Time dilation in classical physics

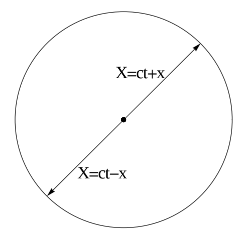
Abstract

'Time dilation' appears to be an experimental fact. We show that if Einstein had not invented the multiple time dimensions and multiple ethers of relativity, time dilation could have been derived from absolute time (defined as universal simultaneity) and the constant speed of light in absolute space.

Prior to 1905 physics was formulated with absolute time, defined as universal simultaneity; essentially it is *NOW* throughout the universe, independently of position, or velocity. If the speed of light is constant, then $x = ct$ where x is the distance traveled by light during time t. Obviously $x' = ct'$ where t' is a time measurement differing from time t. Time and space are independent of each other; it is the same time everywhere in space and locations in space do not depend on time. Therefore we choose Hestenes' *geometric algebra* and represent the two basic entities as the four-vector

$$X = ct + \vec{x} \tag{1}$$

where ct is a scalar distance and $\vec{x}$ is a three-position vector. The location of either origin $(t = 0, \vec{x} = 0)$ is irrelevant. In the figure at right we invert the $x = ct$ ray in the origin to obtain $-\vec{x} = ct$ which we can represent as the conjugate four-vector $\tilde{X} = ct - \vec{x}$. These entities support the relations:



$$\vec{x} = \frac{X - \tilde{X}}{2}, \qquad t = \frac{X + \tilde{X}}{2c}, \qquad X\tilde{X} = c^2t^2 - \vec{x}^2 = 0, \tag{2}$$

The product $X\tilde{X}$ is invariant and holds for all rays: $X\tilde{X} = 0 = X'\tilde{X}'$. Invariants are key physics and engineering entities: without some unchanging relation it is generally impossible to formulate a theory of anything. The invariant relations are such that the origin of the coordinate system $(\vec{r} = 0)$ is irrelevant. We generalize to the case in which one coordinate system is in uniform motion with respect to the other, with velocity $\vec{v}$. For constant $\vec{v}$ we can replace $x = ct$ with $\vec{x} = \vec{v}t$ in invariance (2) which we can rewrite

$$(ct + \vec{v}t)(ct - \vec{v}t) = t^2(c^2 - v^2) = X\tilde{X} \quad \Rightarrow \quad \frac{d(X\tilde{X})}{dt} \to 2t(c^2 - v^2) \neq 0,$$

but this is not invariant. Can we find a relevant invariant? If $\frac{d}{dt}(X\tilde{X}) \neq 0$ can we show that

$$\frac{d}{dt}\left[X\tilde{X} - X'\tilde{X}'\right] = 0\,? \tag{3}$$

Unlike the constant speed of light, velocities are not necessarily equal; $v \neq v'$, therefore we set $v' = 0$ and perform the derivation:

$$\frac{d}{dt}\left[(c^2 - v^2)t^2 - (c^2 - v'^2)t'^2\right] = 0 \quad \Rightarrow \quad \frac{d}{dt}[(c^2 - v^2)t^2] = \frac{d}{dt}[(c^2)\tau^2] \tag{4}$$

when $\tau \equiv t'$. If the derivatives are equal, then we expect the relation between t and τ to be a function of velocity, $\gamma(v)$, such that $t = \gamma(v)\tau$. Performing the calculation we obtain:

$$2t(c^2 - v^2) = (c^2)2\tau\frac{d\tau}{dt} \quad \Rightarrow \quad \left(1 - \frac{v^2}{c^2}\right) = \frac{\tau}{t}\frac{d\tau}{dt} \quad \Rightarrow \quad \frac{1}{\gamma}\frac{d\tau}{dt} \quad \Rightarrow \quad \frac{1}{\gamma^2} \tag{5}$$

Thus

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{for constant } \; v \neq v(t). \tag{6}$$

The above focuses on math, so let us examine the relevant physics. The obvious interpretation of $t = \gamma(v)\tau$ is that $t \neq \tau$, which seems to violate our key assumption of absolute time. Yet physics is not mathematics; physics relates the universe to mathematics based on measurements, so we ask if t and τ can represent measurements of systems in uniform motion with respect each other. If so, we must examine the measuring devices, the *clocks* that measure time in the two systems in motion with respect to each other. The key question is,

does a clock in motion measure exactly the same as the identical clock at rest?

From (2) and (3) we obtain $(\Delta X)(\Delta\tilde{X}) = c^2(\Delta t)^2 - (\Delta\vec{x})^2$ and $(\Delta X')(\Delta\tilde{X}') = c^2(\Delta\tau)^2 - (\Delta\vec{x}')^2$. We then let $\Delta\vec{x} = \vec{v}\Delta t$ and $\Delta\vec{x}' = \vec{v}'\Delta\tau$. If invariance is preserved, and we set $\vec{v}' = 0$ we obtain

$$(\Delta X')(\Delta\tilde{X}') \;=\; c^2(\Delta\tau)^2 \;=\; (\Delta X)(\Delta\tilde{X}). \tag{7}$$

Divide both sides of the equation by $\Delta\tau$ twice, yielding

$$\left(\frac{\Delta X}{\Delta\tau}\right)\left(\frac{\Delta\tilde{X}}{\Delta\tau}\right) = c^2 \tag{8}$$

If $\Delta\tau \to d\tau$ then four-velocity V is the entity $(dX/d\tau)$ with conjugate $\tilde{V} = d\tilde{X}/d\tau$, hence

$$V\tilde{V} = \left(\frac{dX}{d\tau}\right)\left(\frac{d\tilde{X}}{d\tau}\right) = c^2 \tag{9}$$

As Hestenes observes [5] interestingly, *"Unlike three-velocities, the four-velocity has a constant magnitude independent of the particle history."* This is a key invariance.

Recall that $X(t) = ct + \vec{x}(t)$, $\tilde{X}(t) = ct - \vec{x}(t)$, so

$$\frac{dX}{dt} = c + \frac{d\vec{x}}{dt} = c + \vec{v}\,. \tag{10}$$

If clock time τ is used and $X(\tau) = ct(\tau) + \vec{x}(\tau)$ then

$$\frac{dX}{d\tau} = \frac{dt}{d\tau}\left(c + \frac{d\vec{x}}{dt}\right) = \gamma(c + \vec{v}) \tag{11}$$

Since we know that $t = \gamma\tau$ then $dt = \gamma\, d\tau \Rightarrow \gamma = dt/d\tau$. Let us consider the right-hand term of (11), $\gamma(c+v)$ and let us multiply these by the rest mass m_0 of a particle, $\gamma m_0(c+v)$. The four-vector has the form (*scalar* plus *vector*), with

$$\gamma\, m_0\, c = scalar \qquad \text{and} \qquad \gamma m_0 \vec{v} = vector\,.$$

Since γ is dimensionless, the vector $\gamma m_0 \vec{v}$ has units of momentum:

$$\vec{p} = m\vec{v}\,. \tag{12}$$

If this is to make sense, we define the inertial mass m to be

$$m \equiv \gamma m_0 \tag{13}$$

which implies that the inertial mass increases with velocity, yielding the rest mass at $\vec{v} = 0$. This relation has been confirmed countless times in twentieth century particle physics. Thus

$$P = m_0 V\,.$$

The scalar, $\gamma\, m_0\, c$ has units of momentum, but no obvious interpretation. If we multiply and divide by c we obtain $\gamma\, m_0\, c^2 / c$. Since $\gamma\, m_0\, c^2$ has units of energy, we define $E = \gamma\, m_0\, c^2$ as particle energy. Four-momentum $P = m_0 V = \left(\frac{E}{c} + \vec{p}\right)$ implies a conjugate $\tilde{P} = m_0 \tilde{V} = \left(\frac{E}{c} - \vec{p}\right)$ thus $P\tilde{P} = \frac{E^2}{c^2} - \vec{p}^{\,2}$ while eqn (9) shows that

$$P\tilde{P} = m_0 V m_0 \tilde{V} = {m_0}^2 V\tilde{V} = m_0^2 c^2\,, \tag{14}$$

Hence

$$\frac{E^2}{c^2} - \vec{p}^{\,2} = m_0^2 c^2\,. \tag{15}$$

This dynamical energy-momentum relation is the essence of particle physics. The rest mass is found by setting $\vec{p} = 0 \Rightarrow E(0) = m_0 c^2$. The energy expression $E(\vec{p}) = (m_0^2 c^4 + c^2 \vec{p}^{\,2})^{1/2}$ implies the classical Hamiltonian

$$H(\vec{p}) = E(\vec{p}) = (m_0^2 c^4 + c^2 \vec{p}^{\,2})^{1/2}\,. \tag{16}$$

We now have a four-momentum vector consisting of scalar E plus vector momentum $\vec{p}$, where

$$\vec{p} = m\vec{v} = (\gamma m_0)\vec{v}\,, \qquad \text{and energy} \qquad E = \gamma m_0 c^2 = m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \cong m_0 c^2 + m_0 v^2/2\,.$$

The total energy E is equal to rest mass plus kinetic energy to a first approximation. We use this relation to study moving inertial clocks. We begin with Newton's law $\vec{F} = d\vec{p}/dt$ for the clock at rest where the restoring force on the sprung mass is $\vec{F} = -k\Delta\vec{x}$. We assign an equivalent relation $\vec{F}' = d\vec{p}'/dt' = -k'x'$ to the clock in motion, where $x' = k\Delta x$ is the stretch of the spring and we assume $k' = k$. Velocity v_0 of the rest clock is zero in absolute space while the velocity of sprung mass m_0 of the rest clock is $u = dx/dt = \dot{x}$. Since the moving clock is assumed to be initially at rest, and then to have been accelerated to velocity $\vec{v}$, the velocity of the sprung mass m' of the moving clock is $v + u'$ in absolute space with $\vec{v} = d\vec{x}/dt$ and $u' = dx'/dt = \dot{x}'$. In these terms the momentum of the sprung mass of the rest clock is given by $p = m_0(v_0 + u)$ while the momentum of the sprung mass of the moving clock is $p' = m'(v + u')$ so that

$$\frac{dp}{dt} = m_0 \frac{du}{dt} = -kx = m_0 \ddot{x} \qquad\qquad \frac{dp'}{dt'} = m' \frac{d}{dt'}(v + u') = -kx'\,, \tag{17}$$

hence

$$\frac{d}{dt'} = \frac{d}{(dt/\gamma)} = \gamma \frac{d}{dt} \qquad \Rightarrow \qquad \frac{dp'}{dt'} = m' \frac{d}{dt'}(v + u') = \gamma m' \frac{du'}{dt} = -kx'\,. \tag{18}$$

Since $u' = \dfrac{dx'}{dt}$ and $m' = \gamma m_0$ we have $\dfrac{dp'}{dt'} = \gamma m' \dfrac{d^2 x'}{dt^2} = -kx' \;\Rightarrow \gamma^2 m_0 \ddot{x}' = -kx'$. Summarizing:

$$\frac{dp'}{dt'} \quad \Rightarrow \quad \frac{d^2 x'}{dt^2} + \left(\frac{\omega_0}{\gamma}\right)^2 x' = 0 \qquad \Rightarrow \qquad \ddot{x}' + \omega'^2 x' = 0 \tag{19}$$

$$\frac{dp}{dt} \quad \Rightarrow \quad \frac{d^2 x}{dt^2} + \omega_0{}^2 x = 0 \qquad \Rightarrow \qquad \ddot{x} + \omega_0^2\, x = 0 \tag{20}$$

In terms of universal time t, the equations of motion of the rest clock yields frequency ω_0, while the frequency of the moving clock yields $\omega' = \omega_0/\gamma$. This establishes time dilation for inertial clocks in relative motion in universal time and space.

If one becomes confused over x and x' in the above — the equations of motion for the sprung-mass(es) treat x and x' as the position variables for m and m' respectively — x can be replaced by x_m and x' be replaced by $x'_{m'}$ such that the relevant velocities are denoted by:

$$u = \frac{dx}{dt} \equiv \frac{dx_m}{dt} \qquad \text{and} \qquad u' = \frac{dx'}{dt} \equiv \frac{dx'_{m'}}{dt}\,,$$

and every step can be carried out using these explicit mass-based position coordinates. Of course one obtains the same final results. Note that the above two equations (19) and (20) are *formulated in terms of universal time,* t , as should be expected in a theory of absolute time and space.

Discussion of results

We began by deriving an invariance relation based on $x = ct$ for the constant speed of light in absolute space. We then attempted to generalize the invariance to handle arbitrary velocities and derived an inertial factor $\gamma(v) = (1-(v/c)^2)^{-1/2}$ such that $m = \gamma m_0$ and $dt/d\tau = \gamma$. If dt and $d\tau$ represent time measurements using two clocks in uniform relative motion we see that the inertial factor γ represents a calibration relation. We then make use of the fact that <u>all</u> clocks count oscillations to 'tell time' and the fact that all oscillators involve a linear restoring force $f = -kx$ acting on mass displaced from equilibrium condition by distance x . Based on $f = ma = dp/dt$ we observe that the acceleration for a given force f is f/m, that is, increased inertial mass m reduces the acceleration, slowing the system down. When we specifically analyze a spring-based clock, $\vec{f} = -k\vec{x}$, we derive the frequency of oscillation of the moving clock, $\omega' = \omega_0/\gamma$. Thus the moving clock 'runs slower' than the clock at rest. The increase in inertial mass is essentially the equivalent mass of the kinetic energy of the moving clock gained when the moving clock was accelerated from rest to velocity $\vec{v}$ in the absolute frame.

The key fact of interest is that the analysis of inertial clocks leads to exactly the relationship $\omega' = \omega_0/\gamma \quad \Rightarrow \quad dt/d\tau = \gamma$ that was experimentally determined in the twentieth century.

Summary of 'classical time dilation'

Although 'time dilation' has been considered 'proof' of special relativity, and particularly of 'multiple time dimensions' and 'relativity of simultaneity' – excluding absolute time and space – we find that the classical physics analysis of clocks in relative motion in absolute space and time yields "clock dilation" that perfectly matches the experimental evidence. We thus provide an alternative interpretation of time dilation that contrasts with the century-old interpretation of special relativity.