Fractal for :

$$f(z)=z-\tanh^{-1}(1-z)$$

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abstract
We give a simple formula for Pi

I. Fractal for $f(z) = z - \tanh^{-1}(1-z)$

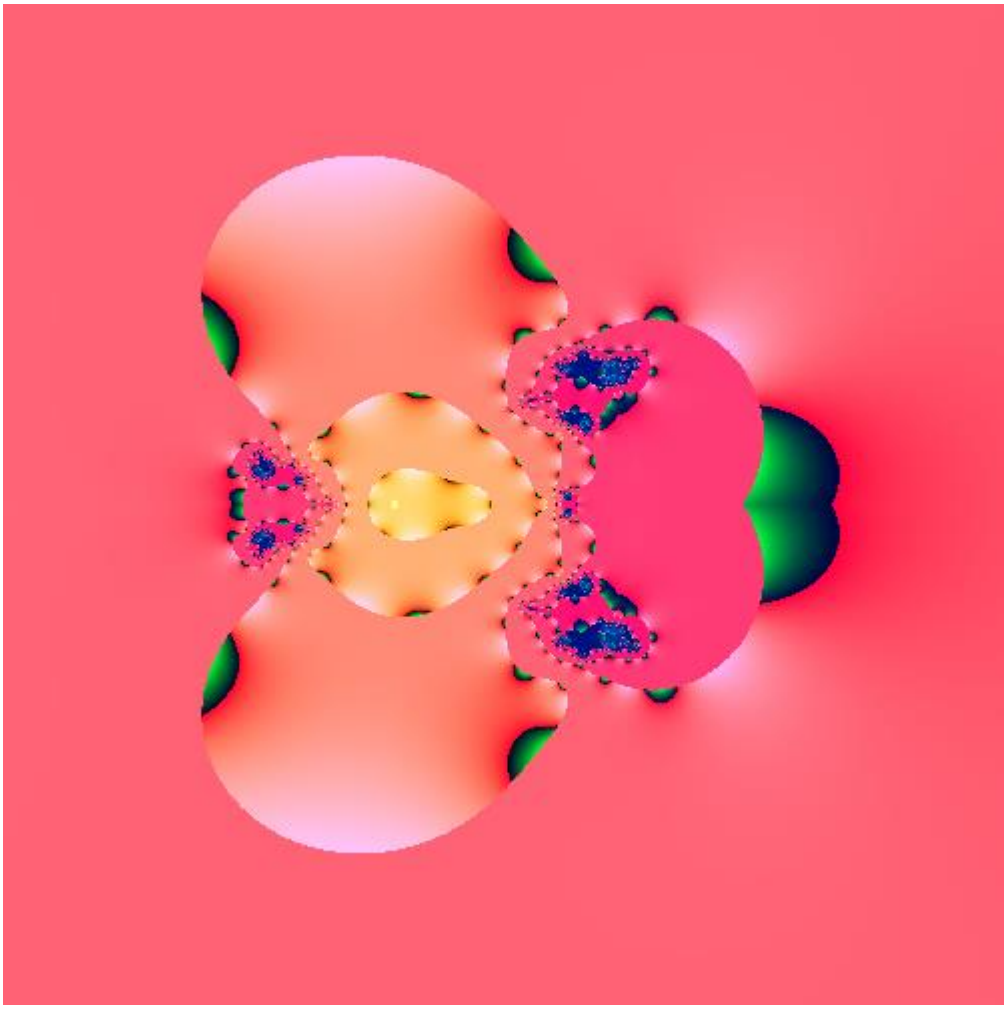

Fig. 1

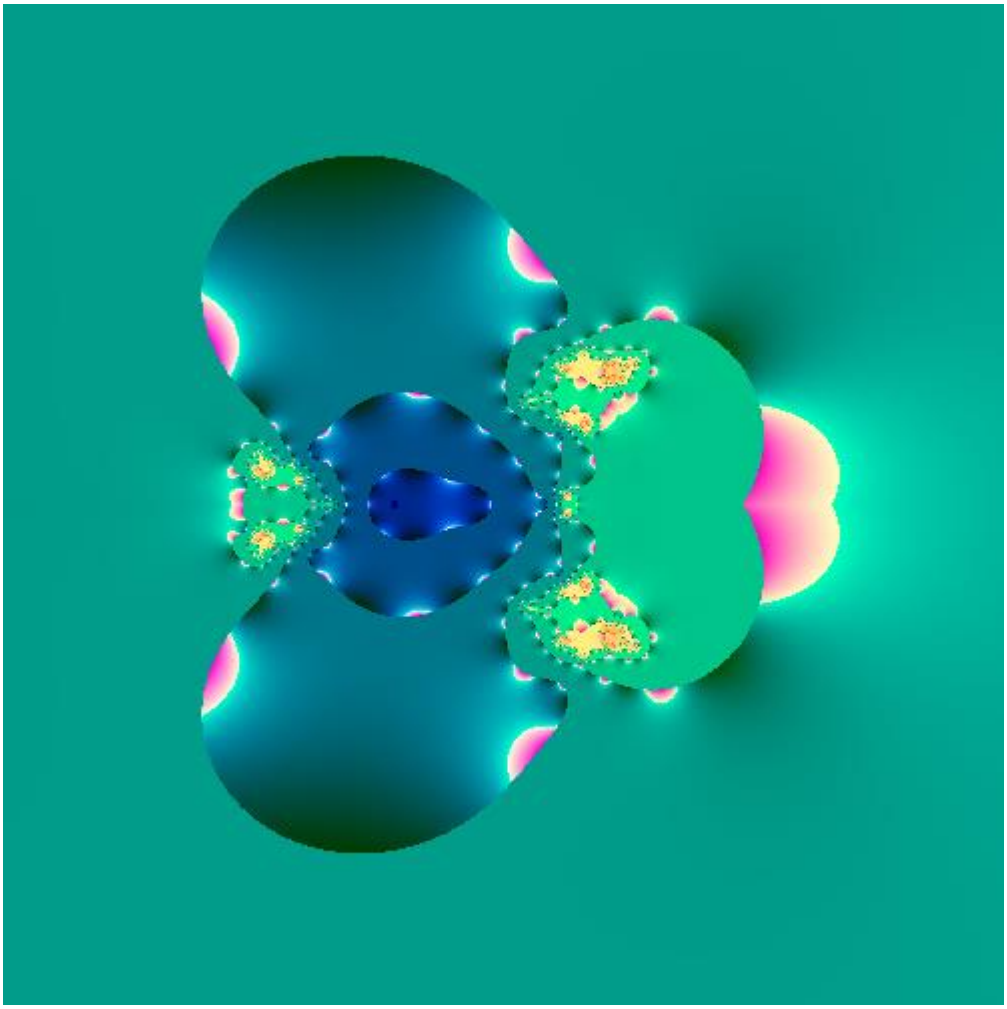

Fig. 2

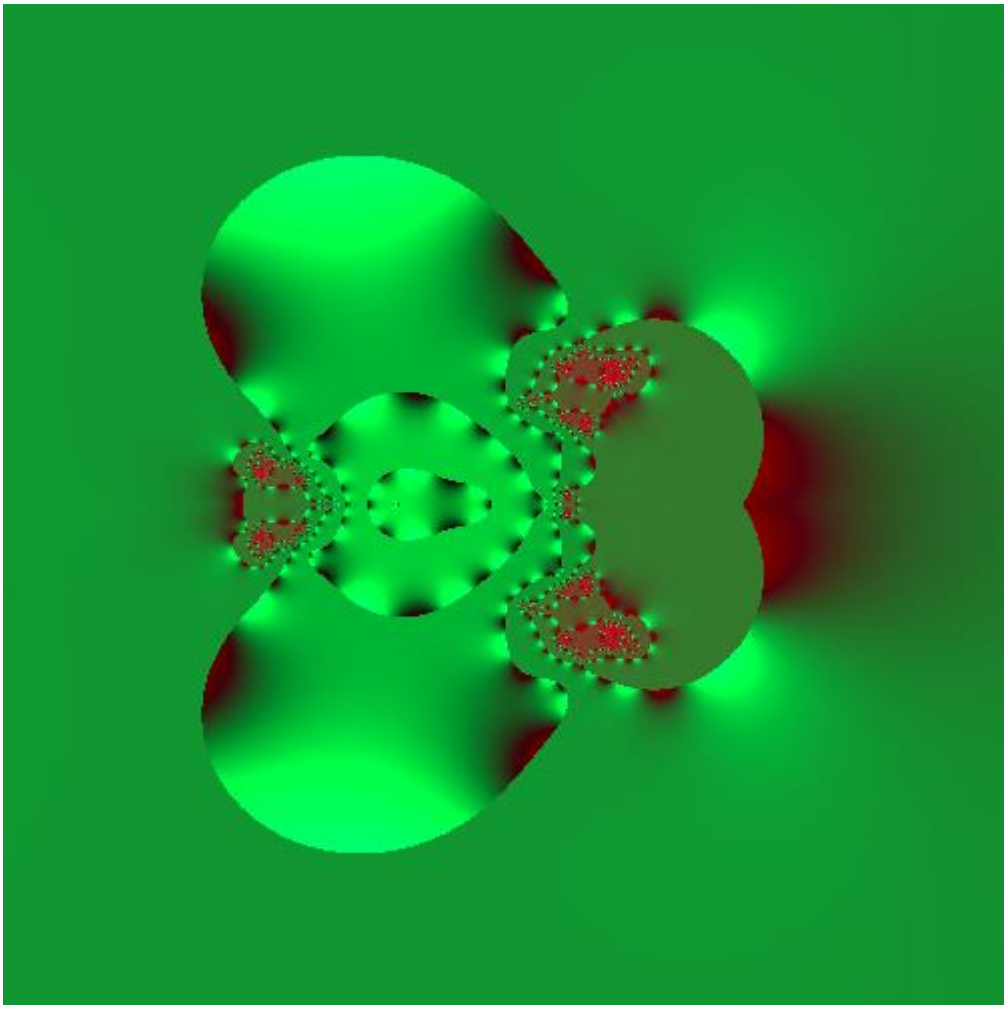

Fig. 3

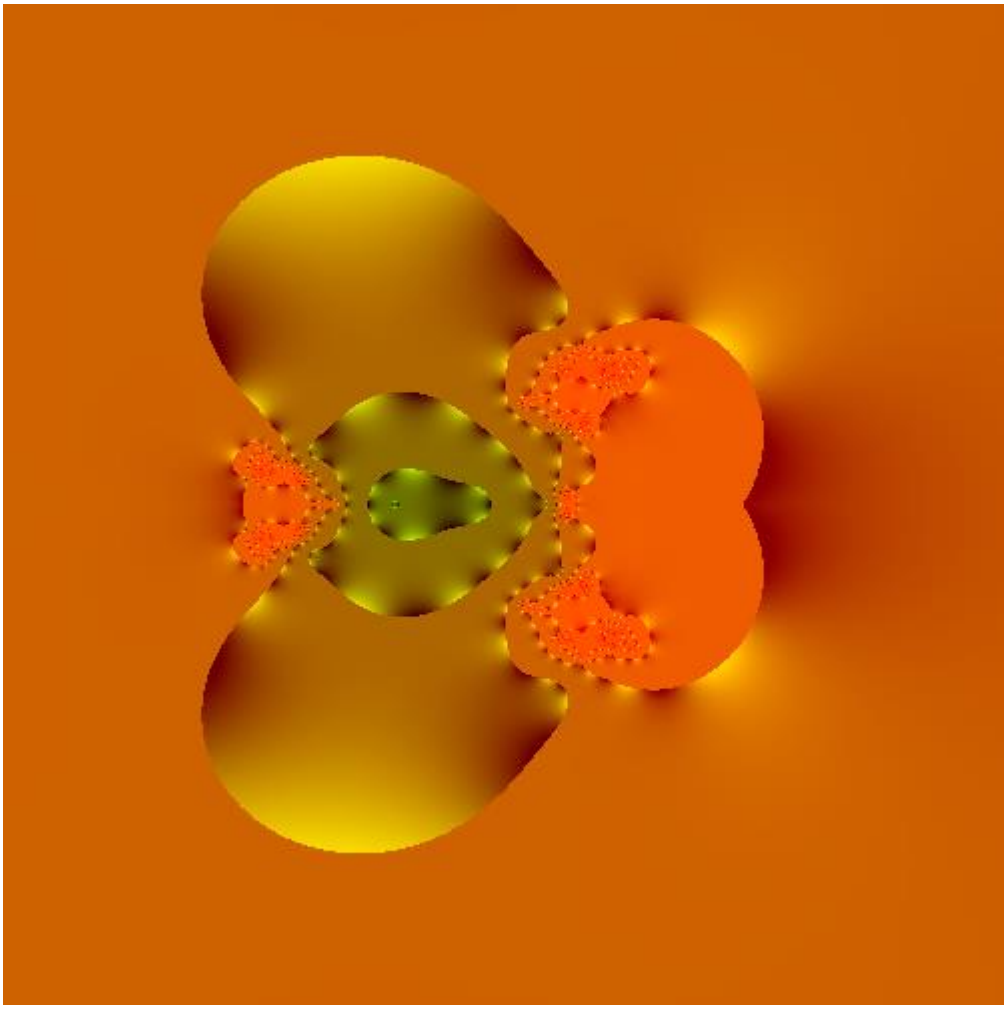

Fig. 4

II. Root of the equation: $x+\tanh x=1$

The unique real root of the equation:

$$x+\tanh x=1$$

is

$$x=0.5212984570002789...$$

$$x=1-\tanh\left(1-\tanh\left(1-\tanh\left(1-...\right)\right)\right)$$

III. A simple formula for Pi

The number Pi is defined by

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.141592...$$

If $\alpha=1-\tanh\left(1-\tanh\left(1-...\right)\right)=0.521298...$, then

$$\pi=4\tan^{-1}\alpha+2\sum_{n=0}^{\infty}\frac{c_n\alpha^{n+1}}{n+1}$$

where

$$c_0=1\quad,c_n=-\sum_{k=1}^{n}\frac{2^{k-3}\left(1+5(-1)^k\right)}{k!}c_{n-k}\quad,n=1,2,3,4,...$$

$$c_n=\left\{1,1,-\frac{1}{2},-\frac{4}{3},-\frac{5}{12},\frac{53}{60},\frac{337}{360},-\frac{38}{315},-\frac{871}{1008},...\right\}$$

IV. References

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