In literature, Carretta et al.[87] calculated that the value of the age of a globular cluster in the Milky Way galaxy is approximately 9.1 Gyr $< t_0 <$ 14.9 Gyr, where as Jimenez et al.[88] constrained the value of 11.5 Gyr $< t_0 <$ 15.5 Gyr. Making use of the white dwarf cooling sequence method, Hansen et al.[89] estimated the age of the globular cluster Messier 4 is approximately 12.0 Gyr $< t_0 <$ 13.4 Gyr. Furthermore, the Wilkinson Microwave Anisotropy Probe (WMAP) observations' estimation[3] on the cosmic age is $t_0 = 13.73 \mp 0.12$ Gyr. Additionally, according to the updated Planck-telescope results[6], the age of the universe is approximately $t_0 = 13.813 \mp 0.038$ Gyr. Although, we have taken into account a specific limit of the PG model with an EoS parameter $\omega = -1$, we get meaningful conclusions in the 5D framework which agrees with the recent observational results. Also, it can be concluded that the ages of the globular clusters are larger than 9 Gyr. FIG. 2 indicates that the cosmic age is predicted by equation (41) as a function of Ω_m^0 and Ω_{pg}^0. Focusing on a vice versa computation way, we find that if t_0 must be larger than 9 Gyr then $\Omega_{pg}^0 > 0.922$.

IV. OTHER COSMOLOGICAL FEATURES

A. EoS parameter

Considering the analysis performed for the 5D spacetime in the previous section and using equations (25), we depict $\omega_{pg}(z)$ relation in FIG. 3. It is seen that the 5D Polytropic gas model cannot cross the phantom line and it behaves like the quintessence DE.

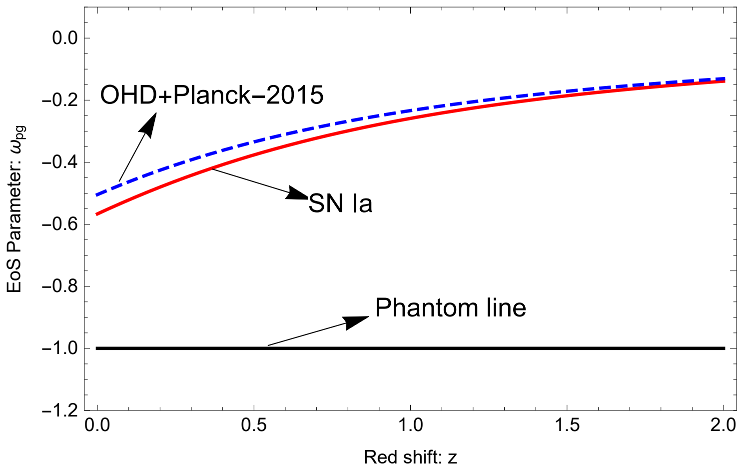

FIG. 3: Graphical analyzes $\omega_{pg}(z)$ for the case $d = 5$.

B. Deceleration parameter

In order to discuss the dD form of the PG model from a cosmological perspective, we can also analyze the deceleration parameter given by

$$q = -1 + \frac{d\ln H(z)}{d\ln(1+z)} = \frac{1}{2} + \frac{3\omega}{2}. \tag{42}$$

Using the result (22) with the above relation yields

$$q = -1 - \frac{3}{2}\frac{c}{\beta(1+z)^{\frac{4}{\xi}} - c}. \tag{43}$$

So, we must have

$$(1+z)^{\frac{4}{\xi}} > \frac{1}{\beta}\left[\frac{3}{2c} + c\right]. \tag{44}$$

Otherwise, we have positive values of q parameter describing a decelerating spacetime. FIG. 4 gives the cosmic evolution of q. It can be concluded that the parameter q in the best-fitted framework is quite different from that for the ΛCDM scenario. Its current value is higher than $q_0^{\Lambda CDM} = -0.55$[38]:

$$(q_0^{SNIa}, q_0^{OHD\&Planck}) = (-0.3498, -0.2566). \tag{45}$$

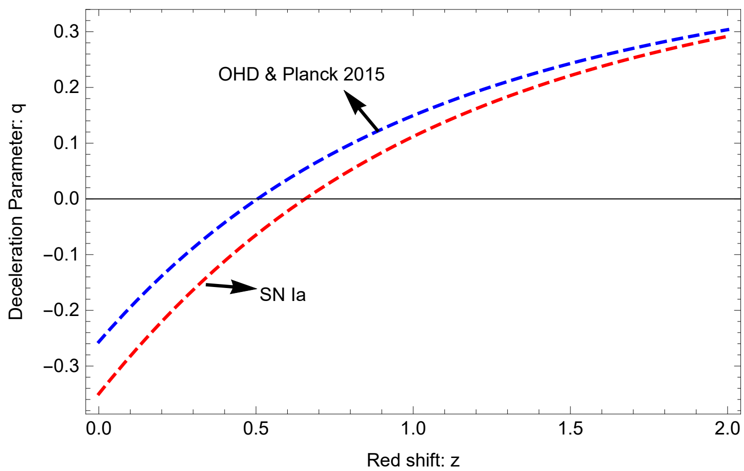

FIG. 4: Graphical analyzes of the $q(z)$ function for the case $d = 5$.

C. Statefinders

The cosmological EoS parameter of some geometrical proposals calculated by using EoM does no longer take a requisite place. From this point of view, a different method is essential to discriminate a theoretical description among various models. Sahni et. al[90] introduced the set of statefinder parameters (r, s) which has qualitatively significant properties[70, 91–95]. The pair is basically given by the following relations

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{46}$$

Moreover, one can rewrite the definition of r in a more convenient form as given below

$$r = q + 2q^2 + (1+z)q'. \tag{47}$$

Making use of calculations given in the previous sections, we plot the statefinders-plane for the 5D PG model in FIG. 6 according to best-fitting values given in TABLE I and TABLE III. It can be seen that (r, s) pair starts from the ΛCDM fixed point $(r, s) \equiv (1, 0)$.

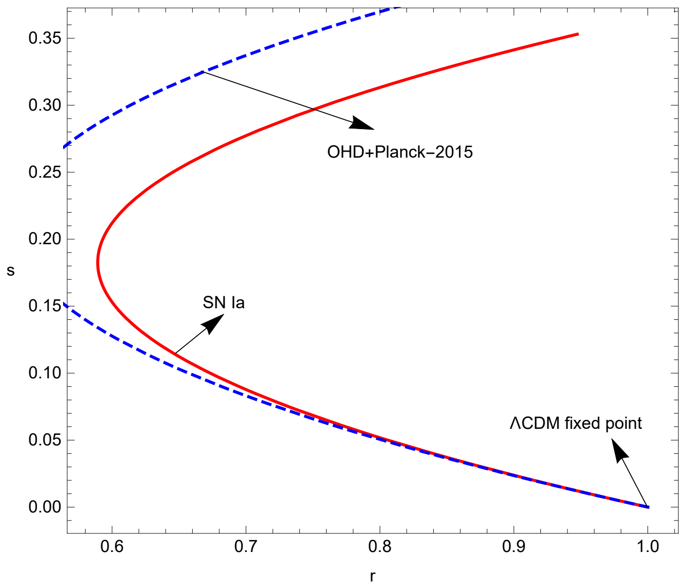

FIG. 5: Numerical analysis of the statefinders pair for the case $d = 5$.

D. Energy conditions

The energy conditions based on quite general physical principles may help to discuss dynamic proposal-independent constraints on the kinematics of the cosmos[102–104] by imposing restrictions on ρ and p. We can transform the corresponding energy conditions into inequalities, which are restricting the possible values of energy density and pressure of a medium. In terms of energy density and pressure, the energy conditions are written in the following forms[105]

- Null energy condition (NEC): $\rho + p \geq 0$,
- Weak energy condition (WEC): $\rho \geq 0$, $\rho + p \geq 0$,
- Strong energy condition (SEC): $\rho + p \geq 0$, $\rho + 3p \geq 0$,
- Dominant energy condition (DEC): $\rho \geq 0$, $\rho + p \geq 0$, $\rho - p \geq 0$.

Because of the above inequalities do not include any definite EoS for the content of the cosmos, energy conditions imply proposal-independent constraints on theoretical energy density and pressure expressions. Subsequently, on the basis of quite general principles, the so-called energy conditions yield a significant opportunity to discuss the evolution of our cosmos. Here, we check whether the PG scenario, representing the dominant substances of the universe, satisfies the constraints by DEC $\rho_{pg} \geq 0$, $\rho_{pg} + p_{pg} \geq 0$ and $\rho_{pg} - p_{pg} \geq 0$. In FIG. 6, we plot the DEC for the fitted PG model and see that it satisfies those energy conditions.

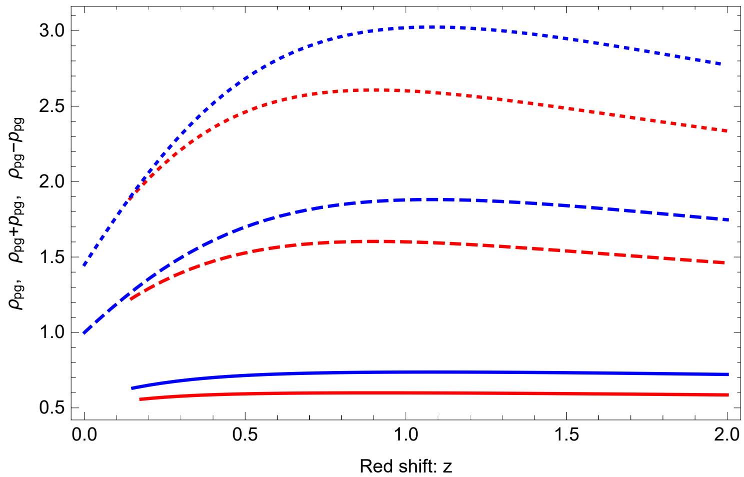

FIG. 6: The DEC analysis of the PG unified dark matter/energy definition according to the best-fitting values obtained by using the SN Ia observations, OHD and the Planck-satellite measurements. Here, the red curves are plotted by using the SN Ia data while the blue curves are depicted by considering the recent OHD and Planck-results. Additionally, the dashed, dotted and solid lines represent ρ_{pg}, $\rho_{pg} - p_{pg}$ and $\rho_{pg} + p_{pg}$, respectively. Next, we assume also that $\rho^0_{pg} = 1$ for the sake of simplicity.

V. CONCLUDING REMARKS

We have mainly investigated the dynamics of the PG model describing a unified dark matter-energy scenario from extra dimensional perspective and concluded that this theoretical candidate of the dark universe may have the ability to explain the speedy expansion phenomenon of the cosmos. While investigating the polytropic cosmology, we have performed the following process. In the first stage, we have obtained an expression describing the energy density in terms of $a(t)$ and then calculated an analytical relation for the cosmic Hubble parameter. Next, making use of those calculations, we have studied some physical features of the model in order to obtain some constraints for the auxiliary parameters given in the proposal. In TABLE V, we have provided theoretical $H(z)$ values, obtained by making use of the SN Ia dataset, OHD and Planck-results.

TABLE V: Comparing sets of $H(z)$ data.

z	$H_{obs}(z)$	H_{th}^{SNIa}	H_{th}^{OHD}	z	$H_{obs}(z)$	H_{th}^{SNIa}	H_{th}^{OHD}
0.0708	69.00±19.68[71]	72.01±0.96	72.60±0.96	0.480	97.00±62.00[78]	88.78±1.18	92.71±1.23
0.1200	68.60±26.20[71]	74.64±0.99	75.70±1.01	0.570	92.40±4.500[79]	91.63±1.22	96.07±1.28
0.1700	83.00±8.000[72]	77.08±1.02	78.55±1.04	0.593	104.0±13.00[73]	92.35±1.23	96.90±1.29
0.1990	75.00±5.000[73]	78.40±1.04	80.13±1.06	0.680	92.00±8.000[73]	95.10±1.26	100.02±1.33
0.2400	79.69±2.650[74]	80.16±1.06	82.25±1.09	0.730	97.30±7.000[80]	96.70±1.28	101.8±1.35
0.2800	88.80±36.60[71]	81.78±1.09	84.21±1.12	0.781	105.0±12.00[73]	98.40±1.30	103.6±1.37
0.3500	84.40±7.000[75]	84.40±1.12	87.41±1.16	0.875	125.0±17.50[73]	101.70±1.35	107.07±1.42
0.3802	83.00±13.50[73]	85.50±1.13	88.70±1.18	0.900	117.0±23.00[72]	102.58±1.36	108.02±1.43
0.4000	95.00±17.00[72]	86.14±1.14	89.53±1.19	1.300	168.0±17.00[81]	120.40±1.59	125.66±1.67
0.4247	87.1011.20±[76]	86.97±1.15	90.54±1.20	1.430	177.0±18.00[81]	127.75±1.69	132.73±1.76
0.4300	86.45±3.680[74]	87.15±1.16	90.75±1.21	1.530	140.0±14.00[72]	133.9±1.77	138.68±1.84
0.4497	92.80±12.90[76]	87.80±1.17	91.53±1.22	1.750	202.0±40.00[82]	149.19±1.98	153.41±2.04
0.4783	80.90±9.000[77]	88.73±1.18	92.65±1.23	1.965	186.5±50.40[73]	166.29±2.21	169.9±2.26
				2.340	222.0±7.000[83]	201.1±2.67	203.9±2.71

In order to check the reliability of our results, we can define a control parameter given by

$$\zeta = \frac{H_{obs}(z_i) - H_{th}(z_i)}{\sigma_i^{obs}}. \tag{48}$$

In FIG. 7, we have depicted the control parameter according to the different values of auxiliary parameters found by focusing on the most recent astrophysical observations. It can be seen that most of the corresponding values are in the 1σ confidence region.

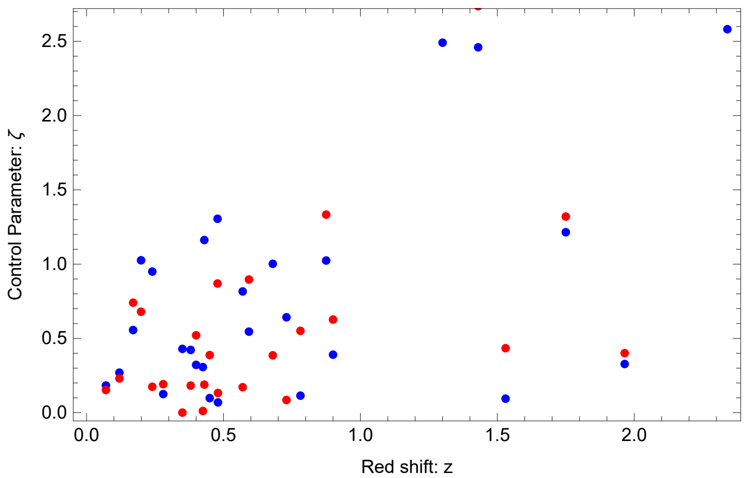

FIG. 7: The evolution of control parameter ζ. Here, the red dots indicates the SN Ia case while the blue ones denotes the case including OHD and Planck 2015 results.

In summary, the PG model is consistent with the recent observational measurements and its 5D form yields better results than the 4D one.

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