Theorem 1 If the real number α satisfies an irreducible polynomial over the field of rational numbers of degree k, and if k is not a power of 2, then α is not constructible.
Proof.
This is Corollary 2 to Theorem 5.4.1 in [1].

Theorem 2 A regular heptagon is not constructible by straightedge and compass.
Proof.
A regular heptagon is a seven-sided polygon with sides of equal length. The construction of a regular heptagon requires the constructibility of the real number $\cos\frac{2\pi}{7}$. Given the formula

$$\cos 7\theta = 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta,$$

putting $\theta = \frac{2\pi}{7}, \alpha = \cos\frac{2\pi}{7}$ and rearranging terms give

$$64\alpha^7 - 112\alpha^5 + 56\alpha^3 - 7\alpha - 1 = 0.$$

Thus α is a root of the polynomial $64x^7 - 112x^5 + 56x^3 - 7x - 1$ over the rational field. Upon factorization,

$$64x^7 - 112x^5 + 56x^3 - 7x - 1 = (x-1)(8x^3 + 4x^2 - 4x - 1)^2.$$

Since $\alpha - 1 \neq 0$,

$$8\alpha^3 + 4\alpha^2 - 4\alpha - 1 = 0.$$

Since $8(\frac{1}{2}x+1)^3 + 4(\frac{1}{2}x+1)^2 - 4(\frac{1}{2}x+1) - 1 = x^3 + 7x^2 + 14x + 7$, this polynomial is irreducible over the rationals by the Eisenstein criterion with the prime number 7. It follows that the polynomial $8x^3 + 4x^2 - 4x - 1$ is irreducible over the rational field and since its degree is 3, which is not a power of 2, α is not constructible by Theorem 1.

Reference

[1] I. N. Herstein, *Topics in Algebra*, John Wiley & Sons, New York, 1975.