

On the maximum \mathcal{X} entropy negation of a complex-valued distribution

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Abstract—In this paper, we propose a generalized model of the negation function, so that it can have more powerful capability to represent the knowledge and uncertainty measure. In particular, we first define a vector representation of complex-valued distribution. Then, an entropy measure is proposed for the complex-valued distribution, called \mathcal{X} entropy. After that, a transformation function to acquire the negation of the complex-valued distribution is exploited. Finally, we verify that the proposed negation method has a maximal entropy.

Index Terms—Knowledge representation, Uncertain information, Negation function, Complex-valued distribution, \mathcal{X} entropy, Uncertainty measure, Decision-making.

I. INTRODUCTION

In this paper, we propose a generalized model of the negation function, so that it can have more powerful capability to represent the knowledge and uncertainty measure. In particular, we first define a vector representation of complex-valued distribution. Then, an entropy measure is proposed for the complex-valued distribution, called \mathcal{X} entropy. After that, a transformation function to acquire the negation of the complex-valued distribution is exploited on the basis of the newly defined \mathcal{X} entropy. Finally, we verify that the proposed negation method has a maximal entropy.

II. VECTOR REPRESENTATION OF COMPLEX-VALUED DISTRIBUTION

Definition 1 (*Complex-valued distribution vector*)

Let \mathbb{C}_k be a complex-valued distribution (CvD) vector on the space $\Psi = \{\psi_1, \psi_2, \dots, \psi_j, \dots, \psi_n\}$, denoted by

$$\mathbb{C}_k = [c_{k1}, c_{k2}, \dots, c_{kj}, \dots, c_{kn}], \quad (1)$$

where c_{kj} represents the complex value of the occurrence of ψ_j , denoted by

$$c_{kj} = x_{kj} + y_{kj}i, \quad (2)$$

where x_{kj} and y_{kj} are real numbers and i is the imaginary unit, satisfying $i^2 = -1$.

In Eq. (2), for c_{kj} , it satisfies the conditions:

$$\begin{aligned} x_{kj} &\geq 0, \\ \sqrt{x_{kj}^2 + y_{kj}^2} &\in [0, 1], \\ \sum_{j=1}^n c_{kj} &= 1. \end{aligned} \quad (3)$$

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Definition 2 (*The inner product between CvDs*)

Let \mathbb{C}_k and \mathbb{C}_h be two complex-valued distribution vectors on the space Ψ . The inner or dot product of \mathbb{C}_k and \mathbb{C}_h is defined by

$$\langle \mathbb{C}_k, \mathbb{C}_h \rangle = \mathbb{C}_k \cdot \mathbb{C}_h = \sum_{j=1}^n c_{kj} \bar{c}_{hj}. \quad (4)$$

Then, the norm of the complex-valued distribution vector is defined as

$$\|\mathbb{C}_k\| = \sqrt{\langle \mathbb{C}_k, \mathbb{C}_k \rangle}. \quad (5)$$

III. ENTROPY OF THE COMPLEX-VALUED DISTRIBUTION

Definition 3 (*Entropy for CvD*)

Let \mathbb{C}_k be a complex-valued distribution vector on the frame of discernment Ψ . The entropy of \mathbb{C}_k , denoted as $\mathcal{X}(\mathbb{C}_k)$ is defined as

$$\mathcal{X}(\mathbb{C}_k) = 1 - \frac{\|\mathbb{C}_k\|^2}{3} = 1 - \frac{\sum_{j=1}^n |c_{kj}|^2}{3}, \quad (6)$$

where

$$0 \leq \mathcal{X}(\mathbb{C}_k) \leq 1 - \frac{1}{3n}. \quad (7)$$

IV. NEGATION OF A COMPLEX-VALUED DISTRIBUTION

Definition 4 (*The negation of the CvD*)

The negation of the complex-valued distribution is defined by

$$\bar{\mathbb{C}}_k = [\bar{c}_{k1}, \dots, \bar{c}_{kj}, \dots, \bar{c}_{kn}], \quad (8)$$

where

$$\bar{c}_{kj} = \frac{1 - c_{kj}}{n - 1}, \quad (9)$$

which satisfies:

$$\sum_{j=1}^n \bar{c}_{kj} = \frac{1}{n-1} \sum_{j=1}^n (1 - c_{kj}) = 1. \quad (10)$$

When \mathbb{C}_k turns into a probability distribution, the proposed negation method degrades into Yager's [1] negation method.

V. ON THE VIEW FROM THE ENTROPY

Example 1 Assume there is a CvD $\mathbb{C} = [0.7 - 0.2i, 0.1 + 0.1i, 0.2 + 0.1i]$.

Fig. 1(a) shows the variation of \bar{c}_j^i with i times negation iteration. Meanwhile, Fig. 1(b) shows the entropy measure of $\bar{\mathbb{C}}^i$. These results verify that the proposed negation method has a maximal entropy.

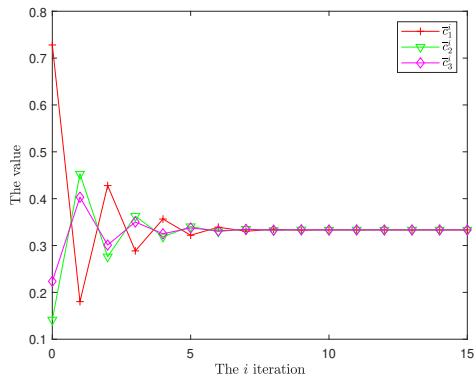
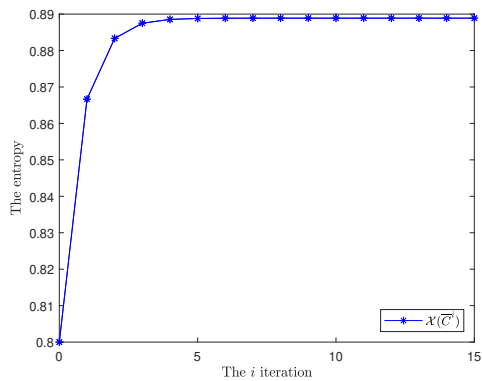
(a) The variation of $|\bar{c}_j^i|$.(b) The variation of $\mathcal{X}(\bar{C}^i)$.

Fig. 1. The measure of entropy in Example 1.

VI. CONCLUSIONS

In this paper, a generalized model of negation function was proposed for complex-valued distributions. In particular, when the complex-valued distributions turn into probability distributions, the proposed negation method degraded into the traditional one.

REFERENCES

- [1] R. R. Yager, "On the maximum entropy negation of a probability distribution," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1899–1902, 2014.